Taxes versus tradable pollution permits: evaluating environmental policies that affect multiple pollutions^{*}

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Abstract

This paper examines the interaction of different policies used to control two types of agricultural pollution. Pollution control is efficient when both pollution types are controlled by taxes, although a tax increase on pollution 1 can increase the quantity of pollution 2 if farm inputs are substitutes, or if farmers switch to pollution 2 intensive activities. However, if pollution 1 is controlled by a local emissions trading scheme, while pollution 2 is taxed, pollution 2 becomes less responsive to changes in its own tax and inefficient levels of pollution will occur unless the emissions trading scheme cap is constantly changed.

Keywords: Pollution Control Instruments; Taxes, Tradable Permits; Theory of Environ-

mental Policy; Water Pollution, Air Pollution; Climate Change; Agri-Environmental Policy;

Economics and Ecological Systems

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1 Introduction

Many governments are interested in adopting environmental policies that reduce agricultural pollution, particularly emissions of greenhouse gases (GHG) and nitrogen (N) in the form of nutrient runoffs into surface and groundwater. Even though the agricultural sector produces multiple types of pollution, environmental policies are typically designed to control one type of pollution at a time and their interactions with other regulations are rarely examined (Goulder 2013). However, the effectiveness by which any single policy achieves its environmental objectives will depend on how it interacts with other policies.

In this paper, we develop a model that explores how environmental policies applied to two forms of pollution affect agricultural pollution levels, input-use, and land-use. The model assumes that profit maximizing farmers with heterogeneous land quality choose one of several agricultural activities and produce output using several different pollution generating inputs. One of the pollution types - GHG - is subject to a tax or a charge, P_G , that is independent of local pollution quantities, while the other - N - may be uncontrolled, subject to a pollution tax, or subject to a local cap and trade program. The model allows for any number of inputs, but many of the formal results are proven only for the case that output is a quadratic function of two inputs, and pollution quantities are a linear function of these two inputs.

The results are presented by tracing out how an increase in the price of one type of pollution (GHG emissions) affects the production of both types of pollution. This paper shows that the effect of a tax or a charge on one type of pollution not only depends on the environmental policies applying to the second type of pollution, but also depends on the extent that the production inputs are complements or substitutes, and the extent that relative pollution intensities differ across the various agricultural activities. These interactions are summarized in Table 1.

	Nitrogen Tax		Nutrient Trading Scheme (NTS)		
	GHG emissions	N runoffs	GHG emissions	N runoffs	
Farm inputs	GHG emissions	N runoffs de-	GHG emissions	N runoffs con-	
are	decline	cline	decline by a	stant; P_N falls.	
complements			smaller amount		
Farm inputs	GHG emissions	N runoffs may	GHG emissions	N runoffs con-	
are substitutes	decline	increase	decline by a	stant; P_N may	
			smaller amount	rise.	

Table 1: The aggregate effect of an increase in P_G due to farm-level input changes

When the price of the nutrient pollution, P_N , is determined exogenously - for example, when nutrient runoffs are taxed - an increase in the price of GHG emissions will induce individual farmers to (i) reduce the intensity in which they use their most carbon producing inputs; (ii) substitute towards techniques and farm inputs that produce less GHG emissions; and (iii) consider whether to switch to a different type of farm production activity (or land-use). Each of these options reduces GHG emissions. Nutrient pollution levels will not necessarily decline, however. If the two farm inputs are complements and the farmer does not change activities, nutrient levels will fall when P_G increases. But if the inputs are substitutes, an increase in P_G can increase a farmers's nutrient runoff, if he or she substitutes towards relatively nutrient intensive inputs to reduce GHG emissions. Nutrient runoffs can also increase if he or she switches to a land-use activity that produces high levels of nutrient runoff but low levels of GHG emissions. The amount nutrient runoff can increase is limited, for once P_G is so high that the farmer no longer uses the most intensive carbon polluting input, further increases in P_G will lead to reductions in the other input. Consequently, both nutrient runoff and GHG emission levels fall to very low levels when P_G is very high.

When a nutrient trading scheme (NTS) is used to limit the maximum amount of nutrient runoff, changes in P_G have a different effect. In this case, the price of nutrient discharge, P_N , is determined endogenously. When P_G is sufficiently low that P_N is positive, increases in P_G have no effect on the aggregate level of nutrient pollution but GHG emissions decrease by a smaller amount than when nutrient runoff is taxed. The reduced responsiveness of GHG emissions to a change in P_G occurs because there is less substitution between inputs (whether inputs are complements of substitutes) and less switching between farm activities. An increase in P_G will change P_N , however; P_N will decrease if the inputs are complements, but P_N can increase if they are substitutes. When P_G is sufficiently high that P_N equals zero and the quantity of nutrient pollution is less than the cap, the NTS has no effect on either nutrient runoffs or GHG emissions. Increases in P_G above this level reduce both nutrient and GHG pollution levels.

The interdependence of P_N and P_G when there is an NTS raises questions about the practicality of adopting an NTS to optimally control pollution. When taxes and emission trading schemes are used to control pollution, socially efficient outcomes are achieved when the tax is equal to the marginal damage caused by the pollution. If the marginal damages incurred by both types of nutrient pollution are constant or vary little with the volume of local emissions, it is likely to be more efficient to tax both types of pollution than to have a local NTS. This is because the nutrient trading scheme permit price P_N depends on P_G as well as the permit cap, so the permit cap will need changing whenever the GHG price changes to ensure P_N is equal to the marginal damage of nutrient pollution. In contrast, if the pollution taxes are set equal to the marginal damage of each type of pollution, a change in the marginal damage of GHG will not require the tax on nutrient pollution to be changed. Thus a pollution tax will require less active management to achieve efficient pollution outcomes than an NTS, and may be preferred for this reason.

In contrast, if the marginal damage of nutrient pollution varies significantly with the quantity of local pollution, an NTS may be more attractive than a nutrient tax. In these circumstances the optimal nutrient pollution tax also depends on P_G , as the quantity of nutrient pollution and thus the marginal damage it causes depends on P_G . Consequently, both an NTS and a nutrient tax will require active management to ensure that pollution charges are equal to the marginal damage costs. A government may be content to have a

NTS in these circumstance to ensure that the quantity of emissions does not exceed a certain level, even if it cannot guarantee the pollution charge is equal to the marginal damage.

The patterns of adjustment described in this paper are complex. GHG emissions always decline when P_G increases, but the amount they decline depends on whether a nutrient tax or an NTS is adopted and whether farm inputs are substitutes or complements. The quantity of nutrient runoffs may rise, fall, or stay the same depending on whether a nutrient tax or an NTS is adopted, and if a tax is adopted, whether the inputs are complements or substitutes and whether a farmer switches to more or less nutrient intensive activities when an increase in P_G makes the initial activity unprofitable. Pollution levels adjust continuously in response to changes in pollution charges as individual farmers change their farm inputs, but these adjustments are punctuated by discrete changes when farmers switch from one type of farm production activity to another. Given the complexity of these interactions, one of the objectives of this paper is to establish in a simplified setting how the effect of different environmental management techniques on pollution levels depends on various technical and economic conditions.

The first part of this paper considers how the basic technological parameters and the prices of farm outputs and farm inputs, including any pollution charges, affect an individual farmer's production activity choices. The results are derived assuming price changes are sufficiently small that farmers do not switch farm activity, in which case farm input use and output levels are continuous functions of prices. The aggregate consequences of farmers' choices are also derived under the assumption that farmers do not switch activities, although they may undertake different farming activities. The second part of the paper examines what happens when an individual farmer switches activities. There is generally a discrete change in the use of inputs and the production of outputs, and aggregate pollution levels (if pollution prices are determined exogenously) or pollution prices (if pollution prices are determined exogenously) can rise or fall discretely depending on the technological structure of the economy.

This paper is organized as follows. Section 2 provides a review of the existing literature that addresses the multi-pollutant control problem. Section 3 develops a model that derives the amount of pollution generated by a set of profit-maximizing farmers who choose different farming activities produced with multiple types of inputs. The general framework is then simplified by considering farmers who produce output using two inputs that generate two types of pollution (nutrient runoffs and GHG emissions). The section examines how exogenous changes in P_G and P_N affect pollution levels when farmers adjust their input use but do not alter land-use change, although different farmers are allowed to choose different activities. Section 4 extends these results by analyzing how aggregate pollution outcomes differ when (i) there is a tax on both types of pollution emissions; and (ii) there is a tax on one type of pollution, but a tradable pollution permit scheme on the other. Again, the results are presented under the assumption that farmers do not change their farm activities. The way that changes in land-use are induced by changes in pollution charges is considered in section 5. The paper concludes with a discussion of the results.

2 Previous literature on multipollutants

This paper examines how two environmental policy instruments designed to control two types of pollution interact when there are multiple ways that pollution can be generated and abated. It extends a large literature recently reviewed by Lehmann (2012) that examines the interaction of multiple policies designed to control for a single type of pollution. It also extends a smaller, but more relevant literature that considers the way a single environmental policy affects multiple pollution externalities.

The multiple pollution problem has been analyzed in different ways. A large number of papers have examined the effects of a single policy instrument when a single production process generates multiple types of pollution. For example, Caplan (2006) examined how multiple correlated pollution externalities (e.g. localized air pollution and global climate change) could be controlled when they are produced by a single activity such as fossil fuels combustion. He found that joint domestic and international emissions taxes or a hybrid involving joint domestic taxes with international tradable permit markets are not efficient means to control correlated externalities. More recently, there have been a number of studies that examined the co-effects of GHG mitigation and water quality in agriculture (Pattanayak et al. 2005; Wilcock et al. 2008; Hartmann et al. 2009; Gasper et al. 2012). Many studies consider the effects of regulating GHG emissions on nutrient emissions or vice versa. For example, Gasper et al. (2012) found that programs designed primarily to improve water quality, such as the water quality trading program in the Chesapeake Bay watershed of the United States, can help the agricultural sector offset half of its greenhouse emissions by 2020. Hartmann et al. (2009) pointed out that policies aimed at improving water quality that reduce GHG emissions as a co-benefit are more effective than policies aimed directly at reducing GHG emissions from agricultural production. Similarly, Pattanayak et al. (2005) found that payments designed to mitigate GHG emissions on US agricultural cropland not only reduced 60 to 70 million tonnes of carbon equivalent nationwide but were also accompanied by a 2 percent improvement in national water quality. Adapted from Smith et al. (2008), Table 2 shows the potential mitigating effects of various proposed measures for GHG emissions and nutrient runoffs abatement in agroecosystems.¹

¹The potential mitigative effects of the list of measures for mitigating nutrient runoffs in Table 2 is based on personal communication with Professor Johan Six at Department of Environmental Systems Science, ETH-Zurich, Zurich, Switzerland.

Measure	Example	CO_2	CH_4	N_2O	Nutrient
Grazing land man-	Grazing intensity	+/-		+/-	+/-
agement/pasture					
improvement					
	Increased productivity (e.g.	+		+/-	+
	fertilization)				
	Nutrient management	+		+/-	+/-
Management of	Avoid drainage of wetlands	+	-	+/-	+
organic soils					
Livestock	Improved feeding practices		+		+
management					
	Specific agents and dietary		+		+
	additives				
Cropland	Nutrient management	+		+	+
management					
	Tillage/residue manage- ment	+		+/-	+

Table 2: Potential mitigative effects of various proposed measures for mitigating GHG emissions and nutrient runoffs from agricultural ecosystems (Adapted from Smith et al. 2008)

While many of these studies consider the co-benefits of GHG mitigation and water quality, there are some agricultural management practices and land-use changes for which the resulting air and water pollution are not positively correlated. Farm management options such as wetland and riparian zone restorations that have been advocated by policy makers to reduce nutrient loading and eutrophication in surface waters have the potential to increase GHG emissions (Compton et al. 2011). In addition to examining co-benefits, Wilcock et al. (2008) also examined the co-costs of management practices identified by the New Zealand government as ways to offset agricultural GHG emissions. The authors pointed out that riparian afforestation near pasture can increase carbon sequestration but can worsen water quality due to logging activities and forest maintenance (Wilcock et al. 2008). Jackson et al. (2005) found that carbon sequestration programs that promote afforestation can lead to a significant decrease in the volume of stream flows, increase soil salinization, and acidification. Given the complexity of agricultural ecosystems, it is not surprising that proposed agricultural management practices for mitigating a particular form of pollution can have differing mitigating effects on other forms of pollution. The current paper provides a framework for analysis of the effect of different environmental control regimes irrespective of whether they have positive or negative effects on different pollution types.

Multiple types of pollution can also be generated as the result of different production processes that makes outputs that are substitutes. Ren et al. (2011) developed a general equilibrium model that examines optimal tax policy in the presence of two pollution externalities (GHG emissions and nutrient leaching) that are related to one another through the demand for a final good. For example, a carbon tax will affect both nutrient leaching and GHG production if nutrient-intensive biofuel production is a substitute for GHG-emissionintensive fossil fuel production. The model in this paper generalizes some of this analysis by allowing each alternative production process to generate both pollutants, and by allowing the producers of each good to vary their production techniques in response to taxes.

Key and Kaplan (2007) is the paper most closely related to this paper. They developed a positive mathematical programming model to capture potential tradeoffs between air and water pollution emissions within the context of livestock waste management. In their model, profit-maximizing producers respond to medium specific and coordinated multi- environmental media policies. Key and Kaplan (2007) modelled three different policy scenarios: (1) a nitrogen application plan that requires growers to apply quantities of manure that deliver less nitrogen than the plants can absorb; (2) a payment from the USDA's Environmental Quality Incentive Program (EQIP) to help producers offset the cost of implementing a nutrient management plan; and (3) a hypothetical ammonia air emissions limit. They find that imposing the ammonia air emissions limit without a nutrient application standard can lead to an increase in nitrogen application. However, constraining the nutrient application rate did not result in any significant changes in ammonia air emissions. This paper also examines conditions where a specific policy instrument targeted at one environmental media may lead to unintended consequences in another environmental media, but instead of regulatory standards, it considers a tradable pollution permit and a pollution tax.

In a different setting, Acemoglu et al. (2012) examined the optimal type of regulation to control the negative environmental externalities associated with non-renewable resource use. They showed the optimal regulation depends on whether non-renewable resources are substitutes or complements for renewable resources. Following their insights, this paper extends the theoretical model developed in Yeo et al. (2013) to allow for inputs that can be substitutes or complements. Yeo et al. (2013) considered how the interaction of two environmental policy instruments affects the production of two pollution types that are the by-products of agricultural industries that each use one input. These results are extended by considering the adjustment mechanisms when industries have multiple ways to reduce pollution. Like Acemoglu et al. (2012) we find that the effect of environmental instruments depends crucially on whether pollution generating inputs are complements or substitutes.

3 Theoretical model

Let $\mathbf{I} = \{1, ..., I\}$ be a set of farmers, let $\mathbf{J} = \{1, ..., J\}$ be a set of different land-uses (e.g. dairy or forestry), and let $\underline{\theta}_{i,j}$ be a set of M farm management options or inputs $\underline{\theta}_{i,j} = \{\underline{\theta}_{i,j}^1, \underline{\theta}_{i,j}^2, ..., \underline{\theta}_{i,j}^M\}$. Each farmer has a particular exogenous characteristic x_i that affects profitability (e.g. land quality). Faced with a particular environmental policy, a farmer $i \in \mathbf{I}$ chooses a type of farm management option $\underline{\theta}_{i,j}$ corresponding to a farming activity $j \in \mathbf{J}$. For example, when $M = 2, \, \underline{\theta}_{i,j}^1$ could be the fertilizer application rate and $\underline{\theta}_{i,j}^2$ could be the animal stocking rate. The farm management options affect output, $Q_j(\underline{\theta}_{i,j}, x_i)$, but also generate nutrient (N) pollution, $N_j(\underline{\theta}_{i,j})$, and GHG emissions, $GHG_j(\underline{\theta}_{i,j})$. Let $\mathbf{P} = \{P_{\theta}, P_{\theta}, P_N, P_G\}$ be a set of prices where P_j is the output price of the agricultural good, $P_{\underline{\theta}} = \{P_{\theta}^1, P_{\theta}^2, ..., P_{\theta}^M\}$ are the price of fertilizer), P_N is the permit price of N if there exists an NTS or a tax on nutrient pollution, and P_G is the carbon tax or the GHG emissions permit price if a GHG ETS is in

place.

It is assumed that the production function, $Q_j(\underline{\theta}_{i,j}, x_i)$, is twice continuously differentiable, increasing and concave in the inputs $\underline{\theta}_{i,j}$ (i.e. $\frac{\partial Q_j}{\partial \theta_{i,j}^m} \geq 0$, $\frac{\partial^2 Q_j}{\partial (\theta_{i,j}^m)^2} \leq 0$). It is also assumed that $N_j(\underline{\theta}_{i,j})$ and $GHG_j(\underline{\theta}_{i,j})$ are increasing and convex in $\underline{\theta}_{i,j}$ (i.e. $\frac{\partial N_j}{\partial \theta_{i,j}^m} \geq 0$ and $\frac{\partial^2 N_j}{\partial (\theta_{i,j}^m)^2} \geq 0$; $\frac{\partial GHG_j}{\partial \theta_{i,j}^m} \geq 0$ and $\frac{\partial^2 GHG_j}{\partial (\theta_{i,j}^m)^2} \geq 0$). The profit for any farmer, *i*, undertaking any farm production activity, *j*, is given by $\Pi_{i,j} = \Pi_j(\underline{\theta}_{i,j}, x_i, \mathbf{P})$:

$$\Pi_{j}(\underline{\theta}_{i,j}, x_{i}, \mathbf{P}) = P_{j}Q_{j}(\underline{\theta}_{i,j}, x_{i}) - P_{\underline{\theta}}\underline{\theta}_{i,j} - P_{N}N_{j}(\underline{\theta}_{i,j}) - P_{G}GHG_{j}(\underline{\theta}_{i,j})$$
(1)

The farmer's decision to adopt a particular type of land-use will depend on prices, \mathbf{P} , and the exogenous variable, x_i . For each type of land-use j, there is an optimal combination of farm management options $\underline{\theta}_{i,j}^*$ that maximizes a farmer's profit $\prod_j(\underline{\theta}_{i,j}, x_i, \mathbf{P})$:

$$\underline{\theta}_{i,j}^* = \arg \max_{\theta_{i,j}} \Pi_j(\underline{\theta}_{i,j}, x_i, \mathbf{P}).$$
(2)

The value of the profit when the farmer uses the optimal values of $\underline{\theta}_{i,j}$ is:

$$\Pi_j^*(x_i, \mathbf{P}) = \Pi_j(\underline{\theta}_{i,j}^*(x_i, \mathbf{P}), x_i, \mathbf{P}).$$
(3)

The farmer than chooses a type of land-use j that maximizes profit $\Pi_j^*(x_i, \mathbf{P})$:

$$j^* = \arg\max_j \{\Pi_j^*(x_i, \mathbf{P})\}.$$
(4)

The set of farmers \mathbf{I} can be partitioned into subsets $\{\mathbf{I}^1, ..., \mathbf{I}^j, ... \mathbf{I}^J\}$ where a farmer i is in \mathbf{I}^j if $j^* = j$.

3.1 Nutrient Trading Scheme

The price of GHG emissions and nutrient runoffs may be exogenous to the farmers' decisions. This would be the case if the regulator imposes a carbon or nutrient tax or if there were a GHG ETS and the country is a price taker in the international carbon market. However, if there is an NTS, the price of nutrient pollution permits P_N is determined endogenously. Let \bar{N} be the nutrient cap for a local catchment and let N^+ be the sum of the individual nutrient leaching from the different farms into the local catchment; then

$$N^{+} = \sum_{j=1}^{J} \sum_{i \in \mathbf{I}^{j}} N_{j}(\theta_{i,j}^{*}(x_{i}, \mathbf{P})).$$
(5)

In equilibrium, the permit price $P_N(\bar{N})$ satisfies:

$$[N^{+} - \bar{N}]P_{N}(\bar{N}) = 0.$$
(6)

According to Equation (6), if not all permits are sold then $P_N = 0$. Otherwise, if $P_N > 0$ then all permits are sold and aggregate N leaching is equal to the nutrient cap \bar{N} .

3.2 A model with two farm management options

This section analyzes an individual farmer's response to a pollution charge keeping land-use j constant. The farmer takes the pollution charges P_N and P_G as exogenously determined. The optimal solution is derived for an example of the general problem where the production function is a quadratic function of two inputs that increase farm output. Pollution abatement requires a reduction in these inputs.

3.2.1 Farmer's profit maximization problem

Suppose the production function $Q_j(\theta_{i,j}^1, \theta_{i,j}^2, x_i)$ is a quadratic function of two farm inputs $\theta_{i,j}^1$ and $\theta_{i,j}^2$ and the exogenous variable x_i :

$$Q(\theta_{i,j}^{1}, \theta_{i,j}^{2}, x_{i}) = x_{i} \left[\alpha_{j}^{0} + \alpha_{j}^{11} (\theta_{i,j}^{1} - \bar{\theta}_{ij}^{1})^{2} + \alpha_{j}^{12} (\theta_{i,j}^{1} - \bar{\theta}_{ij}^{1}) (\theta_{i,j}^{2} - \bar{\theta}_{j}^{2}) + \alpha_{j}^{22} (\theta_{i,j}^{2} - \bar{\theta}_{j}^{2})^{2} \right]$$

$$= x_{i} \left[\alpha_{j}^{0} + (\theta_{i,j} - \bar{\theta}_{ij})' A_{j} (\theta_{i,j} - \bar{\theta}_{ij}) \right]$$
(7)

where

$$A_{j} = \begin{bmatrix} \alpha_{j}^{11} & \frac{1}{2}\alpha_{j}^{12} \\ \frac{1}{2}\alpha_{j}^{12} & \alpha_{j}^{22} \end{bmatrix}, \theta_{i,j} = \begin{bmatrix} \theta_{i,j}^{1} \\ \theta_{i,j}^{2} \end{bmatrix}, \text{ and } \bar{\theta}_{i,j} = \begin{bmatrix} \bar{\theta}_{i,j}^{1} \\ \bar{\theta}_{i,j}^{2} \end{bmatrix},$$

 $\bar{\theta}_{i,j}^1$ and $\bar{\theta}_{i,j}^2$ are the input levels that maximize output. If the production function $Q_j(\theta_{i,j}^1, \theta_{i,j}^2, x_i)$ is concave in the inputs (i.e. $\frac{\partial Q_j}{\partial \theta_{i,j}^m} \ge 0$, and $\frac{\partial^2 Q_j}{\partial (\theta_{i,j}^m)^2} \le 0$), then the matrix A_j is negative definite. This entails the conditions $\alpha_j^{11} < 0$, $\alpha_j^{22} < 0$, and $4\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2 > 0$ (Simon and Blume 1994).

The two inputs can be substitutes or complements. If $\alpha_j^{12} > 0$ then $\theta_{i,j}^1$ and $\theta_{i,j}^2$ are complements and the marginal product of $\theta_{i,j}^2$ increases in the amount of $\theta_{i,j}^1$ (i.e. $\frac{\partial^2 Q_j}{\partial \theta_{i,j}^1 \theta_{i,j}^2} \ge 0$).² For example, Q_j could be the production of milk solids as a function of the number of cows and the amount of fertilizer. An additional cow will increase the production of milk solids by more if there is more fertilizer added to the field. Conversely, if $\alpha_j^{12} < 0$, $\theta_{i,j}^1$ and $\theta_{i,j}^2$ are substitutes and the marginal product of $\theta_{i,j}^2$ decreases in the amount of $\theta_{i,j}^1$ (i.e. $\frac{\partial Q_j^2}{\partial \theta_{i,j}^2 \theta_{i,j}^1} < 0$). A simple dairy example would be when $\theta_{i,j}^1$ and $\theta_{i,j}^2$ are the quantities of different breeds of cows, which produce different amounts of milk solids and have different environmental impacts. They are substitutes as they compete for pastureland; hence an

²Two inputs are complements if the demand for one falls when the price of the other increases, and substitutes if the demand for one rises when the price of the other increases. It is straightforward to show that if the production function of profit maximising producers is concave and has a quadratic form, the inputs will be complements if $\alpha_j^{12} > 0$ and substitutes if $\alpha_j^{12} < 0$.

additional unit of $\theta^1_{i,j}$ decreases the marginal product of $\theta^2_{i,j}.$

Nutrient leaching and GHG emissions are increasing linear functions of the inputs.³

$$N_{i,j} = N_j(\theta_{i,j}^1, \theta_{i,j}^2) = \phi_j^{N0} + \phi_j^{N1} \theta_{i,j}^1 + \phi_j^{N2} \theta_{i,j}^2$$
(8)

and

$$GHG_{i,j} = GHG_{j}(\theta_{i,j}^{1}, \theta_{i,j}^{2})$$

= $\phi_{j}^{G0} + \phi_{j}^{G1}\theta_{i,j}^{1} + \phi_{j}^{G2}\theta_{i,j}^{2}.$ (9)

These equations can be written in matrix form

$$E_{i,j} = \Phi_j^0 + \Phi_j^1 \theta_{i,j}$$

where

$$E_{i,j} = \begin{bmatrix} N_{i,j} \\ GHG_{i,j} \end{bmatrix}, \Phi_j^0 = \begin{bmatrix} \phi_j^{N0} \\ \phi_j^{G0} \end{bmatrix}, \text{ and } \Phi_j^1 = \begin{bmatrix} \phi_j^{N1} & \phi_j^{N2} \\ \phi_j^{G1} & \phi_j^{G2} \end{bmatrix}.$$

For example, if $\theta_{i,j}^1$ is a type of fertilizer, Equation (8) and (9) state that a fraction ϕ_j^{N1} of fertilizer application leaches into water as nutrient pollution and a fraction ϕ_j^{G1} is emitted to the air as GHG emissions. Even if inputs levels are zero, some nutrient pollution, ϕ_j^{N0} , and GHG emissions ϕ_j^{G0} may be produced. If $\theta_{i,j}^1$ is comparatively more nutrient intensive than $\theta_{i,j}^2$, and $\theta_{i,j}^2$ is comparatively more GHG emissions intensive than $\theta_{i,j}^1$, $\phi_j^{N1} > \phi_j^{N2}$ and $\phi_j^{G2} > \phi_j^{G1}$.

 $^{^{3}}$ A complementary analysis is possible when pollution quantities are decreasing in at least one input. In this paper, we assume both inputs generate pollution to simplify the exposition.

For a particular land-use j, each farmer i chooses the management inputs $\theta_{i,j}^1$ and $\theta_{i,j}^2$ that maximize profit subject to the constraints $\theta_{i,j}^1 \ge 0$ and $\theta_{i,j}^2 \ge 0$. The profit function for this farmer is:

$$\Pi_{i,j} = P_j x_i \left[\alpha_j^0 + (\theta_{i,j} - \bar{\theta}_{ij})' A_j (\theta_{i,j} - \bar{\theta}_{ij}) \right] - P_{\theta}' \theta_{i,j} - P_E' E_{i,j}$$

= $P_j x_i \left[\alpha_j^0 + (\theta_{i,j} - \bar{\theta}_{ij})' A_j (\theta_{i,j} - \bar{\theta}_{ij}) \right] - (P_{\theta}' + P_E' \Phi_j^1) \theta_{i,j} - P_E' \Phi_j^0$ (10)

where

$$P_{\theta} = \begin{bmatrix} P_{\theta}^{1} \\ P_{\theta}^{2} \end{bmatrix}, \text{ and } P_{E} = \begin{bmatrix} P_{N} \\ P_{G} \end{bmatrix}$$

The first order condition is

$$\frac{\partial \Pi_{i,j}}{\partial \theta_{i,j}} = 2P_j x_i A_j (\theta_{i,j} - \bar{\theta}_{i,j}) - (P'_{\theta} + P'_E \Phi^1_j) \le 0.$$
(11)

linear as it depends on whether the inputs $\theta_{i,j}^1$ and $\theta_{i,j}^2$ are zero or positive. The following solution is for the case that the farmer ceases using input 2 before input 1 as P_G increases:

$$(\theta_{i,j}^{1}, \theta_{i,j}^{2}) = \begin{cases} (\theta_{i,j}^{1*}, \theta_{i,j}^{2*}), & 0 \le P_{G} < P_{1} \\ (\theta_{i,j}^{1**}, 0), & P_{1} \le P_{G} < P_{2} \\ (0,0), & P_{2} \le P_{G} \end{cases}$$
(12)

where

$$\theta_{i,j}^{1*} = \bar{\theta}_{ij}^{1} + \frac{2\alpha_j^{22}(P_{\theta}^1 + P_N\phi_j^{N1} + P_G\phi_j^{G1}) - \alpha_j^{12}(P_{\theta}^2 + P_N\phi_j^{N2} + P_G\phi_j^{G2})}{P_j x_i (4\alpha_j^{22}\alpha_j^{11} - (\alpha_j^{12})^2)},$$
(13)

$$\theta_{i,j}^{2*} = \bar{\theta}_{ij}^2 + \frac{2\alpha_j^{11}(P_\theta^2 + P_N\phi_j^{N2} + P_G\phi_j^{G2}) - \alpha_j^{12}(P_\theta^1 + P_N\phi_j^{N1} + P_G\phi_j^{G1})}{P_j x_i (4\alpha_j^{22}\alpha_j^{11} - (\alpha_j^{12})^2)},$$
(14)

$$\theta_{i,j}^{1**} = \bar{\theta}_{ij}^{1} + \frac{P_{\theta}^{1} + P_{N}\phi_{j}^{N1} + P_{G}\phi_{j}^{G1} + \alpha_{j}^{12}\bar{\theta}_{i,j}^{2}x_{i}P_{j}}{2x_{i}\alpha_{j}^{11}P_{j}},$$
(15)

and

$$P_{1} = \frac{\alpha_{j}^{12}(P_{\theta}^{1} + P_{N}\phi_{j}^{N1}) - 2\alpha_{j}^{11}(P_{\theta}^{2} + P_{N}\phi_{j}^{N2}) - \bar{\theta}_{ij}^{1}(P_{j}x_{i}(4\alpha_{j}^{22}\alpha_{j}^{11} - (\alpha_{j}^{12})^{2})))}{2\alpha_{j}^{11}\phi_{j}^{G2} - \alpha_{j}^{12}\phi_{j}^{G1}}$$
(16)

$$P_2 = -\frac{1}{\phi_j^{G1}} (P_j x_i (\alpha_j^{12} \bar{\theta}_{ij}^2 + 2\alpha_j^{11} \bar{\theta}_{ij}^1) + P_\theta^1 + P_N \phi_j^{N1}).$$
(17)

3.2.2 Relationship between pollution charges and pollution levels

When both P_N and P_G are exogenous, the relationship between input use and pollution is also piecewise linear:

$$N_{i,j}^{*} = \begin{cases} \phi_{j}^{N0} + \phi_{j}^{N1} \theta_{i,j}^{1*} + \phi_{j}^{N2} \theta_{i,j}^{2*}, & 0 \le P_{G} < P_{1} \\ \phi_{j}^{N0} + \phi_{j}^{N1} \theta_{i,j}^{1**}, & P_{1} \le P_{G} < P_{2} \\ \phi_{j}^{N0}, & P_{2} \le P_{G} \end{cases}$$
(18)

and

$$GHG_{i,j}^{*} = \begin{cases} \phi_{j}^{G0} + \phi_{j}^{G1}\theta_{i,j}^{1*} + \phi_{j}^{G2}\theta_{i,j}^{2*}, & 0 \le P_{G} < P_{1} \\ \phi_{j}^{G0} + \phi_{j}^{G1}\theta_{i,j}^{1**}, & P_{1} \le P_{G} < P_{2} \\ \phi_{j}^{N0}, & P_{2} \le P_{G}. \end{cases}$$
(19)

When $P_G < P_1$, the derivatives are:

$$\frac{\partial N_{i,j}^*}{\partial P_N} = \phi_j^{N1} \left[\frac{2\alpha_j^{22}\phi_j^{N1} - \alpha_j^{12}\phi_j^{N2}}{P_j x_i (4\alpha_j^{22}\alpha_j^{11} - (\alpha_j^{12})^2)} \right] + \phi_j^{N2} \left[\frac{2\alpha_j^{11}\phi_j^{N2} - \alpha_j^{12}\phi_j^{N1}}{P_j x_i (4\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2)} \right]$$
(20)

$$\frac{\partial GHG_{i,j}^*}{\partial P_G} = \phi_j^{G1} \left[\frac{2\alpha_j^{22}\phi_j^{G1} - \alpha_j^{12}\phi_j^{G2}}{P_j x_i (4\alpha_j^{22}\alpha_j^{11} - (\alpha_j^{12})^2)} \right] + \phi_j^{G2} \left[\frac{2\alpha_j^{11}\phi_j^{G2} - \alpha_j^{12}\phi_j^{G1}}{P_j x_i (4\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2)} \right]$$
(21)

$$\frac{\partial N_{i,j}^*}{\partial P_G} = \phi_j^{N1} \left[\frac{2\alpha_j^{22}\phi_j^{G1} - \alpha_j^{12}\phi_j^{G2}}{P_j x_i (4\alpha_j^{22}\alpha_j^{11} - (\alpha_j^{12})^2)} \right] + \phi_j^{N2} \left[\frac{2\alpha_j^{11}\phi_j^{G2} - \alpha_j^{12}\phi_j^{G1}}{P_j x_i (4\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2)} \right]$$
(22)

$$\frac{\partial GHG_{i,j}^*}{\partial P_N} = \phi_j^{G1} \left[\frac{2\alpha_j^{22}\phi_j^{N1} - \alpha_j^{12}\phi_j^{N2}}{P_j x_i (4\alpha_j^{22}\alpha_j^{11} - (\alpha_j^{12})^2)} \right] + \phi_j^{G2} \left[\frac{2\alpha_j^{11}\phi_j^{N2} - \alpha_j^{12}\phi_j^{N1}}{P_j x_i (4\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2)} \right].$$
(23)

When $P_1 \leq P_G < P_2$, the derivatives are:

$$\frac{\partial N_{i,j}^*}{\partial P_N} = \frac{(\phi_j^{N1})^2}{2x_i \alpha_j^{11} P_j} \tag{24}$$

$$\frac{\partial GHG_{i,j}^*}{\partial P_G} = \frac{(\phi_j^{G1})^2}{2x_i \alpha_j^{11} P_j} \tag{25}$$

$$\frac{\partial N_{i,j}^*}{\partial P_G} = \frac{\phi_j^{N1} \phi_j^{G1}}{2x_i \alpha_i^{11} P_j} \tag{26}$$

$$\frac{\partial GHG_{i,j}^*}{\partial P_N} = \frac{\phi_j^{N1}\phi_j^{G1}}{2x_i\alpha_j^{11}P_j}.$$
(27)

These expressions can be combined with an indicator function. Let $I(\theta)$ be an indicator variable that equals 0 if $\theta = 0$ and 1 if $\theta > 0$. Then, ignoring the points where the derivatives do not exist, the partial derivatives linking nutrient pollution to the price of pollution for an an individual farmer *i* choosing activity *j* are

$$\frac{\partial N_{i,j}^{*}}{\partial P_{N}} = 2I(\theta_{1})I(\theta_{2}) \left[\frac{\alpha_{j}^{11}(\phi_{j}^{N2})^{2} - \alpha_{j}^{12}\phi_{j}^{N1}\phi_{j}^{N2} + \alpha^{22}(\phi_{j}^{N1})^{2}}{P_{j}x_{i}(4\alpha_{j}^{11}\alpha_{j}^{22} - (\alpha_{j}^{12})^{2})} \right] + I(\theta_{1})(1 - I(\theta_{2})) \left[\frac{(\phi_{j}^{N1})^{2}}{2\alpha_{j}^{11}x_{i}P_{j}} \right] + I(\theta_{2})(1 - I(\theta_{1})) \left[\frac{(\phi_{j}^{N2})^{2}}{2\alpha_{j}^{22}x_{i}P_{j}} \right],$$
(28)

and

$$\frac{\partial N_{i,j}^{*}}{\partial P_{G}} = 2I(\theta_{1})I(\theta_{2}) \left[\frac{\alpha_{j}^{11}\phi_{j}^{N2}\phi_{j}^{G2} - \alpha_{j}^{12}(\phi_{j}^{N1}\phi_{j}^{G2} + \phi_{j}^{N2}\phi_{j}^{G1}) + \alpha^{22}\phi_{j}^{N1}\phi_{j}^{G1}}{P_{j}x_{i}(4\alpha_{j}^{11}\alpha_{j}^{22} - (\alpha_{j}^{12})^{2})} \right] + I(\theta_{1})(1 - I(\theta_{2})) \left[\frac{\phi_{j}^{N1}\phi_{j}^{G1}}{2\alpha_{j}^{11}x_{i}P_{j}} \right] + I(\theta_{2})(1 - I(\theta_{1})) \left[\frac{\phi_{j}^{N2}\phi_{j}^{G2}}{2\alpha_{j}^{22}x_{i}P_{j}} \right].$$
(29)

The signs of the derivatives depend on the signs of the production parameters $(\alpha_j^{11}, \alpha_j^{12}, \alpha_j^{22})$ and the signs of the pollution parameters $(\phi_j^{N1}, \phi_j^{N2}, \phi_j^{G1}, \phi_j^{G2})$. When the pollution parameters are all positive, (i.e. when an increase in an input increases both types of pollution), the following three results hold.

Proposition 1.

 $\frac{\partial N_{i,j}^*}{\partial P_N} \leq 0$ and $\frac{\partial GHG_{i,j}^*}{\partial P_G} \leq 0$ i.e. the amount of nutrient leaching and GHG emissions are non-increasing functions of their own prices. This result is proved in Appendix A.

Proposition 2.

The cross derivatives $\frac{\partial N_{i,j}^*}{\partial P_G}$ and $\frac{\partial GHG_{i,j}^*}{\partial P_N}$ are equal. This result can be seen by inspection of equations 22 and 23 and 26 and 27, but it is true for general production functions when the quantity of pollution is a linear function of the inputs.⁴

Proposition 3.

 $\frac{\partial N_{i,j}^*}{\partial P_G}$ and $\frac{\partial GHG_{i,j}^*}{\partial P_N}$ are negative if only one input is used. If both inputs are used and the inputs are complements, $\frac{\partial N_{i,j}^*}{\partial P_G}$ and $\frac{\partial GHG_{i,j}^*}{\partial P_N}$ are also negative. However, if both inputs are used and the inputs are substitutes, $\frac{\partial N_{i,j}^*}{\partial P_G}$ and $\frac{\partial GHG_{i,j}^*}{\partial P_N}$ may be positive.

The first part of proposition 3 follows directly from equations 26 and 27, as the denominators of the derivatives are negative. The second part of the proof follows from observing the derivatives of equations 22 and 23 are positive only if

$$\alpha_j^{12} < \frac{2\left(\alpha_j^{11} + \left(\frac{\phi_j^{G1}}{\phi_j^{G2}}\right) \left(\frac{\phi_j^{N1}}{\phi_j^{N2}}\right) \alpha_j^{22}\right)}{\left(\frac{\phi_j^{G1}}{\phi_j^{G2}}\right) + \left(\frac{\phi_j^{N1}}{\phi_j^{N2}}\right)}.$$
(30)

This can only occur if $\alpha_j^{12} < 0$, that is, if the inputs are substitutes. If the inputs are complements, $\frac{\partial N_{i,j}^*}{\partial P_G}$ and $\frac{\partial GHG_{i,j}^*}{\partial P_N}$ are negative. The differences between these two cases are discussed in the next sub-sections.

 $^{{}^{4}}A$ proof is available from the authors. The general result follows directly from applying the implicit function theorem to the profit maximisation problem.

3.3 Farm management options are complements $(\alpha_{12} > 0)$

Suppose farm management inputs are complements. According to proposition 1 and proposition 3, an increase in P_G (holding P_N constant) leads to a reduction in the amount of GHG and quantity of nutrient pollution produced by every individual farmer. The intuition for these results is shown in Figure 1, which contains a pair of diagrams illustrating how the inputs change as P_G changes. As P_G increases, there is an incentive to reduce the most carbon intensive input. Since the inputs are complements, the marginal productivity of the other input also decreases, and in combination with the increase in P_G , the incentive to use this input declines as well. Consequently, the quantity of both inputs are reduced, and production of both pollutants decreases.

Figure 1a shows the direct output effects of changing $\theta_{i,j}^1$ holding $\theta_{i,j}^2$ constant while Figure 1b shows the direct output effects of changing $\theta_{i,j}^2$ holding $\theta_{i,j}^1$ constant.⁵ In both diagrams, the points A_1 and A_2 show the initial equilibrium when $P_G = P_G^0$ and marginal cost is equal to marginal product. In response to an increase in P_G the marginal cost curves shift upward to MC_1^1 and MC_1^2 , by amounts the $\Delta P_G \phi_j^{G1}$ and $\Delta P_G \phi_j^{G2}$ respectively. The price change leads to a direct and an indirect effect on the demand for $\theta_{i,j}^1$ and $\theta_{i,j}^2$. In the upper diagram, holding $\theta_{i,j}^2$ constant, the increase in P_G decreases $\theta_{i,j}^1$ along the curve from point A_1 to point B_1 , while in the bottom diagram, holding $\theta_{i,j}^1$ constant, the increase in P_G decreases $\theta_{i,j}^2$ along the curve from A_2 to B_2 . As the inputs are complements, the decline in $\theta_{i,j}^1$ reduces the marginal product curve $MP(\theta_{i,j}^2|\theta_{i,j}^1)$ by $\alpha_j^{12}\Delta\theta_{i,j}^1$ (in the lower diagram) and the decline in $\theta_{i,j}^2$ reduces the marginal product curve $MP(\theta_{i,j}^2|\theta_{i,j}^1)$ (in the upper diagram), resulting in further decreases in inputs. Eventually, a new equilibrium at point C is reached where $\theta_{i,j}^1 = \theta_{i,j}^{1*}$ and $\theta_{i,j}^2 = \theta_{i,j}^{2*}$.

The two farm management options of most interest to the New Zealand dairy industry are nitrogenous fertilizer application and the animal stocking rate. These two farm management

From Equation 10, the marginal product of $\theta_{i,j}^1 = x_i [2\alpha_j^{11}\theta_{i,j}^1 + \alpha_j^{12}\theta_{i,j}^2]P_j$, the marginal cost of $\theta_{i,j}^1 = (P_{\theta}^1 + P_N \phi_j^{N1} + P_G \phi_j^{G1})$, the marginal product of $\theta_{i,j}^2 = x_i [2\alpha_j^{22}\theta_{i,j}^2 + \alpha_j^{12}\theta_{i,j}^1]P_j$, and the marginal cost of $\theta_{i,j}^2 = (P_{\theta}^2 + P_N \phi_j^{N2} + P_G \phi_j^{G2})$.

Figure 1: The marginal cost and marginal product of the two inputs, θ_1 and θ_2 , when inputs are complements $(\alpha_j^{12} > 0)$



options are complements since the marginal production of milk solids from increasing the livestock stocking intensity increases with higher levels of nitrogenous fertilizer application. With these two choices as inputs, an increase in P_G or P_N will reduce both inputs and both types of pollution levels.

3.4 Farm management options are substitutes ($\alpha_{12} < 0$)

When the two farm management options are substitutes, the amount of nutrient leaching is decreasing in P_N , and the amount of GHG emissions is decreasing in P_G . However, an increase in P_G can lead to an increase in nutrient pollution or an increase in P_N can lead to an increase in GHG emissions if inequality 30 holds; that is $\frac{\partial N_{i,j}^*}{\partial P_G}$ and $\frac{\partial GHG_{i,j}^*}{\partial P_N}$ may be positive if both inputs are used. For intuition, consider a farmer stocking two different breeds of cow, $\theta_{i,j}^1$ and $\theta_{i,j}^2$, where breed 2 has a comparatively higher nutrient leaching than breed 1 (i.e. $\phi_j^{N2} > \phi_j^{N1}$) but breed 1 has a comparatively higher GHG emissions than breed 2 (i.e. $\phi_j^{G1} > \phi_j^{G2}$). The two breeds are substitutes as the additional milk production from an extra cow of breed 1 is decreasing in the numbers of breed 2, as they compete for grazing land. As P_G increases, the farmer can be expected to substitute towards the nutrient pollution intensive breed 2 as they emit smaller amounts of GHG . This substitution makes it possible for total nutrient leaching to increase when P_G increases, i.e. $\frac{\partial N_{i,j}^*}{\partial P_G} > 0$.

Figure 2 and Figure 3 illustrate how an increase in pollution prices can have an ambiguous effect on the demand for θ_{ij}^1 and θ_{ij}^2 . An increase in P_G reduces the incentive to use both inputs, but it reduces the most GHG emissions intensive input $(\theta_{i,j}^1)$ by more than the least GHG intensive input. As the inputs are substitutable, the reduction in $\theta_{i,j}^1$ increases the profitability of using $\theta_{i,j}^2$, and the increase in this indirect demand may exceed the decrease in the demand caused by the higher price P_G . If the two are sufficiently substitutable, the additional nutrient leaching from the higher use of $\theta_{i,j}^2$ may outweigh the reduction in nutrient leaching stemming from the lower use of $\theta_{i,j}^1$, and total nutrient leaching in the catchment can increase.

Figure 2: The marginal cost and marginal product of the two inputs, θ_1 and θ_2 , when inputs are substitutes $(\alpha_j^{12} < 0)$





In Figure 2, an increase in P_G shifts the marginal cost curves up and, holding $\theta_{i,j}^2$ constant, the demand for $\theta_{i,j}^1$ decreases from point A to B. This decrease in $\theta_{i,j}^1$ shifts the marginal product curve $MP(\theta_{i,j}^2|\theta_{i,j}^1)$ upwards by $\alpha_j^{12}\Delta\theta_{i,j}^1$. If α_j^{12} is sufficiently large, the upward movement of the marginal product curve will dominate the upward movement of the marginal cost curve and lead to an increase in $\theta_{i,j}^2$. The increase in $\theta_{i,j}^2$ shifts the marginal product curve of $\theta_{i,j}^1$ down by $\alpha_j^{12}\Delta\theta_{i,j}^2$ decreasing $\theta_{i,j}^1$ further. Eventually, a new equilibrium at point C is reached where $\theta_{i,j}^1 = \theta_{i,j}^{1*}$ has fallen and $\theta_{i,j}^2 = \theta_{i,j}^{2*}$ has increased. If $\theta_{i,j}^2$ is also more nutrient intensive than $\theta_{i,j}^1$, then it is possible that nutrient pollution increases, i.e. $\frac{\partial N_{i,j}^*}{\partial P_G} > 0$.

If the degree of substitutability is sufficiently small, both inputs can decrease when the price of one pollution rises (see Figure 3). $\theta_{i,j}^1$ decreases from point A to point B and shifts the marginal product curve for $\theta_{i,j}^2$ outwards by $\alpha_j^{12}\Delta\theta_{i,j}^1$ when P_G increases. Even though the marginal product curve for $\theta_{i,j}^2$ shifts outwards, the lower diagram of Figure 3 shows $\theta_{i,j}^2$ can decrease from θ_0^2 to θ_1^2 as the outward movement of the marginal product curve is smaller than the upward movement of the marginal cost curve. The decrease in $\theta_{i,j}^2$ shifts the marginal product of $\theta_{i,j}^1$ outwards by $\alpha_j^{12}\Delta\theta_{i,j}^2$. Similarly, while demand curve for $\theta_{i,j}^1$ has shifted outwards, the demand for $\theta_{i,j}^1$ decreases as well if α_j^{12} is sufficiently small. The reduction in both inputs imply that $\frac{\partial N_{i,j}^*}{\partial P_G} < 0$, if both inputs increase nutrient pollution, even if one is comparatively more N intensive than the other.

Equation 30 has two special cases, described by Corollary 1 and Corollary 2 below. The first corollary shows that it is impossible for pollution levels to increase in response to higher pollution prices if $\frac{\phi_j^{N1}}{\phi_j^{N2}} = \frac{\phi_j^{G1}}{\phi_j^{G2}}$. The second corollary shows that it is possible for pollution to increase if the two inputs have the same impact on one type of pollution but have a different impact on another.

Corollary 1: Suppose $\alpha_j^{12} < 0$, and $\phi_j^{N1} > 0$; $\phi_j^{N2} > 0$; $\phi_j^{G1} > 0$; $\phi_j^{G2} > 0$. If $\frac{\phi_j^{N1}}{\phi_j^{N2}} = \frac{\phi_j^{G1}}{\phi_j^{G2}}$, then $\frac{\partial N_{i,j}^*}{\partial P_G} < 0$. (See Appendix B for a proof).

Figure 3: The marginal cost and marginal product of the two inputs, θ_1 and θ_2 , when inputs are substitutes and α_j^{12} is sufficiently small



Corollary 2: Suppose $\alpha_j^{12} < 0$, and $\phi_j^{N1} > 0$; $\phi_j^{N2} > 0$; $\phi_j^{G1} > 0$; $\phi_j^{G2} > 0$. If $\phi_j^{G1} = \phi_j^{G2}$, but one of the inputs is sufficiently nutrient intensive, $\frac{\partial N_{i,j}^*}{\partial P_G}$ may exceed zero. (see Appendix B for a proof).

4 The aggregate response to pollution charges

Section 3 shows how an individual farmer with a particular land-use changes inputs in response to pollution price changes. This section examines how aggregate pollution levels change in response to pollution prices. It is assumed that the price of GHG emissions is exogenously determined, but two separate management options for nutrient pollution are considered: either a tax or a NTS. The results are presented by showing how GHG quantities and N prices and quantities respond to an increase in P_G . This section assumes the change in P_G is sufficiently small that farmers adjust input levels and do not change their farm activity, although the results are calculated under the assumption that different farmers may have different land-uses. We maintain the assumption that both inputs generate non-negative amounts of both types of pollution for all land use types i.e. $\phi_j^{N1}, \phi_j^{N2}, \phi_j^{G1}, \phi_j^{G2} \ge 0 \forall j$.

4.1 Environmental management option: two pollution taxes

4.1.1 Complementary inputs $(\alpha_j^{12} > 0)$

All farmers reduce their use of inputs that produce GHG emissions, as $\frac{\partial \theta_{i,j}^1}{\partial P_G} \leq 0$ and $\frac{\partial \theta_{i,j}^2}{\partial P_G} \leq 0$ when $\alpha_j^{12} > 0$ (by inspection of equations (12) to (15).) Since the inputs reduce, all farmers reduce their production of both types of pollution (equations (21), (22), (25), and (26).) Consequently, aggregate levels of GHG and nutrient levels decrease as P_G increases. Figure 4 shows how input levels and nutrient pollution levels depend on P_G for each farmer. Similarly, an exogenous increase in P_N will reduce aggregate GHG and nutrient pollution levels.

Figure 4: Changes in N, P_N , and inputs given a change in P_G and resulting change in farm activity at thresholds (dashed line) for the case of complement inputs $(\alpha_j^{12} > 0)$ and exogenously determined P_N and P_G



4.1.2 Substitutable inputs $(\alpha_i^{12} < 0)$

Section 3 shows that $\frac{\partial GHG_{i,j}^*}{\partial P_G} \leq 0$ for all farmers irrespective of whether the inputs are complements or substitutes. Consequently, aggregate levels of GHG emissions cannot increase in response to an increase in P_G . However, it is possible that nutrient pollution levels on some farms can increase in response to an increase in P_G when farmers switch from the most carbon intensive input to the most nutrient intensive input (proposition 3). It follows that aggregate nutrient pollution could increase in response to an increase in P_G , if there are enough individual farmers whose nutrient pollution increases to offset any whose nutrient pollution decreases. This possibility ceases when P_G rises to a sufficiently high level that no farmers use both inputs, for then $\frac{\partial N_{i,j}^*}{\partial P_G} \leq 0$ for every farmer (equation (26).) Figure 5 shows how input use and nutrient pollution levels change in the circumstances where condition (30) holds. As P_G increases a farmer eventually reduces first one and then the other input level to zero, so the aggregate level of N must eventually decrease.

Figure 5: Changes in N, P_N , and inputs given a change in P_G and resulting change in farm activity at thresholds (dashed line) for the case of substitute inputs ($\alpha_j^{12} < 0$) and exogenously determined P_N and P_G



4.2 Environmental management option: a tax and a nutrient trading scheme

Suppose nutrient pollution is regulated by an NTS with a cap \bar{N} . From Equation (6), the level of nutrients is equal to the cap set by the regulator if $P_N > 0$, and a change in P_G leads to a change in P_N . In these circumstances the relationship between P_N and P_G can be derived using the implicit function theorem. (If the aggregate sum of nutrient pollution, N^+ , is less than \bar{N} , $P_N = 0$ and thus $\frac{dP_N}{dP_G} = 0$). Let

$$F(x_i, \mathbf{P}) = \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} N_j(\theta_{i,j}^*(x_i, \mathbf{P})) - \bar{N}.$$
(31)

By the Implicit Function Theorem,

$$\frac{\partial P_N}{\partial P_G} = -\frac{\frac{\partial F}{\partial P_G}}{\frac{\partial F}{\partial P_N}} = -\frac{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \frac{\partial N_{i,j}^*}{\partial P_G}}{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \frac{\partial N_{i,j}^*}{\partial P_N}}.$$
(32)

The denominator is always negative as $\frac{\partial N_{i,j}^*}{\partial P_N}$ is always negative (if the derivative exists.) The numerator can be positive or negative (Equation 29). When inputs on all land use types are complements, P_N decreases when P_G increases. However, if sufficient numbers of farmers are undertaking activities where inputs are substitutes, P_N can increase when P_G increases.

4.2.1 Effect on GHG emissions

The effect of a change in P_G on aggregate GHG emissions comprises a direct effect reflecting farmers' responses to the change in P_G and an indirect effect reflecting their responses to the induced change in P_N that is needed to keep aggregate nutrient emissions equal to the cap. It can be calculated by summing the GHG emissions (Equation 19) across all farmers *i* and farm activities *j*, and by taking the total derivative with respect to P_G :

$$\frac{dGHG^*}{dP_G} = \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \frac{dGHG^*_{i,j}}{dP_G}
= \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \frac{\partial GHG^*_{i,j}}{\partial P_G} + \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \frac{\partial GHG^*_{i,j}}{\partial P_N} \cdot \frac{\partial P_N}{\partial P_G}$$
(33)

The first term of equation (33) is equal to the change in emissions that occurs as a direct response to the change in P_G . It is equal to the effect when P_N is determined exogenously and is non-positive because $\frac{\partial GHG_{i,j}^*}{\partial P_G} \leq 0$ for all farmers. The sign of the second term is always

non-negative:

$$\begin{split} \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial GHG_{i,j}^{*}}{\partial P_{N}} \cdot \frac{\partial P_{N}}{\partial P_{G}} &= \left(\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N_{i,j}^{*}}{\partial P_{G}} \right) \cdot \left(-\frac{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N_{i,j}^{*}}{\partial P_{G}}}{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N_{i,j}^{*}}{\partial P_{N}}} \right) \\ &= - \left(\frac{\left(\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N_{i,j}^{*}}{\partial P_{G}} \right)^{2}}{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N_{i,j}^{*}}{\partial P_{N}}} \right) \\ &\geq 0 \end{split}$$

While the second term is non-negative, it is proved in Appendix C that $\frac{dGHG^*}{dP_G} \leq 0$ irrespective of the type of land use or the quantity of inputs that are used. Consequently, GHG emissions decline when there is an NTS in place, but by a smaller amount than when P_N is determined by a tax.

4.2.2 Effect on N emissions

When P_G is in a range where $P_N > 0$, the total amount of nutrient runoff equals the cap on N, \overline{N} , so a change in P_G will not change the level of N emissions at the aggregate level. When P_G is sufficiently high, the cap on N leaching is not binding, P_N falls to zero and aggregate N decreases as P_G increases (see Figure 6).⁶

4.2.3 The efficiency of taxes and an NTS

When taxes and emission trading schemes are used to control pollution, socially efficient outcomes are achieved when the price of pollution is equal to the marginal externality damage caused by the pollution. If the marginal damage associated with each type of pollution varies little with the amount of pollution, but can change through time, the appropriate response to an increase in the damage caused by one type of pollution is to change the tax or charge on that pollution type, but leave other pollution charges unchanged. If a country adopts a tax on nutrient pollution as well as a tax or exogenous charge on GHG emissions, optimal

⁶Since P_N is zero, $\frac{dN^+}{dP_G} = \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \overline{\frac{\partial N_{i,j}^*}{\partial P_G}}$, which is negative as $\frac{\partial N_{i,j}^*}{\partial P_G} < 0$ for all farmers.

Figure 6: Changes in N, P_N , and inputs given a change in P_G and resulting change in farm activity at thresholds (dashed line) for the case of complement inputs $(\alpha_j^{12} > 0)$ and endogenously determined P_N but exogenous P_G



pollution control is straightforward to implement: one simply changes one tax and leaves the other unchanged. But when a country adopts a local NTS to control nutrient pollution, it is more difficult to achieve an efficient solution. This is because the endogenous permit price P_N depends on P_G as well as the number of permits that are issued. The above results show an increase in P_G will reduce P_N if inputs are complements but may increase it if they are substitutes. Either way, P_N will no longer equal the marginal damage done by nutrient pollution, so neither the quantity of inputs nor the level of pollution is efficient. There will be too much GHG pollution and too much nitrogen pollution if the inputs are complements, and too much GHG pollution but too little nutrient pollution if the inputs are substitutes.

The inefficiency of an NTS in these circumstances arises because P_N endogenously changes when P_G changes. An NTS can generate an efficient solution if the nutrient cap is changed to offset the change in P_G , thereby keeping P_N equal to the marginal damage of nutrient pollution. Put differently, an NTS requires active management to ensure it achieves efficient outcomes when there are taxes on other types of pollution. When inputs are complements, the government needs to reduce the number of permits when the price P_G increases to ensure the permit price does not fall below the marginal damage level, perhaps by purchasing the permits on market. If P_G declines, however, or if the inputs are substitutes and P_G rises, additional permits would need to be sold to induce a price fall. This continuous intervention is not needed when both types of pollution are subject to environmental taxes, as the tax levels can be set independently.

If the marginal damage of nutrient pollution varies significantly with the quantity of local pollution, the price P_N will require changing when P_G changes irrespective of whether a tax or an NTS is used to control nutrient pollution. When nutrient pollution is taxed, a change in P_G changes the quantity of nutrient pollution that is produced and thus changes the marginal damage it causes. To achieve efficient outcomes the nutrient pollution tax P_N will therefore need to be changed. When there is an NTS, a change in P_G changes the price P_N even though the marginal damage of nutrient pollution is unchanged as the level of pollution is unchanged. Consequently, both an NTS and a nutrient tax will require active management to ensure that pollution charges are equal to the marginal damage costs. A government may be content to have an NTS in these circumstances to ensure the quantity of emissions does not exceed a certain level, even if it cannot guarantee the pollution charge is equal to the marginal damage.

5 Land use changes

Sections 3 and 4 analyze the outcomes when farmers adjust their inputs in response to changes in P_G or P_N , but keep their activity or land use the same. At some point, however, prices will adjust sufficiently that a farmer switches land use. This section describes what happens when these switches occur. Since farmers are different, each farmer switches at a different price. Consequently, the section examines what happens when P_G increases just above the point where a single farmer is indifferent between two land uses, and switches from one to the other, but no other farmers switch activities.

5.1 Individual farmer nitrogen levels when the farmer switches land-use

It is convenient to rewrite the profit equation 10 for an individual farmer in matrix form as:

$$\Pi_{i,j} = P_j x_i \alpha_j^0 + \left[(\theta_{i,j} - \bar{\theta}_{ij})' A_j (\theta_{i,j} - \bar{\theta}_{ij}) \right] P_j x_i - P'_{Hj} \theta_{i,j} - P'_E \Phi_j^0 \tag{34}$$

where

$$P_{Hj} = (P'_{\theta} + P'_E \Phi^1_j) = \begin{bmatrix} P_{\theta 1} + P_N \phi^{N1}_j + P_G \phi^{G1}_j \\ P_{\theta 2} + P_N \phi^{N2}_j + P_G \phi^{G2}_j \end{bmatrix}$$
(35)

and the associated level of pollution is

$$E_{i,j} = \Phi_j^0 + \Phi_j^1 \theta_{i,j}.$$
 (36)

The maximum level of profits is found by setting the first order condition to zero, which implies

$$\theta_{i,j}^* = \bar{\theta}_{i,j} + \frac{1}{2P_j x_i} A_j^{-1} P_{Hj}, \qquad (37)$$

and thus

$$P_j x_i (\theta_{i,j}^* - \bar{\theta_j})' A_j (\theta_{i,j}^* - \bar{\theta_j}) = \frac{1}{4P_j x_i} P'_{Hj} A_j^{-1} P_{Hj}.$$
(38)

The maximum profit level is therefore a quadratic function of the prices P_N and P_G

$$\Pi_{i,j}^* = (P_j x_i \alpha_j^0 - P'_{Hj} \bar{\theta}_j - P'_E \Phi_j^0) - \frac{1}{4P_j x_i} P'_{Hj} A_j^{-1} P_{Hj}.$$
(39)

A farmer will be indifferent between two different types of land-use j and k when the profit associated with each type of land-use is the same. From Equation (39), this will occur

at a set of prices such that:

$$(P_{j}x_{i}\alpha_{j}^{0} - P'_{Hj}\bar{\theta}_{j} - P'_{E}\Phi_{j}^{0}) - \frac{1}{4P_{j}x_{i}}P'_{Hj}A_{j}^{-1}P_{Hj}$$

$$= (P_{k}x_{i}\alpha_{k}^{0} - P'_{Hk}\bar{\theta}_{k} - P'_{E}\Phi_{k}^{0}) - \frac{1}{4P_{k}x_{i}}P'_{Hk}A_{k}^{-1}P_{Hk}.$$

$$(40)$$

Given P_N , the price P_G^* at which the farmer is indifferent between the two types of production activity can be found by solving (40). The levels of inputs $(\theta_{i,j}^1, \theta_{i,j}^2)$ and $(\theta_{i,k}^1, \theta_{i,k}^2)$ and the levels of pollution $(N_{i,j}^*, GHG_{i,j}^*)$ and $(N_{i,k}^*, GHG_{i,k}^*)$ can be calculated in turn from the profit equalizing price P_G^* . Appendix D provides more details of the solution.

The solution shows there are discrete jumps in GHG and N pollution levels at the prices at which a farmer switches activities. When a farmer switches from land use j to land use k, carbon pollution levels must decrease, but the level of nitrogen leaching can be higher or lower than the initial levels, depending on the model parameters. Appendix D provides an example where nutrient levels drop when a farmer switches activities if ϕ_k^{N1} is sufficiently low, but increase if ϕ_k^{N1} is sufficiently high, holding other parameters constant. For intuition, suppose j is a profitable but intensive carbon pollution producing activity, and k is a less profitable and less intensive carbon pollution activity. The farmer will switch from j to k when P_G is sufficiently high, but the effect on nutrient pollution will depend on whether activity k or more or less nitrogen intensive than activity j. Either outcome is possible.⁷ For example, a dairy farmer might reduce GHG emissions in response to a higher GHG price by switching to sheep and beef farming; but the initial level of N leaching on the sheep farm could be lower or higher than level of leaching on the dairy farm, particularly if the cost of N leaching is relatively low.

⁷The coefficients of the quadratic are sufficiently complex that it is not possible to find a simple analytical expression for the conditions when a switch from one land use to another increases nutrient leaching when P_G increases. An analytic expression is available in the case that there is a single input: see Yeo et al. (2013).

5.2 Aggregate implications of farm land-use changes

When farmers only adjust their input use in response to an increase in P_G , there are continuous changes in aggregate outputs and pollution levels. This is not the case when an individual farmer alters his or her land use, however. First, suppose that P_N is fixed. Suppose at price P_{Gi}^* farmer is just indifferent between land use j and k, but switches to activity k when the price increases to $P_{Gi}^* + \epsilon$ for some arbitrarily small $\epsilon > 0$. Assume that at this price no other farmers switch activity.⁸ As there is an infinitesimally small response to the arbitrarily small increase in P_G by all other farmers, the aggregate change in output and GHG and N pollution levels at the price P_{Gi}^* will equal the discrete change by farmer *i*. Consequently, aggregate output and pollution levels adjust discretely at this price. Now, suppose there is an NTS in place. In this case, there will be a discrete change in the individual GHG and N pollution levels of farmer *i* when he or she switches activities at P_{Gi}^* . The change in N pollution by farmer *i* will induce a discrete change in P_N that in turn induces all other farmers to change their inputs by a discrete amount. This will lead to discrete changes in aggregate GHG pollution levels, but the total amount of N leaching will still equal the level dictated by the original cap.

To summarize, sections 3 and 4 provide a complete description of the market adjustment when P_G is sufficiently high that no farmers change land use in response to further increases in P_G . Below this level the continuous adjustment process that occurs as farmers change inputs is punctuated by discrete changes in inputs and outputs as one farmer or another switches activity. GHG pollution levels always decrease in response to an increase in P_G that induces a change in farmer activity, but N pollutions levels (if there is a N tax) or prices (if there is an NTS) can either increase or decrease in discrete jumps.

⁸This assumption is not strictly necessary. In general, if x_i is different for different farmers, then the switching price $P_{G_i}^*$ will be different for different farmers. If some farmers have identical x_i , then several farmers will switch from one activity to another simultaneously, and the consequent change in output and/or prices will reflect the simultaneous but discrete change by all affected farmers.

6 Conclusion

This paper develops a theoretical model to systematically analyze how the interaction of environmental policies influences farmers' pollution generating input choices and how this affects the level of two separate but related types of pollution. While the model is developed to analyze the production of GHG emissions and nutrient leaching to surface or groundwater by the agricultural sector in circumstances that inputs produce both types of pollution, its applicability is quite general. The model allows farmers to alter inputs or change land-use, and key results are calculated when several types of farming activity simultaneously occur. Parameter changes allow results to be derived for the case that an input increases one type of pollution while reducing another, although this case is not analyzed in this paper. The model assumes that one type of pollution, e.g. GHG, is controlled by charging farmers an exogenously determined price for every unit of GHG they produce, while the other type of pollution can either be controlled by a nutrient tax or a local NTS with an endogenously determined pollution permit price.

The comparative static results have focused on the effect of an increase in the charge for GHG emissions, P_G , on the quantities of inputs and pollution outputs. When there are two exogenously determined taxes, we show that while an increase in P_G always reduces GHG emissions, and will also reduce nutrient leaching if P_G is sufficiently high, nutrient pollution levels can either increase or decrease in response to an increase in P_G when P_G is sufficiently low. They will decrease if pollution generating inputs are complements, but may increase if pollution generating inputs are substitutes, or if farmers switch to relatively nutrient intensive activities in response to an increase in P_G . If the price of each pollution charge is set equal to the marginal damage of each pollution type, and if the marginal damages of each pollution type varies little with the quantity of pollution, taxes can be chosen so that efficient levels of inputs and pollution can be produced; moreover, in these circumstances socially efficient levels of pollution can be achieved when one tax is changed without needing to change the other. This is not true if marginal damages vary significantly with the quantity of pollution, as the level of nutrient leaching and therefore its marginal damage changes when P_G changes. In these circumstances, socially efficient outcomes will only be achieved if the tax on one pollution is changed when the tax on the other changes.

When there is a local NTS as well as an exogenously determined charge on GHG, the outcomes are quite different. In these circumstances an increase in P_G reduces GHG emissions, but only reduces nutrient pollution if P_G is so high that P_N falls to zero. When P_G is lower than this level, an increase in P_G has no effect on nutrient levels, but P_N changes, falling if inputs are complements, but possibly increasing if inputs are substitutes or farmers switch to relatively nutrient intensive activities. The change in P_N means that the rate at which GHG emissions decline in response to an increase in P_G is smaller than when there are two taxes, irrespective of whether inputs are complements or substitutes, as farmers reduce the speed at which they substitute away from intensive GHG producing inputs. This causes socially inefficient outcomes, even if marginal damages vary little with the quantity of pollution. If P_G changes in response to a change in the marginal damages associated with nutrient pollution. Consequently a nutrient trading scheme will require active management to ensure socially efficient outcomes are achieved, even when the marginal damage associated with nutrient pollution.

The result that a NTS reduces the responsiveness of GHG abatement to a change in the price of GHG emissions if the number of N permits is constant is a major finding of this paper. This result holds irrespective of whether pollution generating inputs are complements or substitutes. This finding suggests that if a local NTS is adopted, the total number of N permits distributed will need to take into account the damages done by multiple types of pollution, not just the target type. Moreover, the cap will need to be adjusted in response to the price of other types of pollution. This does not mean that an ETS should not be used to control a particular type of pollution. However, it does suggest the permit cap may need to be actively managed to attain broader environment goals.

The choice of a tradable pollution permit scheme rather than a pollution tax will depend in part on a society's environmental goals, for example, its level of concern over climate mitigation versus water quality. This paper suggests that it should also depend on the nature of the production processes and on the extent to which pollution generating farm inputs are substitutes or complements. Since a tradable pollution permit scheme reduces the responsiveness of GHG emissions to a change in the price of GHG emissions, employing both a NTS and a tax on GHG emissions might not be the most effective approach to attain environmental objectives if climate mitigation is the primary concern. On the other hand, if water quality is the primary concern, a tax on nutrient emissions may not be the best approach since conditions exist where a charge on GHG emissions can increase nutrient pollution. In this case, a risk-adverse regulator might choose to implement a NTS as it ensures that aggregate pollution levels are restricted to the cap specified by the regulator.

Appendix A: Proof of Proposition 1

Proposition 1. $\frac{\partial N_{i,j}^*}{\partial P_N} \leq 0 \text{ and } \frac{\partial GHG_{i,j}^*}{\partial P_G} \leq 0.$

Proof. First recall $\alpha_j^{11} < 0, \alpha_j^{22} < 0$ and $4\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2 > 0$.

(i)When $P_1 \leq P_G \leq P_2$ and only one input (θ_{ij}^1) is positive, inspection of equations 24 and 25 shows the derivatives are negative.

(ii) Suppose $P_G < P_1$ and consider $\frac{\partial N_{i,j}^*}{\partial P_N}$. (The case of $\frac{\partial GHG_{i,j}^*}{\partial P_G}$ is similar.) Note the denominator of equation 20 is positive. The numerator of equation 20 can be rearranged as a quadratic in ϕ_j^{N1} :

$$F(\phi_j^{N1}) = 2\alpha_j^{22}(\phi_j^{N1})^2 - 2\alpha_j^{12}(\phi_j^{N1}\phi_j^{N2}) + 2\alpha_j^{11}(\phi_j^{N2})^2$$
(41)

As $\alpha_j^{22} < 0$ $F(\phi_j^{N1})$ will be positive only if the roots of $F(\phi_j^{N1}) = 0$ are real. The roots of $F(\phi_j^{N1}) = 0$ are

$$\begin{split} \phi_j^{N1} &= \frac{2\alpha_j^{12}\phi_j^{N2} \pm \sqrt{(2\alpha_j^{12}\phi_j^{N2})^2 - 4(4\alpha_j^{11}\alpha_j^{22}(\phi_j^{N2})^2)}}{4\alpha_j^{22}} \\ &= \frac{2\alpha_j^{12}\phi_j^{N2} \pm 2\phi_j^{12}\sqrt{(\alpha_j^{12})^2 - 4\alpha_j^{11}\alpha_j^{22}}}{4\alpha_j^{22}} \end{split}$$

The solution is complex as $4\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2 > 0$. Consequently, the numerator is negative and so $\frac{\partial N_{i,j}^*}{\partial P_N} \leq 0$

Appendix B: Proof of Corollary 1 and 2

Corollary 1.

Suppose
$$\alpha_j^{12} < 0$$
, and $\phi_j^{N1} > 0$; $\phi_j^{N2} > 0$; $\phi_j^{G1} > 0$; $\phi_j^{G2} > 0$. If $\frac{\phi_j^{N1}}{\phi_j^{N2}} = \frac{\phi_j^{G1}}{\phi_j^{G2}}$, then $\frac{\partial N_{i,j}^*}{\partial P_G} < 0$

Proof. From equation 30, $\frac{\partial N_{i,j}^*}{\partial P_G} > 0 \text{ if } \alpha_j^{12} < \frac{2\left(\alpha_j^{11} + \left(\frac{\phi_j^{G1}}{\phi_j^{G2}}\right)\left(\frac{\phi_j^{N1}}{\phi_j^{N2}}\right)\alpha_j^{22}\right)}{\left(\frac{\phi_j^{G1}}{\phi_j^{G2}}\right) + \left(\frac{\phi_j^{N1}}{\phi_j^{N2}}\right)}$

Let $\beta = \frac{\phi_j^{G1}}{\phi_j^{G2}} = \frac{\phi_j^{N1}}{\phi_j^{N2}}$. Then $\frac{\partial N_{i,j}^*}{\partial P_G} > 0$ if $\alpha_j^{12} < \frac{2(\alpha_j^{11} + \beta^2 \alpha_j^{22})}{2\beta} = \frac{1}{\beta} \alpha_j^{11} + \beta \alpha_j^{22}$ Let $\alpha_j^{12} = \frac{1}{\beta} \alpha_j^{11} + \beta \alpha_j^{22} + \epsilon$

Since the concavity of the production function requires $4\alpha_j^{11}\alpha_j^{22}-(\alpha_j^{12})^2>0$

$$\left(\frac{1}{\beta}\alpha_j^{11} + \beta\alpha_j^{22} + \epsilon\right)^2 < 4\alpha_j^{11}\alpha_j^{22}$$

$$\Rightarrow \quad \left(\left(\frac{1}{\beta}\alpha_j^{11}\right)^2 + \left(\beta\alpha_j^{22}\right)^2 + 2\alpha_j^{11}\alpha_j^{22}\right) + \epsilon^2 + 2\epsilon \left(\frac{1}{\beta}\alpha_j^{11} + \beta\alpha_j^{22}\right) < 4\alpha_j^{11}\alpha_j^{22}$$

$$\Rightarrow \qquad \left(\frac{1}{\beta}\alpha_j^{11} - \beta\alpha_j^{22}\right)^2 + \epsilon^2 + 2\epsilon \left(\frac{1}{\beta}\alpha_j^{11} + \beta\alpha_j^{22}\right) < 0$$

As the first two terms are positive, the last term must be negative, so $\epsilon > 0$. Hence $\alpha_j^{12} > \frac{1}{\beta} \alpha_j^{11} + \beta \alpha_j^{22}$ and $\frac{\partial N_{i,j}^*}{\partial P_G} > 0$.

Corollary 2. Suppose $\alpha_j^{12} < 0$ and $\phi_j^{N1} > 0$, $\phi_j^{N2} > 0$, $\phi_j^{G1} > 0$, $\phi_j^{G2} > 0$. If $\phi_j^{G1} = \phi_j^{G2}$, then $\frac{\partial N_{i,j}^*}{\partial P_G}$ may exceed zero if $|\phi_j^{N1} - \phi_j^{N2}|$ is sufficiently large.

 $\text{Proof. From Equation 22 , } \frac{\partial N_{i,j}^*}{\partial P_G} > 0 \text{ if } \alpha_j^{12} < \frac{2 \left(\alpha_j^{11} + \left(\frac{\phi_j^{G1}}{\phi_j^{G2}}\right) \left(\frac{\phi_j^{N1}}{\phi_j^{N2}}\right) \alpha_j^{22}\right)}{\left(\frac{\phi_j^{G1}}{\phi_j^{G2}}\right) + \left(\frac{\phi_j^{N1}}{\phi_j^{N2}}\right)}$

Let $\beta = \frac{\phi_j^{N1}}{\phi_j^{N2}}$. Then $\frac{\partial N_{i,j}^*}{\partial P_G} > 0$ if $\alpha_j^{12} < \frac{2(\alpha_j^{11} + \beta \alpha_j^{22})}{1+\beta} = 2(\gamma \alpha_j^{11} + (1-\gamma)\alpha_j^{22})$ where $\gamma = \frac{1}{1+\beta}$.

Since the concavity of the production function requires $(\alpha_j^{12})^2 - 4\alpha_j^{11}\alpha_j^{22} < 0$, let $\alpha_j^{12} =$

$$\begin{aligned} 2\left(\gamma\alpha_{j}^{11}+(1-\gamma)\alpha_{j}^{22}\right)+\epsilon, \text{ and let} \\ F(\epsilon,\gamma,\alpha_{j}^{11},\alpha_{j}^{22})&=\left(2\left(\gamma\alpha_{j}^{11}+(1-\gamma)\alpha_{j}^{22}\right)+\epsilon\right)^{2}-4\alpha_{j}^{11}\alpha_{j}^{22} \\ \text{(i) When } \epsilon&=0, \text{ the limit of } F(\epsilon,\gamma,\alpha_{j}^{11},\alpha_{j}^{22}) \text{ as } \frac{\phi_{j}^{N1}}{\phi_{j}^{N2}}=\beta\to0 \text{ is} \\ F(\epsilon,\gamma,\alpha_{j}^{11},\alpha_{j}^{22})\to F(0,1,\alpha_{j}^{11},\alpha_{j}^{22})&=4\alpha_{j}^{11}(\alpha_{j}^{11}-\alpha_{j}^{22})<0 \text{ if } \alpha_{j}^{11}>\alpha_{j}^{22}. \\ \text{By continuity, } F(\epsilon,\gamma,\alpha_{j}^{11},\alpha_{j}^{22})&<0 \text{ for some } \epsilon<0 \text{ if } \alpha_{j}^{11}>\alpha_{j}^{22} \text{ and } \beta=\frac{\phi_{j}^{N1}}{\phi_{j}^{N2}} \text{ is sufficiently} \\ \text{close to zero. Consequently, if } \alpha_{j}^{11}>\alpha_{j}^{22} \text{ there exists values of } \phi_{j}^{N2} \text{ and } \phi_{j}^{N1}<\phi_{j}^{N2} \text{ such that } \\ \alpha_{j}^{12}<2\left(\gamma\alpha_{j}^{11}+(1-\gamma)\alpha_{j}^{22}\right) \text{ and } \frac{\partial N_{i,j}^{*}}{\partial P_{G}}>0. \\ \text{(ii) Similarly, when } \epsilon&=0, \text{ the limit of } F(\epsilon,\gamma,\alpha_{j}^{11},\alpha_{j}^{22}) \text{ as } \frac{\phi_{j}^{N2}}{\phi_{j}^{N1}}\to0 \text{ is} \\ F(\epsilon,\gamma,\alpha_{j}^{11},\alpha_{j}^{22})\to F(0,0,\alpha_{j}^{11},\alpha_{j}^{22})&=4\alpha_{j}^{22}(\alpha_{j}^{22}-\alpha_{j}^{11})<0 \text{ if } \alpha_{j}^{22}>\alpha_{j}^{11}. \\ \text{In this case, continuity means } F(\epsilon,\gamma,\alpha_{j}^{11},\alpha_{j}^{22})<0 \text{ for some } \epsilon<0 \text{ if } \alpha_{j}^{22}>\alpha_{j}^{11} \text{ and } \frac{\phi_{j}^{N2}}{\phi_{j}^{N1}} \text{ is sufficiently close to zero. Consequently, if } \alpha_{j}^{22}>\alpha_{j}^{11} \text{ there exists values of } \phi_{j}^{N1} \text{ and } \phi_{j}^{N2}<\phi_{j}^{N1} \text{ such that } \alpha_{j}^{12}<2\left(\gamma\alpha_{j}^{11}+(1-\gamma)\alpha_{j}^{22}\right) =4\alpha_{j}^{22}(\alpha_{j}^{22}-\alpha_{j}^{11})<0 \text{ if } \alpha_{j}^{22}>\alpha_{j}^{11} \text{ and } \frac{\phi_{j}^{N2}}{\phi_{j}^{N1}} \text{ is sufficiently close to zero. Consequently, if } \alpha_{j}^{22}>\alpha_{j}^{11} \text{ there exists values of } \phi_{j}^{N1} \text{ and } \phi_{j}^{N2}<\phi_{j}^{N1} \text{ such that } \alpha_{j}^{12}<2\left(\gamma\alpha_{j}^{11}+(1-\gamma)\alpha_{j}^{22}\right) \text{ and } \frac{\partial N_{i,j}^{*}}{\partial P_{G}}>0. \end{cases}$$

Appendix C: Proof that an increase in P_G always reduces GHG emissions in the presence of a nutrient trading scheme

Proposition: When there is an NTS,

$$\frac{dGHG^*}{dP_G} = \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \frac{dGHG^*_{i,j}}{dP_G} \le 0.$$

Proof.

From Equation 31,

$$\frac{\partial P_N}{\partial P_G} = -\frac{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \frac{\partial N_{i,j}^*}{\partial P_G}}{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^j} \frac{\partial N_{i,j}^*}{\partial P_N}}.$$

From Equation 32,

$$\frac{dGHG^{*}}{dP_{G}} = \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{dGHG^{*}_{i,j}}{dP_{G}}$$

$$= \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial GHG^{*}_{i,j}}{\partial P_{G}} + \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial GHG^{*}_{i,j}}{\partial P_{N}} \cdot \frac{\partial P_{N}}{\partial P_{G}}$$

$$= \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial GHG^{*}_{i,j}}{\partial P_{G}} - \sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial GHG^{*}_{i,j}}{\partial P_{N}} \cdot \frac{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N^{*}_{i,j}}{\partial P_{N}}}{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N^{*}_{i,j}}{\partial P_{G}}}$$

$$= \frac{\left(\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial GHG^{*}_{i,j}}{\partial P_{G}}\right) \left(\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N^{*}_{i,j}}{\partial P_{N}}\right) - \left(\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N^{*}_{i,j}}{\partial P_{G}}\right)^{2}}{\sum_{j \in \mathbf{J}} \sum_{i \in \mathbf{I}^{j}} \frac{\partial N^{*}_{i,j}}{\partial P_{N}}}$$

$$(42)$$

since $\frac{\partial N_{i,j}^*}{\partial P_G} = \frac{\partial GHG_{i,j}^*}{\partial P_N} \ \forall i.$

The denominator in Equation (42) is negative. Hence, if the numerator is positive, $\frac{dGHG^*}{dP_G} < 0$. Note that as $\frac{\partial N_{i,j}^*}{\partial P_N} \leq 0 \quad \forall i$ and $\frac{\partial GHG_{i,j}^*}{\partial P_G} \leq 0 \quad \forall i$, the first two terms in the numerator are always negative.

The proof is done by induction by considering how the numerator of Equation (42) changes when moving from M farmers to M + 1 farmers by adding an extra farmer s. The proof has two stages. First, we show that for any individual farmer s, $\frac{\partial GHG_{s,j}^*}{\partial P_G} \frac{\partial N_{s,j}^*}{\partial P_N} \ge \left(\frac{\partial N_{s,j}^*}{\partial P_G}\right)^2$. Then we prove that:

1. if
$$\left(\sum_{i=1}^{M} \frac{\partial GHG_{i,j}^{*}}{\partial P_{G}}\right) \left(\sum_{i=1}^{M} \frac{\partial N_{i,j}^{*}}{\partial P_{N}}\right) - \left(\sum_{i=1}^{M} \frac{\partial N_{i,j}^{*}}{\partial P_{G}}\right)^{2} \ge 0$$
; and
2. if $\frac{\partial GHG_{s,j}^{*}}{\partial P_{G}} \frac{\partial N_{s,j}^{*}}{\partial P_{N}} \ge \left(\frac{\partial N_{s,j}^{*}}{\partial P_{G}}\right)^{2}$ for the additional $M + 1$ farmer s ; then
3. $\left(\sum_{i=1}^{M+1} \frac{\partial GHG_{i,j}^{*}}{\partial P_{G}}\right) \left(\sum_{i=1}^{M+1} \frac{\partial N_{i,j}^{*}}{\partial P_{N}}\right) - \left(\sum_{i=1}^{M+1} \frac{\partial N_{i,j}^{*}}{\partial P_{G}}\right)^{2} \ge 0.$

Stage 1

We show that for any farmer s, $\frac{\partial GHG_{s,j}^*}{\partial P_G} \frac{\partial N_{s,j}^*}{\partial P_N} \ge \left(\frac{\partial N_{s,j}^*}{\partial P_G}\right)^2$ whether the farmer uses 1 or 2 inputs. First, consider a farmer that uses only 1 input (e.g. input 2). From equations 24 - 27, $\frac{\partial N_{s,j}^*}{\partial P_N} = \frac{(\phi_j^{N2})^2}{2\alpha_j^{22}x_s P_j}$; $\frac{\partial GHG_{s,j}^*}{\partial P_G} = \frac{(\phi_j^{G2})^2}{2\alpha_j^{22}x_s P_j}$. Hence,

$$\frac{\partial GHG_{s,j}^*}{\partial P_G}\frac{\partial N_{s,j}^*}{\partial P_N} - \left(\frac{\partial N_{s,j}^*}{\partial P_G}\right)^2 = \frac{1}{2\alpha_j^{22}x_sP_j}\left[(\phi_j^{G2})^2(\phi_j^{N2})^2 - (\phi_j^{N2}\phi_j^{G2})^2\right] = 0$$

Secondly, consider a farmer who uses both inputs. Using the expressions for $\frac{\partial GHG_{s,j}^*}{\partial P_G}$, $\frac{\partial N_{s,j}^*}{\partial P_N}$, and $\frac{\partial N_{s,j}^*}{\partial P_G}$ (equations 20-23), it can be shown that

$$\frac{\partial GHG_{s,j}^*}{\partial P_G} \frac{\partial N_{s,j}^*}{\partial P_N} - \left(\frac{\partial N_{s,j}}{\partial P_G}\right)^2 = \frac{\left[4(\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2)((\phi_j^{N1}\phi_j^{G2}) - (\phi_j^{N2}\phi_j^{G1}))^2\right]}{4(\alpha_j^{11}\alpha_j^{22} - (\alpha_j^{12})^2)x_sP_j} \ge 0$$

Hence, whether farmer s uses one or two inputs $\frac{\partial GHG_{s,j}^*}{\partial P_G} \frac{\partial N_{s,j}^*}{\partial P_N} \ge \left(\frac{\partial N_{s,j}^*}{\partial P_G}\right)^2$.

Stage 2

To prove the second stage, suppose for some collection of M farmers,

$$\left(\sum_{i=1}^{M} \frac{\partial GHG_{i,j}^{*}}{\partial P_{G}}\right) \left(\sum_{i=1}^{M} \frac{\partial N_{i,j^{*}}}{\partial P_{N}}\right) - \left(\sum_{i=1}^{M} \frac{\partial N_{i,j^{*}}}{\partial P_{G}}\right)^{2} \ge 0. \text{ Let}$$

$$1. A_{M} = \sum_{i=1}^{M} \frac{\partial GHG_{i,j}^{*}}{\partial P_{G}}; B_{M} = \sum_{i=1}^{M} \frac{\partial N_{i,j}^{*}}{\partial P_{N}}; \text{ and } C_{M} = \sum_{i=1}^{M} \left(\frac{\partial N_{i,j}^{*}}{\partial P_{G}}\right)^{2}$$

$$2. a_{m} = \frac{\partial GHG_{s,j}^{*}}{\partial P_{G}}; b_{m} = \frac{\partial N_{s,j}^{*}}{\partial P_{N}}; \text{ and } c_{m} = \left(\frac{\partial N_{s,j}^{*}}{\partial P_{G}}\right)^{2}.$$

Note $a_m A_M \ge 0$, $b_m B_M \ge 0$, and $c_m C_M \ge 0$, and let $\epsilon = A_M B_M - C_M^2 \ge 0$, $\mu = a_m b_m - c_m^2 \ge 0$

0. Then,

$$(A_{M} + a_{m})(B_{M} + b_{m}) = A_{M}B_{M} + a_{m}b_{m} + a_{m}B_{M} + A_{M}b_{m}$$

$$= (C_{M}^{2} + \epsilon) + (c_{m}^{2} + \mu) + \frac{(c_{m}^{2} + \mu)B_{M}}{b_{m}} + \frac{(C_{M}^{2} + \epsilon)b_{m}}{B_{M}}$$

$$= (C_{M}^{2} + \epsilon) + (c_{m}^{2} + \mu) + c_{m}C_{M}\left(\frac{c_{m}B_{M}}{C_{M}b_{m}} + \frac{C_{M}b_{m}}{c_{m}B_{M}}\right) + \frac{\mu B_{M}}{b_{m}} + \frac{\epsilon b_{m}}{B_{M}}$$

$$= C_{M}^{2} + c_{m}^{2} + c_{m}C_{M}\left(\frac{c_{m}B_{M}}{C_{M}b_{m}} + \frac{C_{M}b_{m}}{c_{m}B_{M}}\right) + (B_{M} + b_{m})\left(\frac{\mu}{b_{m}} + \frac{\epsilon}{B_{M}}\right)$$

$$\geq C_{M}^{2} + c_{m}^{2} + 2c_{m}C_{M} + (B_{M} + b_{m})\left(\frac{\mu}{b_{m}} + \frac{\epsilon}{B_{M}}\right)$$

$$\geq (C_{M} + c_{M})^{2}$$
(43)

as $\left(\frac{c_m B_M}{C_M b_m} + \frac{C_M b_m}{c_m B_M}\right) \ge 2$ and $\epsilon, \mu \ge 0$.

Hence, if a collection of M farmers satisfies condition 1, any set of M+1 farmers also satisfies it. Since in stage 1 we showed condition 1 holds when M = 1, it must hold for any collection size M = 2; hence it also holds for M = 3; and by induction it must hold for any arbitrary set of farmers. Thus the numerator of equation 42 is non-negative and so $\frac{dGHG^*}{dP_G} \leq 0$. Appendix D: Derivation of the formula for calculating the amount of pollution when farmers switch land use activities.

From equation 40,

$$\Pi_{i,j}^* = (P_j x_i \alpha_j^0 - P'_{Hj} \bar{\theta}_j - P'_E \phi_j^0) - \frac{1}{4P_j x_i} P'_{Hj} A_j^{-1} P_{Hj}$$
(44)

where

$$P_{Hj} = \begin{bmatrix} P_{\theta 1} + P_N \phi_j^{N1} + P_G \phi_j^{G1} \\ P_{\theta 2} + P_N \phi_j^{N2} + P_G \phi_j^{G2} \end{bmatrix};$$

and

$$A_j^{-1} = \left[\begin{array}{cc} a^{11} & a^{12} \\ a^{21} & a^{22} \end{array} \right].$$

Expanding and rearranging the profit function as a quadratic in P_G ,

$$\begin{split} \Pi_{i,j}^* &= \frac{-1}{4P_j x_i} \left[a_j^{11} (\phi_j^{G1})^2 + 2a_j^{12} \phi_j^{G1} \phi_j^{G2} + a_j^{22} (\phi_j^{G2})^2 \right] (P_G)^2 \\ &+ \frac{-2}{4P_j x_i} \left((P_{\theta 1} + P_N \phi_j^{N1}) (a_j^{11} \phi_j^{G1} + a_j^{12} \phi_j^{G2}) + (P_{\theta 2} + P_N \phi_j^{N2}) (a_j^{12} \phi_j^{G1} + a_j^{22} \phi_j^{G2}) \right) \\ &- (\phi_j^{G0} + \phi_j^{G1} \bar{\theta}_j^1 + \phi_j^{G2} \bar{\theta}_j^2) P_G \\ &+ (P_j x_i \alpha_0 - (P_{\theta 1} + P_N \phi_j^{N1}) \bar{\theta}_j^1 - (P_{\theta 2} + P_N \phi_j^{N2}) \bar{\theta}_j^2 - P_N \phi_j^{N0}) \\ &- \frac{1}{4P_j x_i} (a_j^{11} (P_{\theta 1} + P_N \phi_j^{N1})^2 + 2a_j^{12} (P_{\theta 1} + P_N \phi_j^{N1}) (P_{\theta 2} + P_N \phi_j^{N2}) \\ &+ a_j^{22} (P_{\theta 2} + P_N \phi_j^{N2})^2). \end{split}$$

A farmer will be indifferent between two different types of land-use j and k when the profits associated with each activity are equal. This level can be calculated by solving the quadratic equation $\Pi_{i,j}^* - \Pi_{i,k}^* = 0$ to give the profit-equalizing GHG price P_G^* , which is a function of all the basic parameters of the model. The levels of inputs $(\theta_{i,j}^1, \theta_{i,j}^2)$ and $(\theta_{i,k}^1, \theta_{i,k}^2)$ and the levels of pollution $(N_{i,j}^*, GHG_{i,j}^*)$ and $(N_{i,k}^*, GHG_{i,k}^*)$ can be calculated in turn from the profit-equalizing price P_G^* .

Figure 7 shows how the profit equalizing price P_G^* and the levels of nutrient pollution $(N_{i,j}^*, N_{i,k}^*)$ vary with the parameter ϕ_k^{N1} in circumstances where activity j is more profitable than activity k at low GHG prices because the price $P_j > P_k$ and activity k produces less GHG than activity j for any levels of inputs θ^1 and θ^2 .⁹ The figure shows (i) that the profit equalizing price P_G^* is increasing in ϕ_k^{N1} ; (ii) nutrient pollution falls when farmers switch from one activity to the other for small values of ϕ_k^{N1} ; and (iii) nutrient pollution increases when farmers switch from one activity to the other for large values of ϕ_k^{N1} . A similar result holds when ϕ_k^{N0} is varied. These results clearly demonstrate that individual farmer nutrient pollution can either decline or increase when farmers switch from one activity to the other.

 $[\]overline{ \begin{array}{c} {}^{9}\text{The parameters are } a_{j}^{11} = 1; a_{j}^{12} = -\frac{1}{3}; a_{j}^{22} = 1; A_{k} = A_{j}; \bar{\theta_{j}^{1}} = \bar{\theta_{j}^{2}} = \bar{\theta_{k}^{1}} = \bar{\theta_{k}^{2}} = 50; \alpha_{j}^{0} = \alpha_{k}^{0} = 6666.67; x_{1} = 1; \phi_{j}^{N0} = \phi_{k}^{N0} = \phi_{j}^{G0} = \phi_{k}^{G0} = 0; \phi_{j}^{N1} = 2; \phi_{j}^{N2} = 1; \phi_{j}^{G1} = 1; \phi_{j}^{G2} = 3; \phi_{k}^{N2} = 1; \phi_{k}^{G1} = 0.5; \phi_{j}^{G2} = 1.5; P_{N} = 1; P_{\theta}^{1} = P_{\theta}^{2} = 10; P_{j} = 2; P_{k} = 1.9.$



Figure 7: How profit equalizing price P_G^* and the levels of nutrient pollution $(N_{i,j}^*, N_{i,k}^*)$ vary with the parameter ϕ_k^{N1}

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