The Elasticity of Taxable Income and the Tax Revenue Elasticity

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Abstract

This paper examines the elasticity of tax revenue with respect to a marginal rate change, at both the individual and aggregate level. The roles of the elasticity of taxable income (the effect on taxable income of a tax rise) and the revenue elasticity (the effect on revenue of a change in taxable income) are highlighted. The revenue elasticity is the central concept in examining fiscal drag, but it has an additional role in the context of the revenue effects of tax changes when incomes respond to rate changes. Illustrations are provided using changes to the New Zealand income tax structure in the 2010 Budget. This reduced all marginal tax rates while leaving income thresholds unchanged.

Keywords: Income Tax Revenue; Elasticity of taxable income; revenue elasticity.

JEL Codes: H24; H31; H26

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1 Introduction

In the analysis of income tax structures, two elasticities play an important role at individual and aggregate levels. First, the tax revenue elasticity – the elasticity of tax revenue with respect to a change in gross income – is the central concept in the literature on 'fiscal drag', which is concerned with the extent to which the non-indexation of tax thresholds leads to increasing average tax rates over time.[1] In this context, the change in gross income is considered to be exogenous and any consequent feedback disincentive effect on income arising from the change in the average tax rate is ignored. Indeed, at the individual level the literature concentrates on changes which do not involve a movement across tax thresholds, which would otherwise lead to a change in the marginal tax rate.[2]

Second, the elasticity of taxable income – the response of taxable income to a change in the marginal net-of-tax rate (one minus the marginal rate) – captures the net effect of all incentive effects associated with the marginal rate change. This approach to grouping all the various responses, such as labour supply, income shifting, under-reporting of income and so on, in a reduced-form specification has attracted much attention.[3] It avoids the considerable complexities of attempting to combine these effects into a structural model, as well as providing (under certain assumptions) a convenient method of measuring the marginal excess burden arising from tax changes. The elasticity can be influenced by policy changes concerning, for example, regulations regarding income shifting and the timing of income receipts and tax payments.

Hence, one elasticity concerns the way tax revenue changes in response to exogenous income changes while the other elasticity measures the extent to which income declared for tax purposes adjusts when the income tax rate varies.[4] The first elasticity

[1] See the survey in Creedy and Gemmell (2002). The revenue elasticity is also used in discussions of local measures of tax progressivity.

[2] In simulations generating aggregate elasticities from individual elasticities, care is also needed to avoid such movements because they can involve huge individual revenue elasticities for tiny changes in gross income. Labour supply incentive effects, in the context of the revenue elasticity with respect to wage rate changes, are examined by Creedy and Gemmell (2005).

[3] The seminal paper is Feldstein (1995), with important evidence for the US by Auten and Carroll (1995, 1999) and Auten *et al.* (2008). Giertz (2007) and Saez, Slemrod and Giertz (2009) provide comprehensive reviews of evidence, while Creedy (2010) provides an introduction to the underlying analytics.

[4] The tax rate may vary as a result of a deliberate policy change, or it may change as individuals move across income thresholds, particularly as a result of fiscal drag. As mentioned earlier, such

is concerned only with the nature of the income tax structure itself and, when considering aggregation over individuals, the form of the income distribution. The second elasticity is concerned with a wide range of behavioural adjustments associated with tax rate changes, captured in a single measure. Hence there is no direct connection between the two elasticity concepts. However, there is another associated elasticity in which the two elasticities have a role. The elasticity of tax revenue with respect to a change in the marginal tax rate is influenced, first, by the extent to which taxable income adjusts to the tax rate change and, second, by the way tax revenue adjusts to the taxable income change.

When discussing revenue changes resulting from marginal rate changes, the existing literature on the elasticity of taxable income has not generally identified a separate role for the revenue elasticity. Changes in total tax obtained from the top marginal rate in a multi-rate structure are examined in Saez *et al.* (2009), in the course of deriving the aggregate excess burden. But they do not consider revenue elasticities. It is shown below how the revenue elasticity has a clear role at the individual level in influencing the change in tax resulting from a rate change. In considering aggregate revenue over all individuals, changes are shown to depend on the revenue elasticity at the arithmetic mean income level within each tax bracket in a multi-rate income tax structure.

The aim of this paper is to explore the precise relationships among the three elasticities for the tax functions mentioned above. Section 2 examines the individual tax revenue elasticity, the individual elasticity of taxable income and presents the way in which the two elasticities combine to determine the elasticity of tax with respect to a change in the marginal tax rate. The relationships are examined for a completely general tax function. However, in view of the ubiquitous nature of the multi-step tax function, and the focus of the previous literature on the 'top rate', results are also given for this special cases. Section 3 looks at aggregation over individuals when a single marginal rate changes in a multi-rate tax structure. To illustrate the orders of magnitude involved section 3 applies the aggregate analysis to the New Zealand income tax system. This provides a convenient 'natural experiment' since the New Zealand government's May 2010 Budget involved changes to all marginal income tax rates whilst holding all thresholds constant. (Analytical results for the effects of changes in income thresholds are examined in the Appendix). Brief conclusions are in Section 4.

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transitions across thresholds are typically not considered in producing revenue elasticities.

2 Relationships Among Elasticities

This section demonstrates, at the individual level, how the revenue elasticity and the elasticity of taxable income combine to generate the elasticity of tax with respect to the marginal rate. Subsections 2.1 and 2.2 are for a general tax function, but subsection 2.3 explores the special case of the ubiquitous multi-step tax function. In these sections, the distinction between gross income and taxable income is ignored, though this distinction is likely to be important for countries with extensive income tax deductions.[5] Subsection 2.4 therefore extends the results to deal with the case where there are endogenous, income-related deductions.

2.1 The Revenue Elasticity

The literature on the tax revenue elasticity concentrates on the effects of changes in taxation resulting from exogenous changes in taxable income, *with tax rates and thresholds held constant*.[6] Let $T(y)$ denote the tax paid by an individual with income of y, and let τ denote the marginal tax rate facing the individual. Totally differentiating $T(y)$ gives:

$$dT = \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial \tau} d\tau \tag{1}$$

This assumes that any deductions and tax thresholds in the tax structure remain unchanged. The existence of separate deductions, which may depend on y, are discussed below. Using the general notation, $\eta_{b,a} = \frac{a}{b}\frac{db}{da}$, to denote a 'total' elasticity and letting $\eta'_{b,a} = \frac{a}{b}\frac{\partial b}{\partial a}$, so that a prime indicates that the elasticity is partial, rearrangement of (1) gives:

$$\eta_{T,y} = \eta'_{T,y} + \eta'_{T,\tau}\eta_{\tau,y} \tag{2}$$

However, it can be seen that the partial elasticity $\eta'_{T,\tau} = 1$, so that:

$$\eta_{T,y} = \eta'_{T,y} + \eta_{\tau,y} \tag{3}$$

[5] For discussion of the empirical importance of income-related deductions in personal income tax regimes in OECD countries, see Caminada and Goudswaard (1996) and Wagstaff and van Doorslaer (2001). For the US, Feldstein (1999, p. 675) estimated that total income tax deductions in 1993 amounted to about 60% of estimated taxable income.

[6] The restriction to exogenous income changes is easily controlled in considering individual elasticity values but of course the nature of the overall distribution of income, which is needed to obtain aggregate values, may well be influenced by the incentive effects of the consequent tax changes.

In obtaining expressions for the revenue elasticity, it is usual in the literature to assume that the exogenous change in income does not cause the individual to move into a higher tax bracket. Such a movement, where the tax function involves discrete changes in marginal rates, gives rise to a large jump in the elasticity, and this can – when carrying out appropriately tax-share weighted aggregation – distort the aggregate elasticity. Hence $\eta_{\tau,y}$ is considered to be zero, and it may be said that the literature concentrates on $\eta'_{T,y}$.

It may be tempting here to rewrite $\eta_{\tau,y}$ as $1/\eta_{y,\tau}$ and think of the latter as reflecting an incentive effect of a change in the marginal tax rate. However, this is not legitimate: for example the assumption that the individual does not move into a higher tax bracket when income rises is not consistent with an infinitely large response of income to a change in the tax rate.

2.2 The Elasticity of Taxable Income

The individual elasticity of taxable income, $\eta_{y,1-\tau}$, measures the behavioural response of taxable income to a change in the marginal net-of-tax rate, $1-\tau$, facing the individual. This is positive, because increases in the net-of-tax rate are expected to lead to increases in y. Expressed instead as the elasticity of taxable income with respect to the *tax rate*, this simply becomes:

$$\eta_{y,1-\tau} = -\left(\frac{1-\tau}{\tau}\right)\eta_{y,\tau} \ , \tag{4}$$

which is negatively signed. Consider a change in the individual's tax liability resulting from an exogenous increase in the marginal tax rate. From (1), dividing by $d\tau$ gives:

$$\frac{dT}{d\tau} = \frac{\partial T}{\partial \tau} + \frac{\partial T}{\partial y}\frac{dy}{d\tau} \tag{5}$$

The first term may be said to reflect a pure 'tax rate' effect of a rate change, with unchanged incomes, while the second term reflects the net 'tax base' effect resulting from the incentive effects on taxable income combined with the revenue consequences of that income change. When discussing the effect on total revenue of a change in the top income tax rate, Saez *et al.* (2009) refer to these as the 'mechanical' and 'behavioural' effects respectively.[7] In terms of elasticities, equation (5) becomes:

$$\eta_{T,\tau} = \eta'_{T,\tau} + \eta'_{T,y}\eta_{y,\tau} \tag{6}$$

Furthermore, using (4), along with the fact, mentioned above, that $\eta'_{T,\tau} = 1$, it can be seen that:

$$\eta_{T,\tau} = 1 - \left(\frac{\tau}{1-\tau}\right)\eta'_{T,y}\eta_{y,1-\tau} \tag{7}$$

This result links the two relevant elasticities. The total response of tax revenue to a change in the marginal tax rate is one, minus $\tau/(1-\tau)$ multiplied by the product of the revenue elasticity and the elasticity of taxable income. The latter elasticity governs the way income changes when the marginal tax rate varies, while the first elasticity reflect the consequent change in revenue as a result of that income change. If there is no incentive effect, the total change $\eta_{T,\tau}$ is thus equal to the partial change $\eta'_{T,\tau} = 1$.

Tax paid by the individual increases, when the marginal rate increases, only if:

$$\frac{1-\tau}{\tau} > \eta'_{T,y}\eta_{y,1-\tau} \tag{8}$$

Or, in terms of $\eta_{y,\tau}$, tax increases if:

$$\left|\eta_{y,\tau}\right| < \frac{1}{\eta'_{T,y}} \tag{9}$$

In any progressive income tax structure, the partial elasticity, $\eta'_{T,y}$, exceeds 1, unless (as shown in the following section) allowances vary sufficiently with income. Hence revenue increases only if the absolute value of the elasticity, $\eta_{y,\tau}$, is sufficiently small.

2.3 The Multi-Step Tax Function

Consider the case of the multi-step tax function, which is defined by a set of income thresholds, a_k, for $k = 1, ..., K$, and marginal income tax rates, τ_k, applying in tax brackets, that is between adjacent thresholds a_k and a_{k+1}. The function can be written as:[8]

$$\begin{array}{rcllll} T(y) & = & \tau_1(y-a_1) & & & a_1 < y \leq a_2 \\ & = & \tau_1(a_2-a_1) & + & \tau_2(y-a_2) & a_2 < y \leq a_3 \end{array} \tag{10}$$

[7] Thus, their 'behavioural effect' combines the revenue elasticity and elasticity of taxable income effects. Saez *et al.* (2009, p. 5) do not discuss the separate role of the revenue elasticity in this context. Discussion of the rate and base effects is often discussed in the context of a simple proportional tax structure, with constant average and marginal rate, t, where the revenue elasticity is everywhere unity. Thus if $\bar{y}$ is arithmetic mean income, $\frac{dT}{dt} = \bar{y} + \frac{td\bar{y}}{dt}$ and in terms of elasticities, $\eta_{T,t} = 1 + \eta_{\bar{y},t}$.

[8] This is examined in more detail in Creedy and Gemmell (2006).

and so on. If y falls into the kth tax bracket, so that $a_k < y \leq a_{k+1}$, $T(y)$ can be written for $k \geq 2$ as:

$$T(y) = \tau_k (y - a_k) + \sum_{j=1}^{k-1} \tau_j (a_{j+1} - a_j) \tag{11}$$

This expression for $T(y)$ can be rewritten as:

$$T(y) = \tau_k (y - a_k^*) \tag{12}$$

where:

$$a_k^* = \frac{1}{\tau_k} \sum_{j=1}^{k} a_j (\tau_j - \tau_{j-1}) \tag{13}$$

and $\tau_0 = 0$. Thus the tax function facing any individual taxpayer in the kth bracket is equivalent to a tax function with a single marginal tax rate, τ_k, applied to income measured in excess of a single threshold, a_k^*. Therefore, unlike a_j, a_k^* differs across individuals depending on the marginal income tax bracket into which they fall. For this structure, and supposing that the income thresholds, a_k, remain fixed, the revenue elasticity is:

$$\begin{aligned} \eta'_{T,y} &= \frac{y}{y - a_k^*} \\ &= 1 + \frac{a_k^*}{y - a_k^*} \end{aligned} \tag{14}$$

and the individual partial elasticity must exceed unity.

For this tax function, appropriate substitution gives the result that the elasticity of revenue with respect to the marginal rate faced by an individual in the kth tax bracket is given by:

$$\eta_{T,\tau} = 1 - \left(\frac{y}{y - a_k^*}\right) \left(\frac{\tau_k}{1 - \tau_k}\right) \eta_{y,1-\tau} \tag{15}$$

2.4 Endogenous Allowances

Suppose that, instead of having only fixed income thresholds, there is a range of deductions which are income related. In the case of the multi-rate tax schedule discussed in the previous subsection, the deductions can be integrated into the income thresholds, a_k, which are then considered to adjust when income varies.

Consider first the revenue elasticity. In this case, again assuming that the individual does not cross into a higher-rate tax bracket when income increases, it can be shown that the total revenue elasticity is given by:

$$\eta_{T,y} = \eta'_{T,y} + \eta'_{T,a^*}\eta_{a^*,y} \tag{16}$$

where $\eta_{a^*,y}$ measures the extent to which the 'effective threshold', a^*, varies when income changes (a k subscript is suppressed here for convenience). In the case of the multi-rate schedule, the elasticity of tax with respect to the effective threshold, η'_{T,a^*}, is given by:

$$\eta'_{T,a^*} = -\frac{a^*}{y - a^*} \tag{17}$$

Hence:

$$\eta_{T,y} = \eta'_{T,y}\left(1 - \frac{a^*}{y}\eta_{a^*,y}\right) \tag{18}$$

Next, consider the elasticity of taxable income. With endogenous deductions, a modification must also be made to the expression given above for $\eta_{T,\tau}$, such that:

$$\begin{aligned} \eta_{T,\tau} &= \eta'_{T,\tau} + \eta'_{T,y}\eta_{y,\tau} + \eta'_{T,a^*}\eta_{a^*,\tau} \\ &= 1 + \left(\eta'_{T,y} + \eta'_{T,a^*}\eta_{a^*,y}\right)\eta_{y,\tau} \end{aligned} \tag{19}$$

and using (4):

$$\eta_{T,\tau} = 1 - \left(\frac{\tau}{1-\tau}\right)\left(\eta'_{T,y} + \eta'_{T,a^*}\eta_{a^*,y}\right)\eta_{y,1-\tau} \tag{20}$$

Hence, the total effect on revenue as a result of a change in the marginal tax rate is modified by the endogenous effect of an income change on the deductions from gross income. Indeed, to the extent that an increase in the marginal tax rate reduces declared income, it also reduces the level of deductions claimed, so that the fall in tax revenue is not as large as it otherwise would be if deductions were fixed.

For the multi-rate schedule, appropriate substitution gives (again neglecting the k subscript), for the individual elasticity:

$$\eta_{T,\tau} = 1 - \left(\frac{y}{y-a^*}\right)\left(\frac{\tau}{1-\tau}\right)\left(1 - \frac{a^*}{y}\eta_{a^*,y}\right)\eta_{y,1-\tau} \tag{21}$$

This can alternatively be expressed in terms of responses of 'taxable income', $y_T = y - a^*$, as follows:

$$\begin{aligned}
\eta_{T,\tau} &= 1 - \left(\frac{\tau}{1-\tau}\right)\left(\frac{y}{y-a^*}\right)\left(\frac{d(y-a^*)}{dy}\right)\eta_{y,1-\tau} \\
&= 1 - \left(\frac{\tau}{1-\tau}\right)\eta_{y_T,y}\eta_{y,1-\tau} \\
&= 1 - \left(\frac{\tau}{1-\tau}\right)\eta_{y_T,1-\tau} \qquad (22)
\end{aligned}$$

Hence the elasticity of interest, $\eta_{T,\tau}$ can be expressed simply in terms of the marginal tax rate, τ, and the responsiveness of *taxable* income, y_T, to the net-of-tax rate, $1 - \tau$.

3 Aggregate Revenue

For tax policy purposes attention is often devoted to aggregate revenue and its variation as component tax rates are changed. This section therefore examines aggregation over individuals. Emphasis is on the effect on total income tax revenue of a change in a single tax rate, and the effect of a simultaneous similar change in all rates. In order to obtain clear results, attention is restricted to the case of the multi-rate tax function. It is assumed that all individuals face the same income thresholds, so that endogenous allowances are not considered here. First, components of total revenue are examined in subsection 3.1. Aggregate elasticities are derived in subsection 3.2 and, in subsection 3.3, these are compared with an earlier result produced by Saez *et al.* (2009). The potential orders of magnitude involved are then examined in subsection 3.4.

3.1 Components of Total Revenue

When dealing with population aggregates it is necessary to distinguish various tax and revenue terms, for both clarity and succinctness. In the previous section, the tax liability facing an individual with an income of y was denoted by $T(y)$. In the multi-tax form, if y is in the kth tax bracket a distinction can be made between $T(y) = \tau_k (y - a_k^*)$ and the tax paid by the individual at the marginal rate, τ_k, thereby ignoring tax paid on income falling into lower thresholds.

For aggregate revenue amounts defined over populations, or population sub-groups, R is used. Thus, in this section R represents aggregate revenue, while R_k refers to the aggregate revenue obtained from all individuals whose incomes fall in the kth tax bracket: that is, R_k is the aggregate over individuals in the kth bracket of $\tau_k\left(y - a_k^*\right)$ values. Let $R_{(k)}$ denote the aggregate amount raised only at the rate k from individuals who fall into the kth bracket: that is, $R_{(k)}$ is the sum over individuals in the kth bracket of $\tau_k\left(y - a_k\right)$ values. Furthermore, $R_{(k)}^+$ refers to the aggregate revenue obtained at the kth rate from individuals whose incomes fall into higher tax brackets: that is, $R_{(k)}^+$ is the number of all individuals in higher tax brackets multiplied by $\tau_k\left(a_{k+1} - a_k\right)$.

In the multi-step tax function with K brackets, suppose P_k people are in each bracket, for $k = 1, ..., K$, and the arithmetic mean income in each bracket is $\bar{y}_k$. Then aggregate revenue is:

$$\begin{aligned} R &= \sum_{k=1}^{K} R_k \\ &= \tau_1\left(\bar{y}_1 - a_1\right) P_1 \qquad (23) \\ &\quad + \left\{\tau_2\left(\bar{y}_2 - a_2\right) + \tau_1\left(a_2 - a_1\right)\right\} P_2 \\ &\quad + \left\{\tau_3\left(\bar{y}_3 - a_3\right) + \tau_1\left(a_2 - a_1\right) + \tau_2\left(a_3 - a_2\right)\right\} P_3 \\ &\quad + \text{etc} \qquad (24) \end{aligned}$$

Let $P_k^+ \equiv \sum_{j=k+1}^{K} P_j$ denote the number of people above the kth tax bracket. For the top marginal rate, where $k = K$, clearly $P_K^+ = 0$. Thus aggregate revenue can be written more succinctly as:

$$R = \sum_{k=1}^{K} \tau_k\left(\bar{y}_k - a_k\right) P_k + \sum_{k=1}^{K-1} \tau_k\left(a_{k+1} - a_k\right) P_k^+ \qquad (25)$$

Thus, using $R_{(K)}^+ = 0$:

$$R = \sum_{k=1}^{K} R_{(k)} + \sum_{k=1}^{K-1} R_{(k)}^+ = \sum_{k=1}^{K} \left(R_{(k)} + R_{(k)}^+\right) \qquad (26)$$

In this expression, $R_{(k)} + R_{(k)}^+ \neq R_k$, although their sums over $k = 1, ..., K$ are equal.

The various components of aggregate revenue can be illustrated as in Figures 1 and 2. Individuals are ranked in ascending order by taxable income. Let $F\left(y\right)$ and $F_1\left(y\right) = \int_0^y u dF\left(u\right)/\bar{y}$ denote the distribution function and first moment distribution function (giving the cumulative proportion of total income) of y respectively. The

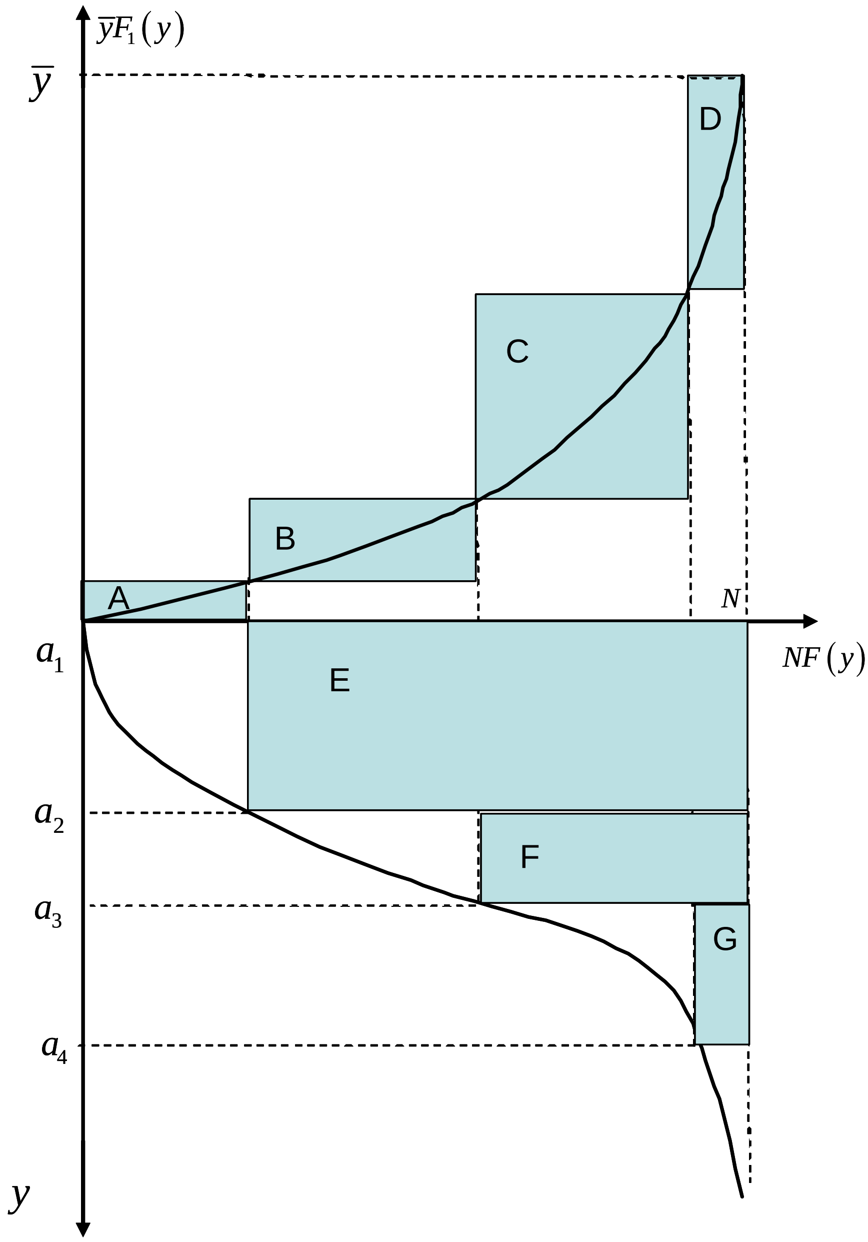

Figure 1: Aggregate Revenue Components: Part A

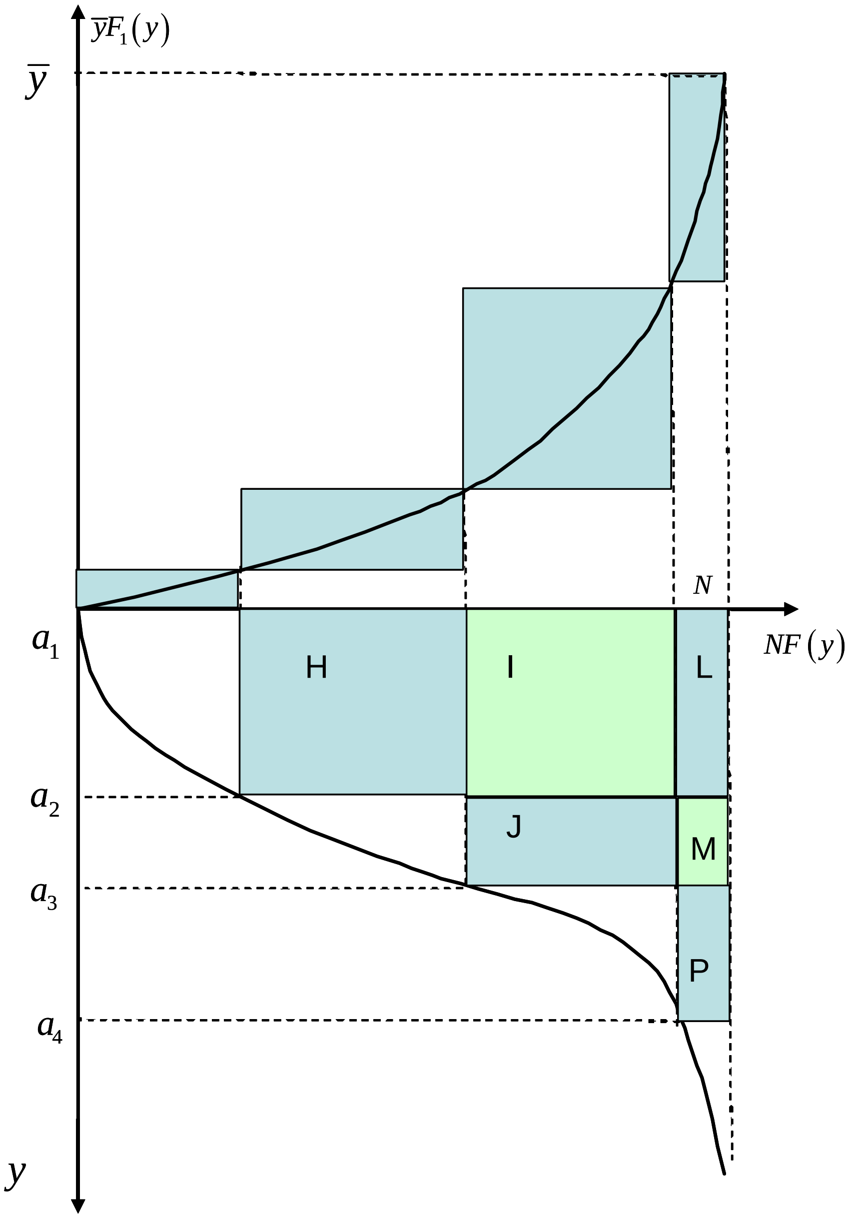

Figure 2: Aggregate Revenue Components: Part B

standard Lorenz curve of the income distribution shows the relationship between $F(y)$ and $F_1(y)$. However, in the top part of each of these figures, a simple transformation is used such that the cumulative number of people, $NF(y)$, is measured on the horizontal axis, and $\bar{y}F_1(y)$ is measured on the vertical axis. The lower part of the figures plots the relationship between taxable income, y, and the number (rather than proportion) of people below or equal to y.

The value on the vertical axis corresponding to N (where $F(y) = 1$) is therefore the arithmetic mean taxable income, $\bar{y}$, over all individuals. For any range of taxable income, the corresponding distance measured on the vertical axis of the top section of each diagram gives the arithmetic mean of those individuals within the range. Hence corresponding areas in the top section measure total taxable income (the arithmetic mean multiplied by the number of individuals) in the income range.

The diagrams illustrate the case of a tax function having four marginal tax rates, with the first marginal rate applying to income above the threshold of $a_1 = 0$. The income thresholds in the lower section of each figure translate, as indicated above, into numbers of people and arithmetic means within each tax bracket, shown in the top section. In each diagram, the letters A, B, C and so on refer to the respective shaded areas. For example, the area A is the total income of all those whose taxable income falls into the first tax bracket.

For those in the first tax bracket, where the tax system is effectively proportional, at the rate τ_1, the aggregate revenue is simply aggregate income multiplied by the tax rate, as is $R_1 = \tau_1 A$. The aggregate amount of tax raised from those within the next bracket is the sum of tax raised from the rate, τ_1, plus the tax from τ_2, and is thus $R_2 = \tau_2 B + \tau_1 H$. Similarly, the revenue from those in the third and fourth brackets are given respectively by $R_3 = \tau_3 C + \tau_1 I + \tau_2 J$ and $R_4 = \tau_4 D + \tau_1 L + \tau_2 M + \tau_3 P$. Thus aggregate revenue is expressed as:

$$\begin{aligned} R &= \tau_1 A + \tau_2 B + \tau_3 C + \tau_4 D \\ &\quad + \tau_1 (H + I + L) + \tau_2 (J + M) + \tau_3 P \end{aligned} \tag{27}$$

Furthermore, the aggregate amounts of tax raised by those in the kth bracket, from τ_k alone, are given simply by: $R_{(1)} = \tau_1 A$, $R_{(2)} = \tau_2 B$, $R_{(3)} = \tau_3 C$, and $R_{(4)} = \tau_4 D$. Finally, it is required to obtain for each bracket, k, the tax raised at the rate, k, by all those whose taxable income falls into a higher band. These amounts are given by

$R^+_{(1)} = \tau_1 E$, $R^+_{(2)} = \tau_2 F$, $R^+_{(3)} = \tau_3 G$, and $R^+_{(4)} = 0$. Adding the $R_{(k)}$ and $R^+_{(k)}$ values over all tax brackets gives:

$$\begin{aligned}\sum_{k=1}^{K}\left(R_{(k)} + R^+_{(k)}\right) &= \tau_1 A + \tau_2 B + \tau_3 C + \tau_4 D \\ &\quad + \tau_1 E + \tau_2 F + \tau_3 G \qquad (28)\end{aligned}$$

From the diagrams, comparison of areas shows clearly that $H + I + L = E$, $J + M = F$ and $P = G$. Hence, (27) and (28) are equal, as in (26). From the diagrams it is also clear that the component parts, $R_{(k)} + R^+_{(k)}$ are not equal to R_k, although their sums over k are equal.

3.2 Changes in Aggregate Revenue

First, it is useful to clarify the general relationship between the elasticity of aggregate revenue with respect to a single marginal rate change, and the elasticity with respect to changes in all rates. Suppose all marginal tax rates change, but income thresholds remain fixed. Totally differentiating R gives:

$$dR = \sum_{k=1}^{K} \frac{\partial R}{\partial \tau_k} d\tau_k \qquad (29)$$

Hence if all rates change by the same proportion, $\partial\tau_k/\tau_k = d\tau/\tau$ for all k and:

$$\eta_{R,\tau} \equiv \frac{\tau}{R}\frac{dR}{d\tau} = \sum_{k=1}^{K} \frac{\tau_k}{R}\frac{\partial R}{\partial \tau_k} = \sum_{k=1}^{K} \eta'_{R,\tau_k} \qquad (30)$$

Thus the elasticity of aggregate revenue with respect to a simultaneous equal proportional change in all tax rates is the sum of the separate elasticities, η'_{R,τ_k}, over all $k = 1, ..., K$.

Consider next the response of aggregate revenue to a change in the kth marginal tax rate. This has two basic components. First, there is the direct effect of the change in the kth tax rate on tax from that bracket alone. From previous sections above, this is made up of the behavioural effect of the tax rate change on the incomes of those in the kth bracket, along with the revenue elasticity effect (which is not a reflection of behaviour but depends on the tax structure). Second, there is an indirect effect on individuals in higher tax brackets, as a result of the term $\tau_k(a_{k+1} - a_k)$. Assume first that there are no behavioural responses. It can be seen that, letting $\eta'_{R,\tau_k} \equiv \frac{\tau_k}{R}\frac{\partial R}{\partial \tau_k}$:

$$\eta'_{R,\tau_1} = \frac{\tau_1}{R}\left\{(\bar{y}_1 - a_1) P_1 + (a_2 - a_1) P_1^+\right\} \tag{31}$$

$$\eta'_{R,\tau_2} = \frac{\tau_2}{R}\left\{(\bar{y}_2 - a_2) P_2 + (a_3 - a_2) P_2^+\right\} \tag{32}$$

and so on. Hence in general:

$$\eta'_{R,\tau_k} = \frac{\tau_k}{R}\left\{(\bar{y}_k - a_k) P_k + (a_{k+1} - a_k) P_k^+\right\} \tag{33}$$

For this 'no behavioural response' case, these elasticities sum to:

$$\eta_{R,\tau} = \sum_{k=1}^{K} \eta'_{R,\tau_k} = 1 \tag{34}$$

and the elasticity of total revenue with respect to an equal proportional change in all rates, in (30), is unity. Any behavioural response clearly reduces the elasticity below 1, as shown below.

In the case where there are behavioural effects of marginal rate changes, it is convenient to assume that all those in a given bracket have the same elasticity, η_{y,τ_k}. In this case, it can be shown that an appropriate adjustment to the average income level within the tax bracket gives:

$$\eta'_{R,\tau_k} = \frac{\tau_k}{R}\left[\left\{\bar{y}_k\left(1 + \eta_{y,\tau_k}\right) - a_k\right\} P_k + (a_{k+1} - a_k) P_k^+\right] \tag{35}$$

The expression in (35), while quite straightforward, does not bring out the separate elements influencing η'_{R,τ_k} in a transparent way. First, rewrite this as:

$$\eta'_{R,\tau_k} = \frac{\tau_k P_k}{R}\left[(\bar{y}_k - a_k) - \bar{y}_k\left(\frac{\tau_k}{1-\tau_k}\right)\eta_{y,1-\tau_k} + (a_{k+1} - a_k)\frac{P_k^+}{P_k}\right] \tag{36}$$

Then multiplying and dividing by $(\bar{y}_k - a_k^*)$ gives:

$$\eta'_{R,\tau_k} = \frac{R_k}{R}\left[\frac{R_{(k)} + R_{(k)}^+}{R_k} - \left(\frac{\bar{y}_k}{\bar{y}_k - a_k^*}\right)\left(\frac{\tau_k}{1-\tau_k}\right)\eta_{y,1-\tau_k}\right] \tag{37}$$

Hence, in the case where $\eta_{y,1-\tau_k} = 0$, the sum $\sum_{k=1}^{K} \eta'_{R,\tau_k}$ is, using (26), equal to 1, as in (34).

From equation (14), $\bar{y}_k / (\bar{y}_k - a_k^*)$ is the revenue elasticity at arithmetic mean income in the kth bracket. This expression therefore shows how the elasticity, η'_{R,τ_k}, depends on the elasticity of taxable income of those in the kth tax bracket, $\eta_{y,1-\tau_k}$, along with the revenue elasticity at $\bar{y}_k$, and various tax-share terms. Furthermore, it can be shown that $\eta'_{R,\tau_k} > 0$ if:

$$\eta_{y,1-\tau_k} < \left(\frac{1-\tau_k}{\tau_k}\right)\left[1 - a_k\left(1 - \frac{P_k^+}{P_k}\right) + a_{k+1}\left(\frac{P_k^+}{P_k}\right)\right]\frac{1}{\bar{y}_k} \tag{38}$$

For the top bracket, the final term within square brackets in equation (38) is zero and the elasticity is positive if:

$$\eta_{y,1-\tau_K} < \left(\frac{1-\tau_K}{\tau_K}\right)\left(\frac{\bar{y}_K - a_K}{\bar{y}_K}\right) \tag{39}$$

and although the first term in brackets exceeds 1 as long as the tax rate, τ_K, is less than 0.5, the second term in brackets is likely to be well below 1. Hence the elasticity of taxable income must be relatively low for a tax rate increase to increase aggregate revenue.

3.3 Comparison with Earlier Results

The above result for any tax rate in a multi-rate structure may be compared with that given by Saez *et al.* (2009, p. 5, equation 4) for the top marginal rate. They consider changes in taxation paid at the top rate only. When converted to the present notation and written in elasticity form, their result thus refers not to actual tax paid but to the elasticity of tax paid at the rate τ_K, which can be defined as $\eta'_{R(K),\tau_K}$. Hence:

$$\eta'_{R(K),\tau_K} = \left[1 - \left(\frac{\bar{y}_K}{\bar{y}_K - a_K}\right)\left(\frac{\tau_K}{1-\tau_K}\right)\eta_{y,1-\tau_K}\right] \tag{40}$$

Saez *et al.* (2009) discuss the term $\bar{y}_K / (\bar{y}_K - a_K)$, which is constant if the income distribution above the top threshold follows the Pareto form. Their expression therefore does not indicate the separate role for the revenue elasticity at $\bar{y}_K$. Furthermore, their 'behavioural response' actually includes both the behavioural response and the revenue elasticity effect, which depends on the full tax structure as well as average income above the top threshold. It is useful to convert (40) into an expression which does separate the these two elasticity effects, by writing:

$$\eta'_{R_{(K)},\tau_K} = \frac{R_K}{R_{(K)}} \left[\frac{R_{(K)}}{R_K} - \left(\frac{\bar{y}_K}{\bar{y}_K - a^*_K} \right) \left(\frac{\tau_K}{1-\tau_K} \right) \eta_{y,1-\tau_K} \right] \tag{41}$$

and remembering that $\bar{y}_K / (\bar{y}_K - a^*_K)$ is the revenue elasticity at $\bar{y}_K$.

From the general result above, the value of η'_{R,τ_K} is given by:

$$\eta'_{R,\tau_K} = \frac{R_K}{R} \left[\frac{R_{(K)}}{R_K} - \left(\frac{\bar{y}_K}{\bar{y}_K - a^*_K} \right) \left(\frac{\tau_K}{1-\tau_K} \right) \eta_{y,1-\tau_K} \right] \tag{42}$$

For comparison with (41), it is necessary to use the general relationship between $\eta'_{R_{(K)},\tau_K}$ and η'_{R,τ_K}. For the top rate, this takes the simple form:

$$\eta'_{R_{(K)},\tau_K} = \left(\frac{R}{R_{(K)}} \right) \eta'_{R,\tau_K} \tag{43}$$

Thus multiplication of (42) by $R/R_{(K)}$ gives the rearranged form of the Saez *et al.* result in (41). Hence, as expected, the Saez result is a special case of the more general result derived above. But instead of focussing on a term such as $\bar{y}_K / (\bar{y}_K - a_K)$, which depends purely on the form of the distribution of income, the present formulation emphasises the *joint* role of the elasticity of taxable income and the appropriate revenue elasticity (at the income level, $\bar{y}_K$), which depends on the nature of the tax function (the lower rates and thresholds, not simply the top threshold) as well as the income distribution (which, together with a_K, affects $\bar{y}_K$).

3.4 Illustrative Examples

In order to provide an illustration of the nature of the relationships involved and the sensitivity to variations in the elasticity of taxable income, it is useful to consider the change to the income tax structure in New Zealand, made in the 2010 Budget. Table 1 provides summary information regarding the distribution of annual personal taxable incomes in the 2008/09 tax year, the most recent year for which data are available.[9] For comparison purposes the tax rates and thresholds shown in the table relate to the structure in 2009/10. The overall arithmetic mean taxable income is \$35,507.

[9] The table is obtained from unpublished Inland Revenue Department data covering 3,304,210 individuals.

Table 1: The Distribution of Taxable Income in New Zealand: 2008/09 Tax Year

k	a_k	$\bar{y}_k$	Prop of people	Prop of income
1	1	6748.82	0.241	0.046
2	14000	24080.76	0.434	0.294
3	48000	52414.34	0.224	0.331
4	70000	115480.70	0.101	0.329

Table 2 provides summary information about the pre- and post-2010 Budget tax structures, for the taxable income distribution of Table 1. In the 2010 Budget, all the income thresholds were left unchanged, but the marginal tax rates were reduced, in particular the top marginal rate. Given the relatively low value of the income threshold above which the top rate applies, this tax bracket contributes a higher proportion of total income tax revenue than the other brackets, even though it contains only ten per cent of taxpayers. This compares with the second tax bracket which contains over forty per cent of all taxpayers. The final column of the table reports the revenue elasticity, $\eta_{T,y}$, in each tax bracket, evaluated at arithmetic mean income within the bracket. For each tax structure, this elasticity is highest in the third tax bracket because the value of $\bar{y}_3$ is relatively closer to the effective income threshold, a_3^* than for the other brackets. For those in the first tax bracket, the average and marginal tax rates are equal and hence the revenue elasticity is unity. The Budget change in the marginal tax rates has little effect on the revenue elasticities.

Table 2: The New Zealand Income Tax Structure Before and After the 2010 Budget

k	τ_k	a_k^*	R_k/P_k	R_k/R	$\eta_{T(\bar{y}_k),y}$
Tax rates pre-2010 Budget					
1	0.125	1.00	843.48	0.027	1.000
2	0.210	5667.26	3866.83	0.222	1.308
3	0.330	21060.99	10346.61	0.306	1.672
4	0.380	27500.33	33432.53	0.446	1.313
Tax rates post-2010 Budget					
1	0.105	1.00	708.52	0.026	1.000
2	0.175	5600.60	3234.03	0.217	1.303
3	0.300	23267.02	8744.20	0.303	1.798
4	0.330	27515.47	29028.52	0.454	1.313

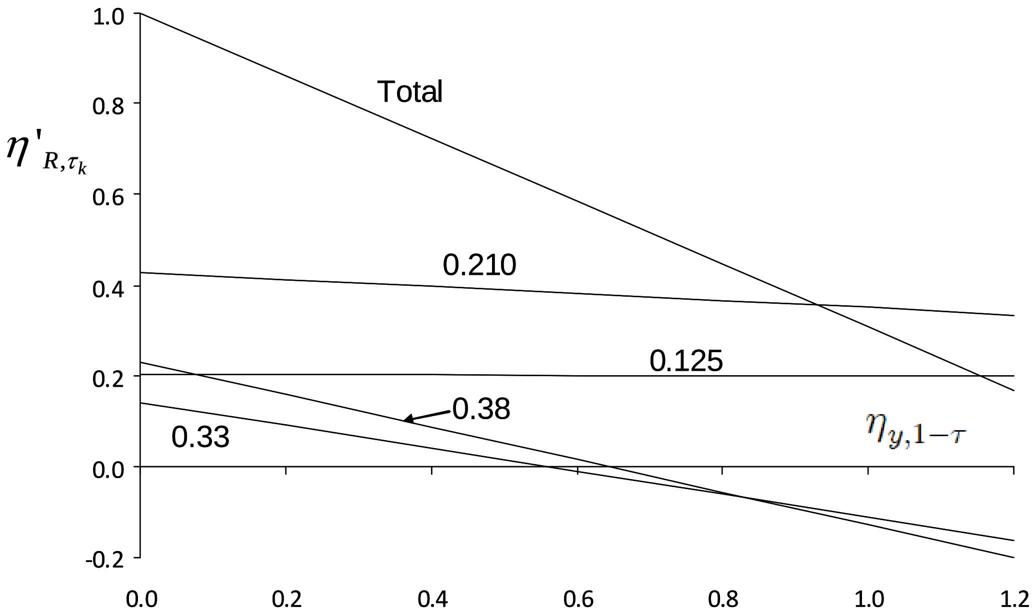

Figure 3: Elasticity of Total Tax Revenue wrt Tax Rates: Pre-2010 Budget

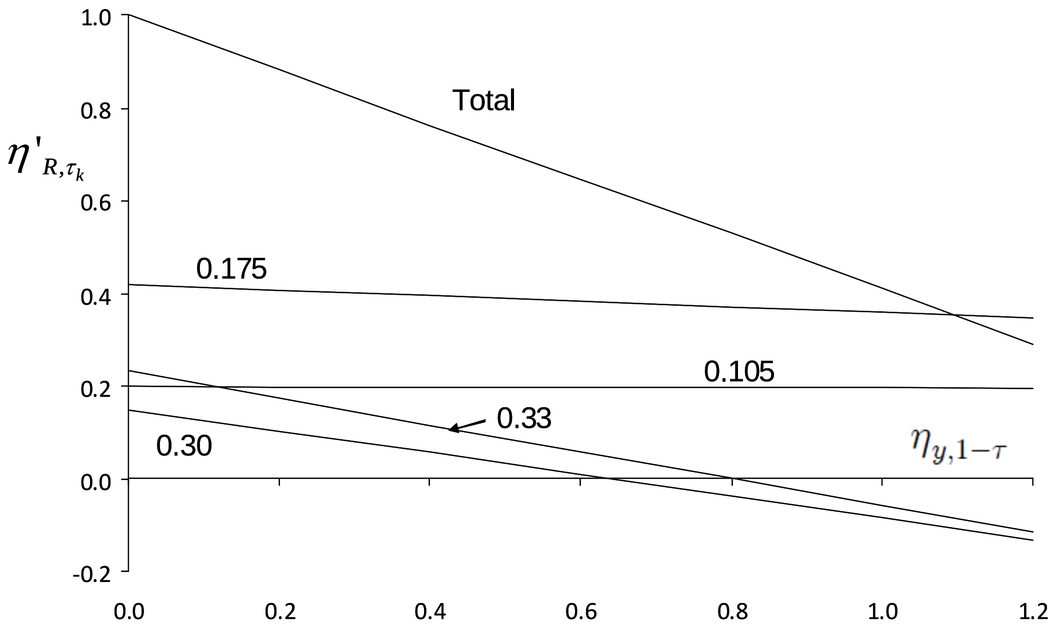

Figure 4: Elasticity of Total Tax Revenue wrt Tax Rates: Post-2010 Budget

Figures 3 and 4 show the variations in the elasticity, η'_{R,τ_k}, for each tax bracket, as the elasticity of taxable income, $\eta_{y,1-\tau}$, increases. As demonstrated above, the value of each η'_{R,τ_k} falls linearly with $\eta_{y,1-\tau}$, but the rate of decrease is less in the post-2010 Budget structure. In each case the elasticity, η'_{R,τ_k}, for the lowest income tax bracket remains approximately constant. Although the elasticity $\eta_{T(\bar{y}),y}$ is highest in the third tax bracket, the value of η'_{R,τ_k} falls slightly faster in the top marginal rate bracket. This is because the value of $\tau/(1-\tau)$ is higher for the higher marginal tax rate, along with the fact that the top-rate bracket contributes a higher proportion of aggregate tax revenue. The reduction in the higher marginal tax rates, resulting from the 2010 Budget, implies that the revenue elasticity, η'_{R,τ_k}, continues to be positive, for higher values of the elasticity of taxable income.

Some evidence regarding the elasticity of taxable income of New Zealand taxpayers is reported in Claus *et al.* (2010). They found that for those in the lower tax brackets, the estimated elasticities were very small, but for the top marginal tax rate the responses were substantial. For top-rate taxpayers, the values were mainly in the range 0.5 to 1.2.

4 Conclusions

This paper has examined the joint role of the elasticity of taxable income (which refers to the effect on taxable income of a marginal tax rate rise) and the revenue elasticity (which reflects the effect on revenue of a change in taxable income) in influencing the revenue effects of tax rate changes. Traditionally, the revenue elasticity has been the central concept in examining fiscal drag, and obtaining local measures of tax progressivity. But it has an additional role in the context of the revenue effects of tax changes when incomes respond to rate changes. This separate effect has not previously been discussed explicitly. The elasticity of tax revenue with respect to a rate change was examined at both the individual and aggregate level.

When a single marginal tax rate in a multi-rate income tax structure is changed, those in the relevant tax bracket adjust their incomes in accordance with the elasticity of taxable income, and this affects the tax paid via the revenue elasticity. There is also a revenue effect on those individuals who are in higher tax brackets, since marginal rate changes in lower tax brackets imply a change in their effective income threshold.

But there are no incentive effects on higher-rate taxpayers because only their average tax rate changes. If there were no incentive effects, an equal proportional change in all marginal tax rates would produce the same proportional increase in total revenue – the elasticity is unity. However, this rapidly falls at a linear rate as the elasticity of taxable income increases.

Illustrations were provided using the New Zealand income tax structures before and after the 2010 Budget. This reduced all rates while leaving income thresholds unchanged and, in particular, reduced the top marginal rate substantially. The elasticity of total tax revenue with respect to a single tax rate change was found to be particularly sensitive to the elasticity of taxable income in the top two tax brackets. In the pre-Budget structure, an elasticity of taxable income in excess of about 0.6 was found to produce a negative tax response to an increase in the top two marginal rates. When these rates are lower, as in the post-Budget structure, the elasticity of taxable income needs to be over 0.8 before tax revenue in the highest tax bracket is expected to fall in response to an increase in the marginal rate. However, recent estimates of the elasticity of taxable income in the top tax bracket in New Zealand are in the range (with some estimates in excess of 1) where tax revenue may fall. For New Zealand in particular, these results therefore suggest that further detailed empirical investigation of the elasticity of taxable income for taxpayers at different income levels may be warranted to assess whether cuts in some marginal tax rates are likely to be revenue-enhancing.

Appendix: Changes in Income Thresholds

This appendix considers the effect on total revenue of a small change in one of the thresholds. Differentiating (25) with respect to a_k gives:

$$\begin{aligned}\frac{\partial R}{\partial a_k} &= \tau_k\left(\bar{y}_k - a_k\right)\frac{\partial P_k}{\partial a_k} + \tau_k P_k\left(\frac{\partial \bar{y}_k}{\partial a_k} - 1\right) - \tau_k P_k^{+} \\ &\quad + \tau_{k-1}\left(\bar{y}_{k-1} - a_{k-1}\right)\frac{\partial P_{k-1}}{\partial a_k} + \tau_{k-1} P_{k-1}\frac{\partial \bar{y}_{k-1}}{\partial a_k} \\ &\quad + \tau_{k-1}\left(a_k - a_{k-1}\right)\frac{\partial P_{k-1}^{+}}{\partial a_k}\end{aligned} \tag{A.1}$$

After some rearrangement, this can be expressed as:

$$\begin{aligned}\eta'_{R,a_k} &= -\frac{\tau_k a_k\left(P_k + P_k^{+}\right)}{R} + \frac{\tau_{k-1}\left(a_k - a_{k-1}\right)P_{k-1}^{+}}{R}\eta_{P_{k-1}^{+},a_k} \\ &\quad + \frac{\tau_k(\bar{y}_k - a_k)P_k}{R}\left[\eta_{P_k,a_k} + \frac{\bar{y}_k}{\bar{y}_k - a_k}\eta_{\bar{y}_k,a_k}\right] \\ &\quad + \frac{\tau_{k-1}(\bar{y}_{k-1} - a_{k-1})P_{k-1}}{R}\left[\eta_{P_{k-1},a_k} + \frac{\bar{y}_{k-1}}{\bar{y}_{k-1} - a_{k-1}}\eta_{\bar{y}_{k-1},a_k}\right]\end{aligned} \tag{A.2}$$

where the various elasticities on the right-hand side are understood to be partial elasticities. Using the various definitions of revenue terms, η'_{R,a_k} can be expressed as:

$$\begin{aligned}\eta'_{R,a_k} &= -\frac{\tau_k a_k\left(P_k + P_k^{+}\right)}{R} + \frac{R_{(k-1)}^{+}}{R}\eta_{P_{k-1}^{+},a_k} \\ &\quad + \frac{R_{(k)}}{R}\left[\eta_{P_k,a_k} + \left(\frac{\bar{y}_k}{\bar{y}_k - a_k}\right)\eta_{\bar{y}_k,a_k}\right] \\ &\quad + \frac{R_{(k-1)}}{R}\left[\eta_{P_{k-1},a_k} + \left(\frac{\bar{y}_{k-1}}{\bar{y}_{k-1} - a_{k-1}}\right)\eta_{\bar{y}_{k-1},a_k}\right]\end{aligned} \tag{A.3}$$

The term, $\tau_k a_k\left(P_k + P_k^{+}\right)$ can be interpreted as the tax that would be raised on an income equal to $y = a_k$, at the rate τ_k, from those in and above tax bracket k, considered as a proportional tax. Alternatively, this term can be rewritten as:

$$\tau_k a_k\left(P_k + P_k^{+}\right) = R_{(k)} + R_{(k)}^{+} - \tau_k\left(\bar{y}_k P_k + a_{k+1}P_k^{+}\right) \tag{A.4}$$

where the term $\tau_k\left(\bar{y}_k P_k + a_{k+1}P_k^{+}\right)$ is the tax that would be raised at the rate, τ_k, from all those in and above the kth bracket, if the tax is considered to be proportional for $y < a_{k+1}$. The term $\frac{\bar{y}_k}{\bar{y}_k - a_k}$ is the revenue elasticity at $\bar{y}_k$ if the tax is considered to have a single threshold at a_k. When the threshold, a_k, is increases, the number of those in the kth bracket falls as a result of some individuals near the threshold moving into the lower tax bracket, where the marginal tax rate is lower. The second line of the expression above is the change in revenue due to those shifting out of the bracket above a_k, as a result of the change in a_k, and the third line is the change in revenue due to those shifting into the bracket below a_k.

Yet another way to write this is:

$$\begin{aligned}
\eta'_{R,a_k} &= -\frac{\tau_k a_k P_k}{R} - \frac{\tau_k a_k P_k^+}{R} \\
&\quad + \frac{R^+_{(k-1)}}{R} \eta_{P^+_{k-1},a_k} \\
&\quad + \frac{R_{(k)}}{R} \left[\eta_{P_k,a_k} + \left(\frac{\bar{y}_k}{\bar{y}_k - a_k} \right) \eta_{\bar{y}_k,a_k} \right] \\
&\quad + \frac{R_{(k-1)}}{R} \left[\eta_{P_{k-1},a_k} + \left(\frac{\bar{y}_{k-1}}{\bar{y}_{k-1} - a_{k-1}} \right) \eta_{\bar{y}_{k-1},a_k} \right]
\end{aligned} \tag{A.5}$$

The first line represents simple 'direct' effects on those in the kth bracket and above it. If the numbers of people were actually unchanged, these terms reflect the fact that less income is subject to τ_k. The units of the numerators in each term are revenue, so the terms are 'unit free' as required of an elasticity. The second line reflects the fact that fewer people are above the $(k-1)$th tax bracket: all those people pay tax at the rate τ_{k-1} on their income that lies between the income thresholds, so their actual income (which does not change, on the assumption that it responds only to marginal rate changes, and not average rate changes) is not relevant. The third line reflects the fact that there is movement out of the bracket, and that the average income of those remaining in the kth bracket changes (as a result of the reduced number of people in the bracket as those close to the threshold move into the lower bracket). The final line reflects the increased number of people in the $(k-1)$th tax bracket and the change in average income in that bracket. The latter results not only from the extra people who were previously just above the threshold, but from a behavioural response as they experience a lower marginal tax rate.

Finally, it is possible to express the various terms, involving changes in the Ps, in terms of the distribution of taxable income. Let $f(y)$ denote the density function of income, and let $F(y)$ denote the corresponding distribution function. Then:

$$P_k = N\int_{a_k}^{a_{k+1}} f(y)\,dy = N\{F(a_{k+1}) - F(a_k)\} \tag{A.6}$$

$$P_{k-1} = N\int_{a_{k-1}}^{a_k} f(y)\,dy = N\{F(a_k) - F(a_{k-1})\} \tag{A.7}$$

$$P_{k-1}^{+} = N\int_{a_k}^{\infty} f(y)\,dy = N\{1 - F(a_k)\} \tag{A.8}$$

Hence the required derivatives are $\frac{\partial P_k}{\partial a_k} = -Nf(a_k)$, $\frac{\partial P_{k-1}}{\partial a_k} = Nf(a_k)$ and $\frac{\partial P_{k-1}^{+}}{\partial a_k} = -Nf(a_k)$. Appropriate substitution gives the elasticities as:

$$\eta_{P_k,a_k} = \frac{-a_k f(a_k)}{F(a_{k+1}) - F(a_k)} \tag{A.9}$$

$$\eta_{P_{k-1},a_k} = \frac{a_k f(a_k)}{F(a_k) - F(a_{k-1})} \tag{A.10}$$

$$\eta_{P_{k-1}^{+},a_k} = \frac{-a_k f(a_k)}{1 - F(a_k)} \tag{A.11}$$

Furthermore, the arithmetic mean taxable income in the kth tax bracket, $\bar{y}_k$, is given by:

$$\bar{y}_k = \frac{\int_{a_k}^{a_{k+1}} yf(y)\,dy}{\int_{a_k}^{a_{k+1}} f(y)\,dy} = \frac{\bar{y}\{F_1(a_{k+1}) - F_1(a_k)\}}{F(a_{k+1}) - F(a_k)} \tag{A.12}$$

Hence:

$$\frac{\partial \bar{y}_k}{\partial a_k} = \frac{-\bar{y}a_k f(a_k)}{F(a_{k+1}) - F(a_k)} + \frac{\bar{y}f(a_k)\{F_1(a_{k+1}) - F_1(a_k)\}}{\{F(a_{k+1}) - F(a_k)\}^2} \tag{A.13}$$

and the elasticity is given by:

$$\eta_{\bar{y}_k,a_k} = -\frac{-a_k f(a_k)}{F(a_{k+1}) - F(a_k)}\left[a_k\left(\frac{\bar{y}}{\bar{y}_k}\right) - \frac{\bar{y}\{F_1(a_{k+1}) - F_1(a_k)\}}{\{F(a_{k+1}) - F(a_k)\}}\right] \tag{A.14}$$

Hence:

$$\eta_{\bar{y}_k,a_k} = -\eta_{P_k,a_k}\left[a_k\left(\frac{\bar{y}}{\bar{y}_k}\right) - \bar{y}_k\right] \tag{A.15}$$

Similarly, it can be shown that:

$$\eta_{\bar{y}_{k-1},a_k} = \eta_{\bar{y},a_k} + \eta_{P_{k-1},a_k}\left[a_k\left(\frac{\bar{y}}{\bar{y}_{k-1}}\right) - \bar{y}_{k-1}\right] \tag{A.16}$$

The first term here allows for the possibility that $\bar{y}$ changes as a result of the fact that those who move into the lower tax bracket, when a_k increases, may respond to the lower marginal tax rate. This is likely to be very small, particularly for a small threshold change.

References

[1] Auten, G. and Carroll, R. (1995) Behavior of the affluent and the 1986 Tax Reform Act. In *Proceedings of the 87th Annual Conference on Taxation of the National Tax Association*, Columbus, Ohio, pp. 70-76.

[2] Auten, G. and Carroll, R. (1999) The effect of income taxes on household behavior. *Review of Economics and Statistics*, 81, pp. 681-693.

[3] Auten, G., Carroll, R. and Gee, G. (2008) The 2001 and 2003 tax rate reductions: An overview and estimate of the taxable income response. *National Tax Journal*, 61, pp. 345-364.

[4] Caminada, K. and Goudswaard, K. (1996) Progression and revenue effects of income tax reform. *International Tax and Public Finance*, 3, 57-66.

[5] Claus, I., Creedy, J. and Teng, J. (2010) The elasticity of taxable income in New Zealand. *University of Melbourne Department of Economics Working Paper*, no. 1104.

[6] Creedy, J. and Gemmell, N. (2002) The built-in flexibility of income and consumption taxes. *Journal of Economic Surveys*, 14, pp. 509-532.

[7] Creedy, J. and Gemmell, N. (2005) Wage growth and income tax revenue elasticities with endogenous labour supply. *Economic Modelling*, 22, pp. 21-38.

[8] Creedy, J. (2010) The elasticity of taxable income: an introduction and some basic analytics. *Public Finance and Management*, 10, pp. 556-589.

[9] Feldstein, M. (1995) The effect of marginal tax rates on taxable income: a panel study of the 1986 Tax Reform Act. *Journal of Political Economy*, 103, pp. 551-572.

[10] Giertz, S.H. (2007) The elasticity of taxable income over the 1980s and 1990s. *National Tax Journal*, LX, pp. 743-768.

[11] Saez, E., Slemrod, J.B. and Giertz, S.H. (2009) The elasticity of taxable income with respect to marginal tax rates: a critical review. *National Bureau of Economic Research Working Paper*, no. 15012.

[12] Wagstaff, A. and van Doorslaer, E. (2001) What makes the personal income tax progressive? A comparative analysis for fifteen OECD countries. *International Tax and Public Finance*, 3, pp. 299-316.