LONG RUN AND CYCLICAL DYNAMICS IN THE US STOCK MARKET

Guglielmo Maria Caporale
Brunel University, London, United Kingdom

Luis A. Gil-Alana
University of Navarra, Pamplona, Spain

Abstract

This paper examines the long-run dynamics and the cyclical structure of various series related to the US stock market using fractional integration. We implement a procedure which enables one to consider unit roots with possibly fractional orders of integration both at the zero (long-run) and the cyclical frequencies. We examine the following series: inflation, real risk-free rate, real stock returns, equity premium and price/dividend ratio, annually from 1871 to 1993. When focusing exclusively on the long-run or zero frequency, the estimated order of integration varies considerably, but nonstationarity is found only for the price/dividend ratio. When the cyclical component is also taken into account, the series appear to be stationary but to exhibit long memory with respect to both components in almost all cases. The exception is the price/dividend ratio, whose order of integration is higher than 0.5 but smaller than 1 for the long-run frequency, and is between 0 and 0.5 for the cyclical component. Also, mean reversion occurs in all cases. Finally, we use six different criteria to compare the forecasting performance of the fractional (at both zero and cyclical frequencies) models with others based on fractional and integer differentiation only at the zero frequency. The results show that the former outperforms the others in a number of cases.

Keywords: *Stock Market, Fractional Cycles, Long Memory, Gegenbauer Processes*

JEL classification: *C22, G12, G14*

Corresponding author: *Professor Guglielmo Maria Caporale, Centre for Empirical Finance, Brunel University, Uxbridge, Middlesex UB8 3PH, UK. Tel.: +44 (0)1895 266713. Fax: +44 (0)1895 269770. E-mail:* Guglielmo-Maria.Caporale@brunel.ac.uk

The Efficient Market Hypothesis (EMH) in its weak form states that it is not possible to trade profitably on the basis of historical stock market prices and/or return information (see Fama, 1970). This proposition has been tested in numerous empirical studies by trying to establish whether stock prices are I(1) and consequently stock market returns I(0) series. This is based on the idea that if stock prices fully reflect available information they should follow a random walk process, which implies unpredictable returns, and rules out systematic profits over and above transaction costs and risk premia. Therefore, a finding of mean reversion in returns is seen as inconsistent with equilibrium asset pricing models (see the survey by Forbes, 1996). Note, however, that if risk factors change systematically over the business cycle, expected returns should also be time-varying. Similarly, allowing for business cycle variation and short-range dependence might also result in rejecting long memory in stock prices (see Lo, 1991). In general, as stressed in Caporale and Gil-Alana (2002), the unit root tests normally employed impose too restrictive assumptions on the behaviour of the series of interest, in addition to having low power. That study suggests instead using tests which allow for fractional alternatives, and finds that US real stock returns are close to being I(0) (which raises the further question whether the shocks are autocorrelated, with the implication that markets are not efficient). Fractional integration models (at the long run or zero frequency) have also been used for inflation and interest rates (see, e.g., Shea, 1991; Backus and Zhin, 1993; Hassler and Wolters, 1995; Baillie et al., 1996, etc.).

However, it has become increasingly clear that the cyclical component of economic and financial series is also very important. This has been widely documented, especially in the case of business cycles, for which non-linear (Beaudry and Koop, 1993, Pesaran and Potter, 1997) or fractionally ARIMA (ARFIMA) models (see Candelon and Gil-Alana, 2004) have been proposed. Furthermore, from a pure time series viewpoint, it has been argued that cycles should be modelled as an additional component to the trend and the seasonal structure of the series (see Harvey, 1985, Gray et al., 1989). The available evidence suggests that the periodicity of the series ranges between five and ten years, in most cases a periodicity of about six years being estimated (see, e.g., Baxter and King, 1999; Canova, 1998; King and Rebelo, 1999).

In view of these findings, the present paper extends the earlier work by Caporale and Gil-Alana (2002) by adopting a modelling approach which, instead of considering exclusively the component affecting the long-run or zero frequency, also takes into account the cyclical structure. Furthermore, the analysis is carried out for the US inflation rate, real risk-free rate, equity premium and price/dividend ratio, in addition to real stock returns. More precisely, we use a procedure, which enables one to test simultaneously for roots with possibly fractional orders of integration at both zero and the cyclical frequencies. This approach, due to Robinson (1994), has several distinguishing features compared with other methods, the most noticeable one being its standard null and local limit distributions.[1] Moreover, it does not require Gaussianity (a condition rarely satisfied in financial time series), a moment condition only of order two being sufficient. Also, modelling simultaneously the zero and the cyclical frequencies can solve at least to some extent the problem of misspecification that might arise with respect to these two frequencies. We are able to show that our proposed method represents an appealing alternative to the increasingly popular ARIMA (ARFIMA) specifications found in the literature. It is also consistent with the widely adopted practice of modelling many economic series as two separate components, namely a secular or growth component and a cyclical one. The former, assumed in most cases to be nonstationary, is thought to be driven by growth factors, such as capital accumulation, population growth and technology improvements, whilst the latter, assumed to be covariance stationary, is generally associated with fundamental factors which are the primary cause of movements in the series.[2]

The structure of the paper is as follows. Section 1 briefly describes the statistical model. Section 2 introduces the tests used for the empirical analysis. Section 3 discusses an application to annual data on the US stock market. Section 4 is concerned with model selection for each time series, and the preferred specifications are compared with other more classical representations. Section 5 contains some concluding comments.

1. The statistical model

Let us suppose that $\{y_t, t = 1, 2, \ldots, n\}$ is the time series we observe, which is generated by the model:

$$(1 - L)^{d_1} \, (1 - 2\cos w\, L + L^2)^{d_2} \, y_t \;=\; u_t, \qquad t \;=\; 1, 2, \ldots, \tag{1}$$

where L is the lag operator ($Ly_t = y_{t-1}$), w is a given real number, u_t is I(0)[3] and d_1 and d_2 can be real numbers. Let us first consider the case of $d_2 = 0$. Then, if $d_1 > 0$, the process is said to be long memory at the long-run or zero frequency, also termed 'strong dependent', because of the strong association between observations widely separated in time. The differencing parameter d_1 plays a crucial role from both economic and statistical viewpoints. Thus, if $d_1 \in (0, 0.5)$, the series is covariance stationary and mean-reverting, with shocks disappearing in the long run; if $d_1 \in [0.5, 1)$, the series is no longer stationary but still mean-reverting, while $d_1 \geq 1$ means nonstationarity and non-mean-reversion. It is therefore crucial to examine if d_1 is smaller than or equal to or higher than 1. For example, if $d_1 < 1$, there is less need for policy action than if $d_1 \geq 1$, since the series will return to its original level some time in the future. On the contrary, if $d_1 \geq 1$, shocks will be permanent, and active policies are required to bring the variable back to its original long-term projection. In fact, this is one of the most hotly debated topics in empirical finance. Lo and MacKinlay (1988) and Poterba and Summers (1988) used variance-ratio tests and found evidence of mean reversion in stock returns. On the contrary, Lo (1991) used a generalised form of rescaled range (R/S) statistic and found no evidence against the random walk hypothesis for the stock indices. Other papers examining the persistence of shocks in financial time series are Lee and Robinson (1996), Fiorentini and Sentana (1998) and May (1999).[4]

Let us now consider the case of $d_1 = 0$ and $d_2 > 0$. The process is then said to exhibit long memory at the cyclical frequency. This model was introduced by Andel (1986) and has been studied, among others, by Gray et al. (1989, 1994), who showed that the series is stationary if $|\cos w| < 1$ and $d_2 < 0.50$ or if $|\cos w| = 1$ and $d_2 < 0.25$.[5] They also showed that the second polynomial in (1) can be expressed in terms of the Gegenbauer polynomial C_{j,d_2}, such that, defining $\mu = \cos w$,

$$(1 - 2\mu L + L^2)^{-d_2} \;=\; \sum_{j=0}^{\infty} C_{j,d_2}(\mu) L^j, \tag{2}$$

for all $d_2 \neq 0$, where

$$C_{j,d_2}(\mu) \;=\; \sum_{k=0}^{[j/2]} \frac{(-1)^k (d_2)_{j-k} (2\mu)^{j-2k}}{k!(j-2k)!}; \qquad (d_2)_j \;=\; \frac{\Gamma(d_2+j)}{\Gamma(d_2)},$$

where $\Gamma(x)$ represents the Gamma function. For a formal treatment of Gegenbauer polynomials, see, for example, Szego (1975). Lildholdt (2002) shows that this model can result from cross-sectional aggregation of certain AR(2) processes, while Bierens (2001) concludes that US real GDP can be well characterised as a model of this form with $d_2 = 1$. These processes, for which the crucial issue is to have a spectral density with a peak at $(0, \pi]$, were later extended to the case of a finite number of peaks by Giraitis and Leipus (1995) and Woodward et al. (1998).

Modelling periodicity in stock market returns has been studied by Andersen and Bollerslev (1997). They found evidence of strong intraday periodicity in return volatility in foreign exchange and equity model markets. To model this kind of phenomenon they noted that the lag-j autocovariance was proportional to $\cos(\lambda j)^{2d-1}$ as $j \to \infty$, which has the long memory property of non-summability. However, these autocovariances also oscillate, changing sign every π/λ lags, a property that is satisfied by the Gegenbauer processes described above. The economic implications in (2) are similar to the previous case of long memory at the zero frequency. Thus, if $d_2 < 1$ and $|\mu| < 1$, or if $d_2 < 0.5$ and $|\mu| = 1$, shocks affecting the cyclical part will be mean reverting (see Gray et al., 1989; Smallwood and Norrbin, 2006), while $d_2 \geq 1$ (with $|\mu| < 1$) implies an infinite degree of persistence of the shocks. This type of model for the cyclical component has not been much used for financial time series, (some recent examples are the papers of Bisaglia et al., 2003, and Smallwood and Norrbin, 2006), though Robinson (2001, pp. 212-213) suggests its adoption in the context of complicated autocovariance structures.

Finally, note that the model in (1) (with $w \neq 0$) encompasses many specifications that have been used in financial time series including ARMA, ARIMA and long memory fractional models. Note that the autocovariances not only decay at a hyperbolic rate typical of ARFIMA models, but also exhibit periodic behaviour associated with the cosine function. This is an important feature of the present model, since unlike fractional or ARIMA models, it can capture strong cyclical characteristics that have been observed in the autocorrelation functions of economic and financial data.

2. The testing procedure

Robinson (1994) adopts the following model:

$$y_t = \beta' z_t + x_t \qquad t = 1, 2, \ldots, \tag{3}$$

where y_t is the observed time series; z_t is a (kx1) vector of deterministic regressors that may include, for example, an intercept, (e.g., $z_t \equiv 1$), or an intercept and a linear time trend (in the case of $z_t = (1,t)^T$); β is a (kx1) vector of unknown parameters; and the regression errors x_t are such that:

$$\rho(L;\theta)\, x_t = u_t \qquad t = 1, 2, \ldots, \tag{4}$$

where ρ is a given function which depends on L, and the (px1) parameter vector θ, adopting the form:

$$\rho(L;\theta) = (1-L)^{d_1+\theta_1}\,(1-L^s)^{d^s+\theta^s}\prod_{j=2}^{p-1}(1-2\cos w\,L+L^2)^{d_j+\theta_j}, \qquad (5)$$

for real given numbers d_1, d^s, d_2, … d_{p-1}, integer p, and where u_t is I(0), and thus it can be specified as white noise or any type of weak autocorrelated structure. Note that the second polynomial in (5) refers to the case of seasonality (i.e. s = 4 in case of quarterly data, and s = 12 with monthly observations). Under the null hypothesis, defined by:

$$H_o:\quad \theta = 0 \qquad (6)$$

(5) becomes:

$$\rho(L;\theta=0) = \rho(L) = (1-L)^{d_1}\,(1-L^s)^{d^s}\prod_{j=2}^{p-1}(1-2\cos w\,L+L^2)^{d_j}. \qquad (7)$$

This is a very general specification that makes it possible to consider different models under the null. In this paper we are concerned with both the long run and the cyclical structure of the series, and thus we assume that $d^s = 0$ and p = 3. In such a case (5) can be expressed as:

$$\rho(L;\theta) = (1-L)^{d_1+\theta_1}\,(1-2\cos w\,L+L^2)^{d_2+\theta_2}, \qquad (8)$$

and, similarly, (7) becomes:

$$\rho(L) = (1-L)^{d_1}\,(1-2\cos w\,L+L^2)^{d_2}. \qquad (9)$$

Here, d_1 represents the degree of integration at the long run or zero frequency (i.e., the stochastic trend), while d_2 affects the cyclical component of the series. The functional form of the test statistic, (denoted by $\hat{R}$) is described in Appendix 1.

Based on H_o (6), Robinson (1994) established that, under certain regularity conditions:[6]

$$\hat{R} \to_d \chi_2^2, \quad as \quad n \to \infty, \qquad (10)$$

where n is the sample size and "$\rightarrow_d$" means convergence in distribution. Thus, as shown by Robinson (1994), unlike in other procedures, we are in a classical large-sample testing situation, and furthermore the tests are efficient in the Pitman sense against local departures from the null.[7] Because $\hat{R}$ involves a ratio of quadratic forms, its exact null distribution could have been calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions: a test of (6) will reject H_o against the alternative H_a: $\theta \neq 0$ if $\hat{R} > \chi^2_{2,\alpha}$, where Prob $(\chi^2_2 > \chi^2_{2,\alpha}) = \alpha$. A similar version of Robinson's (1994) tests (with $d_1 = 0$) was examined in Gil-Alana (2001), where its performance in the context of unit-root cycles was compared with that of the Ahtola and Tiao's (1987) tests, the results showing that the former outperforms the latter in a number of cases. Other versions of his tests have been applied to raw time series in Gil-Alana and Robinson (1997, 2001) to test for I(d) processes with the roots occurring at zero and the seasonal frequencies respectively. However, this is the first empirical finance application testing simultaneously for the roots at zero and the cyclical frequencies, a statistical approach which is shown in the present paper to represent a convenient alternative to the more conventional ARIMA (ARFIMA) specifications used for the parametric modelling of many time series.

3. An empirical application for the US stock market

Our dataset includes annual series for US inflation, real risk-free rate, real stock returns, equity premium and price/dividend ratio from 1871 to 1993, and is a slightly updated version of the dataset used in Cecchetti et al (1990) (see that paper for further details on sources and definitions).[8]

Figure 1 contains plots of the original series with their corresponding correlograms and periodograms. All of them, with the exception of the price/dividend ratio, appear to be stationary. However, deeper inspection of the correlograms shows that there are significant values even at some lags relatively distant from zero, along with slow decay and/or cyclical oscillation in some cases, which could indicate not only fractional integration at the zero frequency but also cyclical dependence. Similarly, the periodograms also have peaks at frequencies other than zero. For the price/dividend ratio, the slow decay in the correlogram clearly suggests that the series is not I(0) stationary.

[Insert Figures 1 and 2 about here]

Figure 2 displays similar plots for the first differenced data. The correlograms and periodograms now strongly suggest that all series are overdifferenced with respect to the 0 frequency. On the other hand, there are significant peaks in the periodograms at frequencies different from zero. In view of this, it might be of interest to examine more in depth the behaviour of these series using a fractional model at both the zero and the cyclical frequencies.

As a first step, we focus exclusively on the long-run frequency and implement a simple version of Robinson's (1994) test, which is based on a model given by (3) and (4), with $z_t = (1,t)^T$, $t \geq 1$, $(0,0)^T$ otherwise, and $\rho(L;\theta) = (1-L)^{d+\theta}$. Thus, under H_o (6), we test the model:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, \ldots \tag{11}$$

$$(1-L)^d x_t = u_t, \qquad t = 1, 2, \ldots, \tag{12}$$

for values d = 0, …, (0.01), …, 2, that is, we test from d = 0 to d = 2 with 0.01 increments, and use different types of disturbances. In such a case, the test statistic greatly simplifies, taking the form given by $\hat{R}$ in Appendix 1, with $\psi(\lambda_s)$ being exclusively defined by $\psi_1(\lambda_s)$ and $\hat{u}_t = (1-L)^d y_t - \hat{\beta}' w_t$. The null limit distribution will then be a χ_1^2 distribution. However, if $\rho(L;\theta) = (1-L)^{d+\theta}$, then p = 1, and therefore we can consider one-sided tests based on $\hat{r} = \sqrt{\hat{R}}$, with a standard N(0,1) distribution. Note that testing the null hypothesis with d = 1 means that this becomes a classical unit-root model of the same form as those proposed by

Dickey and Fuller (1979) and others. However, instead of using autoregressive (AR) structures of the form: $(1 - (1+\theta)L)x_t = u_t$, we use fractional alternatives. Moreover, the use of AR alternatives results in a dramatic change in the asymptotic behaviour of the tests: if $\theta < 0$, x_t is stationary; it contains a unit root if $\theta = 0$, and it becomes nonstationary and explosive for $\theta > 0$. On the contrary, under fractional alternatives of the form as in (12), the behaviour of x_t is smooth across d, this being the intuitive reason for its standard asymptotic behaviour.[9]

Table 1 displays the test results. Note that Robinson's (1994) parametric approach does not require preliminary differencing; thus, it allows us to test any real value *d*, encompassing both stationary and nonstationary hypotheses. The numbers in parentheses are the estimates of *d* obtained with the Whittle function. We also report the 95% confidence bands for the non-rejections of *d*. We examine separately the cases of $\beta_0 = \beta_1 = 0$ a priori (i.e., with no regressors in the undifferenced model (11)); β_0 unknown and $\beta_1 = 0$ (with an intercept); and β_0 and β_1 unknown (an intercept and a linear time trend). The inclusion of a linear time trend may appear unrealistic in the case of financial time series. However, it should be noted that in the context of fractional (or integer) differences, the time trend disappears in the long run. For example, suppose that u_t in (12) is white noise. Then, testing H_o (6) in (11) and (12) with $d = 1$, the series becomes, for $t > 1$, a pure random walk process if $\beta_1 = 0$, and a random walk with an intercept if both β_0 and β_1 are unknown.[10] The results differ substantially from one series to another. For instance, for inflation and the real risk-free rate the values are always higher than 0 but smaller than 0.5, oscillating between 0.07 (inflation rate with a linear trend) and 0.49 (real risk-free rate with no regressors). For real stock returns and equity premium, the values of d for which H_o (6) cannot be rejected oscillate widely around 0, ranging between –0.18 (equity premium with a linear trend) and 0.14 (stock returns with no regressors). Finally, for the price/dividend ratio all the non-rejection values are higher than 0.5, implying nonstationarity with respect to the zero frequency.

[Insert Tables 1 and 2 about here]

The significance of the results in Table 1 may be partly due to the fact that I(0) autocorrelation in u_t has not been taken into account. Thus, we also performed the tests imposing AR(1) disturbances (see Table 2). Higher AR orders were also tried and the results were very similar. For all series, except the price/dividend ratio, the values oscillate around 0, implying that the series may be I(0) stationary. However, for the price/dividend ratio, the values are still above 0, ranging from 0.13 (with a linear time trend) to 0.83 (in the case of no regressors). Comparing the results of Table 2 with those of Table 1, one can see that the orders of integration are smaller by about 0.20 when autocorrelation is allowed for. This might reflect the fact that the estimates of the AR coefficients are Yule-Walker, which entails AR roots that, although automatically less than one in absolute value, can be arbitrarily close to one. Hence, they might compete with the order of integration at the zero frequency when describing the behaviour at such a frequency.[11]

We also examined d, independently of the way of modelling the I(0) disturbances, at the same zero frequency. For this purpose, we used two semiparametric methods: an approximate local Whittle approach (Robinson, 1995), and an exact local Whittle estimator recently proposed by Phillips and Shimotsu (2005). In the two cases the conclusions were very similar: for inflation and the real risk-free rate: some estimates are within the I(0) interval, especially if the bandwidth parameter is small; however, for most of values of that parameter, they are not. For real stock returns and the equity premium almost all values are within the I(0) confidence intervals, but not so for the price/dividend ratio. Also, for the latter series, the values are lower than those within the unit root interval, clearly suggesting that d is greater than 0 but smaller than 1. Therefore, the findings are the same as with the parametric procedure, namely there is strong evidence in favour of I(0) stationarity for real stock returns and the equity premium, some evidence of long memory for inflation and the real risk-free rate, and strong evidence of fractional integration for the price/dividend ratio. Of course, stationarity of stock returns and equity premium is not a surprising result, as the absence of long memory in these two series is a well-established fact in the literature (Lo, 1991; Cheung and Lai, 1995, etc.)

The above approach to investigating the long-run behaviour of time series consists in testing a parametric model for the series and estimating two semiparametric ones, relying on the long run-implications of the estimated models. The advantage of the first procedure is the precision gained by providing all the information about the series through the parameter estimates. A drawback is that these estimates are sensitive to the class of models considered, and may be misleading because of misspecification. It is well known that the issue of misspecification can never be settled conclusively in the case of parametric (or even semiparametric) models. However, the problem can be partly addressed by considering a larger class of models. This is the approach used in what follows, where we employ another version of the tests of Robinson (1994) that enables us simultaneously to consider roots at zero and the cyclical frequencies.[12]

Before discussing the test results we describe a small Monte Carlo experiment we have carried out to examine the power properties of the procedure employed below. We suppose that the true model is given by equation (1) with $d_1 = 0.7$; $d_2 = 0.1$ and $w = w_r = 2\pi/6$, implying long memory and nonstationarity at the long run frequency, stationary long memory behaviour of the cyclical component, and cycles with a periodicity of about 6 periods. We also assume that u_t is white noise, though similar conclusions were obtained under weak autocorrelation for the error term.

We perform the procedure described in Section 3, testing the null hypothesis for d_{1o}-values equal to 0, 0.1, …, 2, and d_{2o} = -0.5, -0.4, …, 1.5, and r = 6, for sample sizes T = 120, 240, 360, 480 and 960 observations. We generated Gaussian series using the routines GASDEV and RAN3 of Press, Flannery, Teukolsky and Vetterling (1986), with 10,000 replications in each case.

Table 3 reports the rejection frequencies of the testing procedure at the 5% significance level. Thus, the values corresponding to $d_{1o} = 0.7$ and $d_{2o} = 0.1$ indicate the size of the test.

[Insert Table 3 about here]

One can see that if the sample size is small (e.g. T = 120) the size of the test is slightly above its nominal value, though it approximates the 5% level with T. Looking at local departures from the null (e.g., $d_1 = 0.6$ & $d_2 = 0.1$, and $d_1 = 0.8$ & $d_2 = 0.1$), one finds that the rejection frequencies with T = 120 are 0.202 and 0.207 respectively. For T = 240 the corresponding values are 0.392 and 0.334, and, for T = 480 or 960, they are higher than 0.9 in all cases. For the remaining departures from the null, the rejection probabilities are higher than 0.9 in practically all cases, even for small sample sizes. Similar conclusions were reached with other values of d_1 and d_2.[13]

The procedure is then applied to the five series under examination. For this purpose, let us consider now the model given by (3) and (4), with $\rho(L; \theta)$ as in (8) and $z_t = (1,t)^T$. Thus, under H_o (6), the model becomes:

$$y_t = \beta_0 + \beta_1 t + x_t, \qquad t = 1, 2, ... \qquad (13)$$

$$(1 - L)^{d_1} (1 - 2\cos w L + L^2)^{d_2} x_t = u_t, \qquad t = 1, 2, ..., \qquad (14)$$

and, if $d_2 = 0$, the model reduces to the case previously studied of long memory exclusively at the long-run or zero frequency. We assume that $w = w_r = 2\pi j/n$, $j = n/r$, and r indicating the number of time periods per cycle.

[Insert Table 4 about here]

We first computed the statistic $\hat{R}$ given in Appendix 1 for values of d_1 and d_2 = -0.50, …, (0.10), …, 2, and r = 2, …, (1), …, n/2,[14] assuming that u_t is white noise. For brevity, we do not report the results for all statistics. In brief, the null hypothesis (6) was rejected for all values of d_1 and d_2 if r was smaller than 4 or higher than 9, implying that, if a cyclical component is present, its periodicity is constrained to be between 4 and 9 years. This is consistent with the empirical finding in Canova (1998), Burnside (1998), King and Rebelo (1999) and others that cycles have a periodicity between five and ten years. We report in Table 4 the non-rejection cases at the 5% level only for the case of an intercept and r = 6. The results for the case of a linear time trend were very similar, and the coefficient corresponding to the trend was found to be insignificantly different from zero in virtually all cases. Note that the test statistic is obtained from the null differenced model, which is assumed to be I(0), and therefore standard t-tests apply. Further, we focus on r = 6 since the non-rejection values with r = 4, 5, 7, 8 and 9 formed a proper subset of those non-rejections obtained with r = 6. We see that for inflation and the real risk-free rate the non-rejection values oscillate between 0.10 and 0.40 for d_1, and between 0 and 0.3 for d_2. They are slightly smaller for d_2 in the case of stock returns and the equity premium, in some cases even being negative. Finally, for the price/dividend ratio, the values of d_1 range between 0.5 and 1, while d_2 seems to be constrained between 0 and 0.5.[15]

[Insert Figure 3 about here]

In order to have a more precise view about the non-rejection values of d_1 and d_2, we re-computed the tests but this time for a shorter grid, with d_1, d_2 = -0.50, …, (0.01), …, 2. Figure 3 displays the regions of (d_1, d_2) values where H_o cannot be rejected at the 5% level. It shows that the combination of non-rejection (d_{1o}, d_{2o})-values form clusters, though there are also some values away from the clusters in four of the five series examined. These values of the statistics are in fact close to the critical values of the c_2^2- distribution. Essentially, the series can be grouped into three categories: inflation and the real risk-free rate; real stock returns and the equity premium; finally, the price/dividend ratio. Starting with the first group (inflation and the real risk-free rate), we observe that the values of d_1 range between 0.1 and 0.5 while d_2 seems to lie between 0 and 0.3. Thus, there appears to be a slightly higher degree of integration at the long-run or zero frequency compared to the cyclical one. For real stock returns and equity premium, the values of both orders of integration oscillate around 0. Finally, for the price/dividend ratio the values of d_1 range between 0.5 and 1, while d_2 is between 0 and 0.5, implying nonstationarity with respect to the zero frequency but stationarity with respect to the cyclical component, and mean reversion with respect to both. Consequently, shocks to the latter series will disappear in the long run, with those affecting the cyclical part tending to disappear faster than those affecting its long-run or trending behaviour. This procedure was also applied in the context of autocorrelated (AR(1) and AR(2)) disturbances and the results did not substantially differ from those reported here based on white noise u_t. In the AR(1) case, the AR parameter was not significantly different from zero for most series. The only exception was the price/dividend ratio, for which values of d_1 close to zero are obtained for an AR parameter close to one, suggesting once more that the order of integration at the zero frequency and the AR parameter are in competition. When using AR(2) disturbances the results were again very similar, though with larger regions for the (d_1, d_2)- non-rejection values.

4. Forecasting and comparisons with other models

In this section, we try first to determine the best model specification for each time series. Then, we compare the selected models with other approaches based on I(0) and I(1) hypotheses.

Given the lack of efficient procedures for estimating the parameters in the model in (13) and (14), we use the following strategy: after computing the values of the test statistic for d_1, d_2 = -0.50, …, (0.01), …, 2 and r = 2, …, (1), .., n/2, for the three cases of no regressors, an intercept and an intercept with a linear time trend, we discriminate between these three cases on the basis of the significance of the estimated coefficients in (13), and choose the values of d_1, d_2 and r which produce the lowest statistic. Note that, for each r, the values of d_1 and d_2 producing the lowest statistic should be an approximation to the maximum likelihood estimates since the procedure employed in the paper is based on the LM principle and uses the Whittle function, which is an approximation to the likelihood function. The selected model for each time series is reported in the second column of Table 5. We find that, for the inflation rate and the real risk-free rate, both orders of integration are between 0.10 and 0.30, the order of integration at zero being slightly higher than the cyclical one; for real stock returns and the equity premium, the values of the d's are close to zero, being slightly negative for the zero frequency; finally, the price-dividend ratio appears to be nonstationary at the long-run frequency (d_1 = 0.68), and stationary with d_2 close to zero (d_2 = 0.09) for the cyclical component. Note that in this case all models are based on white noise disturbances, the reason being that, as mentioned in the previous section, the inclusion of autocorrelated disturbances did not alter the conclusions except for the price/dividend ratio - for this series the associated AR coefficient was very close to one, thus making the estimate of d_1 invalid. Moreover, the cyclical fractional polynomial can be considered as an alternative to the ARMA specification when describing the short-run dynamics of the series.

[Insert Table 5 about here]

The third column of the table reports the selected models taking into account only the component affecting the long run or zero frequency, while the fourth refers to the case of integer differentiation with respect to such a frequency. In both cases, we model the cyclical structure using ARMA specifications. Starting with the case of fractional integration, we observe that the highest degree of integration is obtained for the price/dividend ratio (d = 0.73), followed by inflation (d = 0.19). For the remaining three series, the values are practically zero (0.04 for the real risk-free rate; 0.01 for real stock returns, and –0.04 for the equity premium). Here we have followed the same strategy as in the fractional cyclical case, i.e., testing sequentially for a grid of values of d_1, and then choosing the value that produces the lowest statistic in absolute value.[16,17] Imposing integer orders of integration, for the first four variables, we use d = 0 while for the price-dividend ratio we try both d = 0 and 1. For the short-run components we use ARMA(p, q) models, with p, q ≤ 3, and choose the best specification using both LR tests and likelihood criteria (AIC, BIC). We see that, for most of the series, the short-run structure can be described by simple MA models, the only exceptions being the real risk-free rate where an AR(1) process is imposed, and the inflation rate (ARMA(2,1)).

Next, we compare the various models in terms of their forecasting performance. Standard measures of forecast accuracy are the following: Theil's U, the mean absolute percentage error (MAPE), the mean-squared error (MSE), the root-mean-squared error (RMSE), the root-mean-percentage-squared error (RMPSE) and mean absolute deviation (MAD) (Witt and Witt, 1992). These measures are described in Appendix 2.

The three selected time series models (fractional and cyclical differencing, FCD; fractional differencing, FD; and integer differencing, ID) for each of the series were used to generate the following 5-year-ahead out-of-sample forecasts. Each forecast value was calculated and compared with the actual value of the series. Then, the above six criteria were used to rank the three forecasting models for each series. The ranking in terms of forecasting performance is given in Table 6, and is based on the average value of the forecasts for each criterion. We observe that for inflation and the real risk-free rate the FCD model outperforms FD and ID according to all the criteria. For real stock returns and the equity premium, the ID specification seems to be the most adequate, while for the price/dividend ratio the results are mixed. Therefore, on the basis of the MAPE, MSE, RMPSE and RMSE criteria, the fractional and cyclical (FCD) model emerges as the best specification, while the other two criteria, MAD and Theil's U, suggest that the simple fractional model (with d = 0.73) is the most adequate one.

[Insert Table 6 about here]

In Table 7 we focus on the forecasts for inflation and the price/dividend ratio over a longer time-horizon. The reason for focusing on these two series is that they are the two that clearly exhibit non-zero (and fractional) degrees of integration. We consider the forecasting performance of the three types of models discussed above (FCD, FD and ID) over the period 1979 – 1993, based on specifying and estimating the models over the time period 1871 – 1978. The new selected models are displayed in Table 7 and we observe that they are very similar to those presented in Table 5.

[Insert Tables 7 and 8 about here]

Table 8 reports the MSE forecasts for the two series, using the time horizons h = 1, 3, 6, 9, 12 and 15. We observe that for the two series in many cases the lowest MSEs are obtained with the fractional cyclical models. However, the MSE measure used for comparing the relative forecasting performance of our models is a purely descriptive device. There exist several statistical tests for comparing different forecasting models. One of these tests, widely employed in the time series literature, is the asymptotic test for a zero expected loss differential of Diebold and Mariano (1995).[18] However, Harvey, Leybourne and Newbold (1997) note that the Diebold-Mariano test statistic could be seriously over-sized as the prediction horizon increases, and therefore provide a modified Diebold-Mariano test statistic given by:

$$M - DM \; = \; DM \sqrt{\frac{n + 1 - 2h + h(h-1)/n}{n}},$$

where DM is the original Diebold-Mariano statistic, h is the prediction horizon and n is the time span for the predictions. Harvey et al. (1997) and Clark and McCracken (2001) show that this modified test statistic performs better than the DM test statistic (though still poorly in finite samples), and also that the power of the test is improved when p-values are computed with a Student distribution.

Using the M-DM test statistic, we further evaluate the relative forecast performance of the different models by making pairwise comparisons. In Table 8 we indicate with an asterisk, for each prediction-horizon, the rejections of the null hypothesis that the forecast performance of model i and j is equal in favour of the one-sided alternative that model i's performance is superior at the 5% significance level.[19] Given the fact that we have three potential models for each prediction and we make pairwise comparisons, only the preferred model - when there is consistency for all three specifications - is indicated with an asterisk, cases not being chracterised by consistency being left out. We note here that over long horizons the fractional cyclical model produces for both series significantly superior forecasts. Similar results were obtained when using other sets of forecasts based on rolling window statistics.

5. Conclusions

In this paper we have examined the time series behaviour of five series related to the US stock market by means of statistical techniques based on long memory processes. Specifically, we have used a procedure that has enabled us to test for unit roots with integer or fractional orders of integration, not only at zero but also at the cyclical frequencies. These tests have standard null and local limit distributions and can easily be applied to raw time series.[20]

Initially, we focused only on the long-run or zero frequency, applying a suitable version of Robinson's (1994) parametric tests along with various semiparametric estimation procedures. We used these methods because of the distinguishing features that make them particularly relevant in the context of financial time series. Specifically, they do not require Gaussianity (which is an assumption that is not satisfied by most financial series), but only a moment condition of order two. Additionally, they have standard null limit distributions, which is another advantage of these tests compared to other procedures based on AR alternatives. The order of integration estimated using these methods varies considerably from one series to another, but nonstationarity is found only in the case of the price/dividend ratio.

However, the non-rejection values obtained at the zero frequency could be partly due to the fact that attention has not been paid to other possible (cyclical) frequencies of the process. Thus, we adopted a method suitable for simultaneously testing for the presence of roots at the zero and the cyclical frequencies. The results suggest that the periodicity of the series ranges between 5 and 10 years, which is consistent with most of the empirical literature on cycles finding a periodicity of about six years (see, e.g., Baxter and King, 1999, Canova, 1998, and King and Rebelo, 1999). Further, the series can be grouped into three different categories: inflation and the real risk-free rate, with the order of integration at the zero frequency fluctuating between 0 and 0.5 and d_2 (cyclical integration) between 0 and 0.3; real stock returns and the equity premium, with both orders of integration fluctuating around 0; and finally, the price/dividend ratio, with d_1 ranging between 0.5 and 1 and d_2 between 0 and 0.5. Thus, we found evidence of stationary long memory with respect to both components for inflation and the real risk-free rate; I(0) stationarity for stock returns and the equity premium; and nonstationary long memory at the zero frequency but stationarity at the cyclical component for the price/dividend ratio. Finally, the fact that all orders of integration are smaller than 1 suggests that mean reversion takes place with respect to both components for all series, though the rate of adjustment varies across them.

A criticism that could be made of this type of model for the cyclical component is that, unlike seasonal cycles, business cycles are typically weak and irregular and are spread evenly over a range of frequencies rather than peaking at a specific value. A strong counterargument is that, in spite of the fixed frequencies used in this specification, flexibility can be achieved through the first differenced polynomial, the ARMA components and the error term. In fact, Bierens (2001) uses a model of this kind (with $d_2 = 1$) to test for the presence of business cycles in the annual change of monthly unemployment in the UK. Our analysis also yields clear-cut results, which are consistent with earlier findings on the periodicity of cycles.

The selected models for each time series were then compared with other approaches based on fractional and integer differentiation at the zero frequency. Six forecasting criteria were employed and the results showed that the fractional cyclical model outperforms the others in a number of cases.

Clearly, for the sample period examined in this study, structural breaks could also be an issue. Note that fractional integration and structural break are issues which are intimately related (see Bos et al., 1999; Diebold and Inoue, 2001; Granger and Hyung, 2004; Gil-Alana, 2007). However, a theoretical framework for structural breaks and fractional integration at both the zero and the cyclical frequencies has yet to be developed.

It would also be worthwhile to obtain point estimates of the fractional differencing parameters in this context of trends and cyclical models. For the trending component the literature is vast (see, e.g., Fox and Taqqu, 1986; Dahlhaus, 1989; Sowell, 1992; Robinson, 1995; Tanaka, 1999; Phillips and Shimotsu, 2005; Mayoral, 2007 etc.). For the cyclical part, there are fewer contributions such as Arteche and Robinson (2000), Arteche (2002) and Dalla and Hidalgo (2005) and no likelihood estimation methods have been proposed for the joint estimation of the two orders of integration. However, the goal of this paper is to show that a model with fractional orders of integration at both the zero and the cyclical frequencies can be a credible alternative to the conventional ARIMA (ARFIMA) specifications. In fact, our approach produces unambiguous results, with the periodicity ranging between 4 and 10 years and most of the orders of integration within the intervals (0, 0.5) and (0.5, 1) depending on the series and the component under study.

Further research could be carried out using this framework. For instance, the tests can be extended to allow for more than one cyclical component. The existence of multiple cycles in financial series has not yet been examined empirically, and might be of interest in the context of various latent variates. Note that the periodograms displayed in Figures 1 and 2 show in some cases multiple peaks at the cyclical frequencies. However, for real stock returns and equity premium, the estimated order of integration at the zero frequency is extremely close to 0 and the periodograms of the original data (in Figure 1) exhibit a single clear significant peak. Moreover, for the remaining three series the orders of integration at the long run frequency range between 0 and 1, and the periodograms of the first differenced data (in Figure 2) clearly show that type of behaviour along with a single peak at the non-zero frequency. Further, daily data could also be used to examine intraday periodicity, e.g. in the volatility of asset returns. As an alternative to the cyclical fractional approach, Andersen and Bollerslev (1997) modelled periodicity in returns by means of deterministic weights. The inclusion of deterministic components is possible in Robinson's (1994) set-up, and its significance can be tested by means of a joint test of the deterministic regressors and of the order of integration. The univariate nature of the present study is also a limitation in terms of theorising, policy-making or forecasting. Theoretical models and policy-making involve relationships between many variables, and forecast performance can be improved through the use of many variables (e.g., factor-based forecasts based on hundreds of time series beat univariate forecasts, as shown, e.g., in Stock and Watson, 2002). However, the univariate approach taken in the present paper is useful, as it enables one to decompose the series into a long-run and a cyclical component. Moreover, theoretical econometric models for both long-run and cyclical fractional structures in a multivariate framework are not yet available. In this respect, the present study can be seen as a preliminary step in the analysis of financial data from a different time series perspective. Of particular interest in future work would be a more extensive study of the out-of-sample forecasting performance of our preferred model. In order to increase the number of out-of-sample observations and gain power, a rolling design (e.g. McCracken, 2000) with larger samples could be used. Data mining is an additional relevant issue worth exploring.

Appendix 1

We observe $\{(y_t, z_t), t = 1,2,\ldots n\}$, and suppose that the I(0) u_t in (4) have parametric spectral density given by:

$$f(\lambda;\tau) = \frac{\sigma^2}{2\pi}\, g(\lambda;\tau), \qquad -\pi < \lambda \leq \pi,$$

where the scalar σ^2 is known and g is a function of known form, which depends on frequency λ and the unknown (qx1) vector τ. Based on H_o (6), the residuals in (3), (4) and (9) are:

$$\hat{u}_t = (1-L)^{d_1}(1-2\cos w L + L^2)^{d_2}\, y_t - \hat{\beta}' s_t,$$

where

$$\hat{\beta} = \left(\sum_{t=1}^{n} s_t s_t'\right)^{-1} \sum_{t=1}^{n} s_t (1-L)^{d_1}(1-2\cos w L + L^2)^{d_2}\, y_t, \qquad s_t = (1-L)^{d_1}(1-2\cos w L + L^2)^{d_2}\, z_t.$$

Unless g is a completely known function (e.g., $g \equiv 1$, as when u_t is white noise), we need to estimate the nuisance parameter τ, for example by $\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau)$, where T^* is a suitable compact subset of R^q Euclidean space, and

$$\sigma^2(\tau) = \frac{2\pi}{n}\sum_{s=1}^{n-1} g(\lambda_s;\tau)^{-1} I_{\hat{u}}(\lambda_s), \quad \text{with} \quad I_{\hat{u}}(\lambda_s) = \left|(2\pi n)^{-1/2}\sum_{t=1}^{n}\hat{u}_t e^{i\lambda_s t}\right|^2; \qquad \lambda_s = \frac{2\pi s}{n}.$$

The test statistic, which is derived through the Lagrange Multiplier (LM) principle, takes the form:

$$\hat{R} = \hat{r}'\hat{r}; \quad \hat{r} = \left(\frac{\sqrt{n}}{\hat{\sigma}^2}\right)\hat{A}^{-1/2}\,\hat{a},$$

where n is the sample size, and

$$\hat{a} = \frac{-2\pi}{n}\sum_{s}^{*}\psi(\lambda_s)\, g(\lambda_s;\hat{\tau})^{-1} I(\lambda_s); \qquad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{n}\sum_{s=1}^{n-1} g(\lambda_s;\hat{\tau})^{-1} I(\lambda_s),$$

$$\hat{A} = \frac{2}{n}\left(\sum_{s}^{*} \psi(\lambda_s)\psi(\lambda_s)' - \sum_{s}^{*} \psi(\lambda_s)\hat{\varepsilon}(\lambda_s)' \left(\sum_{s}^{*} \hat{\varepsilon}(\lambda_s)\hat{\varepsilon}(\lambda_s)' \right)^{-1} \sum_{s}^{*} \hat{\varepsilon}(\lambda_s)\psi(\lambda_s)' \right)$$

$$\psi(\lambda_s)' = [\psi_1(\lambda_s), \psi_2(\lambda_s)]; \qquad \hat{\varepsilon}(\lambda_s) = \frac{\partial}{\partial \tau} \log g(\lambda_s; \hat{\tau});$$

$$\psi_1(\lambda_s) = \log\left| 2 \sin \frac{\lambda_s}{2} \right|; \qquad \psi_2(\lambda_s) = \log\left| 2\left(\cos \lambda_s - \cos w\right) \right|,$$

and the sums in $\hat{a}$ and $\hat{A}$ in the above expressions are over all frequencies except those which are unbounded.

Appendix 2

Let y_t be the actual value in period t; f_t the forecast value in period t, and n the number of periods used in the calculation. Then:

a) Theil's U: $\dfrac{\sqrt{\sum \left(y_t - f_f\right)^2}}{\sqrt{\sum \left(x_t - x_{t-1}\right)^2}}$;

b) Mean absolute percentage error (MAPE): $\dfrac{\sum \left| (x_t - f_t)/x_t \right|}{n}$;

c) Mean squared error (MSE): $\dfrac{\sum \left(x_t - f_t\right)^2}{n}$;

d) Root-mean-percentage-squared error (RMPSE): $\sqrt{\dfrac{\sum \left(x_t - f_t\right)^2 / f_t}{n}}$;

e) Root-mean-squared error (RMSE): $\sqrt{\dfrac{\sum \left(x_t - f_t\right)^2}{n}}$;

f) Mean absolute deviation (MAD): $\dfrac{\sum \left| x_t - f_t \right|}{n}$.

Footnotes

1. Note that, for example, most of the "classic" unit root tests (i.e., Dickey and Fuller, 1979; Phillips and Perron, 1988; Kwiatkowski et al., 1992; etc.) are non-standard, in the sense that the critical values have to be calculated numerically on a case-by-case simulation approach.

2. Note that, although the model presented in Section 1 only has a single innovation term, this is obtained by combining two fractional processes, one for the long run and the other for the cyclical structure.

3. For the purposes of the present paper, we define an I(0) process as a covariance stationary process with spectral density function that is positive and finite at any frequency.

4. In the context of financial data, Peters (1994) defined the Fractional Market Hypothesis for modelling long-term dependence features in financial time series.

5. Estimation methods in this context have been proposed by Chung (1996a,b) and more recently by Dalla and Hidalgo (2005).

6. These conditions are very mild and concern technical assumptions to be satisfied by $\psi_1(\lambda)$ and $\psi_2(\lambda)$.

7. In other words, if the tests are implemented against local departures of the form: H_a: $\theta = \delta n^{-1/2}$, for $\delta \neq 0$, the limit distribution is a $\chi_2^2(v)$ with a non-centrality parameter v, which is optimal under Gaussianity of u_t.

8. It might be argued that the number of observations used in this application is not sufficiently large to justify the use of fractional integration methods. However, in Gil-Alana and Robinson (1997) similar methods to those employed here were successfully applied to macroeconomic series of smaller sample sizes than in the present study.

9. It is well know that in finite samples it is difficult to distinguish between short and long memory processes, many tests not being sufficiently powerful. Agiakloglou, Newbold and

Wohar (1993) and Agiakloglou and Newbold (1993) show that it is difficult to detect long memory in the presence of AR and MA processes. However, several Monte Carlo experiments conducted in Robinson (1994) suggest that his tests have enough power to detect long memory with weak parametric autocorrelation even with small samples.

10. See Robinson and Iacone (2005) for a recent paper on fractional integration (and cointegration) in the context of deterministic trends.

11. The estimates of d at the long run or zero frequency were also computed using other procedures like Sowell's (1992) maximum likelihood estimation in the time domain, and the results were completely in line with those reported here.

12. Moreover, if cyclical components are present in the series and we do not take them into account, the estimation of d at the zero frequency may create biases in favour of long memory. (See, e.g., Montanari, Rosso and Taqqu, 1996, 1997).

13. If $r = 5$ or 7, with $T = 120$, the rejection frequencies are found to be low for departures close to the null, though when increasing the sample size they tend to 1 in all cases.

14. Note that, in the case of $r = 1$, the model reduces to the case previously studied of long memory exclusively at the long-run frequency.

15. It should be noted that, although $d_2 = 0$ cannot be statistically rejected in most cases, in general, it is "less clearly non-rejected" than for positive values of d_2. (By "less clearly non-rejected" we mean that the value of the test statistic is closer to the critical value. See the results in Table 4).

16. We discriminate between the white noise and the AR specification by looking at the significance of the AR parameter: if it is statistiscally close to 0 or 1, we choose the white noise model for u_t. In fact, this is what we have done for the equity premium and the price/dividend ratio. Also, note that for the real risk-free rate, the inclusion of AR disturbances substantially reduces the order of integration at the zero frequency (from 0.25 in the FCD model to 0.04 in FD).

17. Moreover, the use of standard criteria such as AIC and BIC is not necessarily optimal for applications involving fractional differences, as these criteria focus on the short-term forecasting ability of the fitted model and may not give sufficient attention to the long-run properties of the ARFIMA models (see, e.g. Hosking, 1981, 1984). Another recent paper about model selection in the presence of long and short memory processes is Beran et al. (1998). They propose versions of the AIC, BIC and the HQ (Hannan and Quinn, 1979) which are suitable for fractional autoregressions, but do not consider MA components.

18. An alternative approach is the bootstrap-based test of Ashley (1998), though this method is computationally more intensive.

19. Note that, since the forecasts are measured by MSE, the quadratic loss function is $g(e_{it|t-h}) = e^2_{it|t-h}$. Similar results were obtained when using the absolute values, $\left|e_{it|t-h}\right|$.

20. A diskette containing the FORTRAN programs is available from the authors upon request.

References

Agiakloglou, C., P. Newbold and M. Wohar, 1993, Bias in an estimator of the fractional difference parameter, Journal of Time Series Analysis 14, 235-246.

Agiakloglou, C. and P. Newbold, 1993, Lagrange Multiplier tests for fractional difference, Journal of Time Series Analysis 15, 253-262.

Ahtola, J.. and Tiao, G.C., 1987, Distributions of least squares estimators of autoregressive parameters for a process with complex roots on the unit circle, Journal of Time Series Analysis 8, 1-14.

Andel, J., 1986, Long memory time series models, Kybernetika 22, 105-123.

Andersen, T.G. and T. Bollerslev, 1997, Heterogeneous information arrivals of return volatility dynamics: increasing the long run in high frequency returns, Journal of Finance 52, 975-1006.

Arteche, J., 2002, Semiparametric robust tests on seasonal and cyclical long memory series, Journal of Time Series Analysis 23, 1-35.

Arteche, J. and P.M. Robinson, 2000, Semiparametric inference in seasonal and cyclical long memory processes, Journal of Time Series Analysis 21, 1-25.

Ashley, R., 1998, A new technique for postsample model selection and validation, Journal of Economics Dynamics and Control 22, 647-665.

Backus, D. and S. Zin, 1993, Long memory inflation uncertainty: Evidence of term structure of interest rate, Journal of Money, Credit and Banking 25, 687-700.

Baillie, R.T., C-F. Chung and M.A. Tieslau, 1996, Analysing inflation by the fractionally integrated ARFIMA-GARCH model, Journal of Applied Econometrics 11, 23-40.

Baxter, M. and R.G. King, 1999, Measuring business cycles approximate band-pass filters for economic time series, The Review of Economics and Statistics 81, 575-593.

Beaudry P. and G. Koop, 1993, Do recessions permanently change output?, Journal of Monetary Economics 31, 149-163.

Beran, J., Bhansali, R.J. and Ocker, D., 1998, On unified model selection for stationary and nonstationary short- and long-memory autoregressive processes, Biometrika 85, 921-934.

Bierens, H.J., 2001, Complex unit roots and business cycles. Are they real?, Econometric Theory 17, 962-983.

Bisaglia, L., S. Bordignon and F. Lisi, 2003, k-factor GARMA models for intraday volatility forecasting, Applied Economics Letters 10, 251-254.

Bos, C., Franses, P.H. and Ooms, M., 1999, Long memory and level shifts: Reanalyzing inflation rates, Empirical Economics 24, 427-449.

Burnside, A.C., 1998, Detrending and business cycle facts. A comment, Journal of Monetary Economics 41, 513-532.

Candelon, B. and L. A. Gil-Alana, 2004, Fractional integration and business cycle features, Empirical Economics 29, 1-17.

Canova, F., 1998, Detrending and business cycle facts. A user's guide, Journal of Monetary Economics 41, 533-540.

Caporale, G.M. and L.A. Gil-Alana, 2002, Fractional integration and mean reversion in stock prices, Quarterly Review of Economics and Finance 42, 599-609.

Cecchetti, S.G., Lam, P.-S. and N.C. Mark, 1990, Mean reversion in equilibrium asset prices, American Economic Review 80, 398-418.

Chung, C.-F., 1996a, A generalized fractionally integrated autoregressive moving-average process, Journal of Time Series Analysis 17, 111-140.

Chung, C.-F., 1996b, Estimating a generalized long memory process, Journal of Econometrics 73, 237-259.

Clark, T.E. and M.W. McCracken, 2001, Tests of forecast accuracy and encompassing for nested models, Journal of Econometrics 105, 85-110.

Dahlhaus, R., 1989, Efficient parameter estimation for self-similar process, Annals of Statistics 17, 1749-1766.

Dalla, V. and J. Hidalgo, 2005, A parametric bootstrap test for cycles, Journal of Econometrics 129, 219-261.

Dickey, D. A. and W. A. Fuller, 1979, Distribution of the estimators for autoregressive time series with a unit root, Journal of the American Statistical Association 74, 427-431.

Diebold, F.X. and A. Inoue, 2001, Long memory and regime switching, Journal of Econometrics 105, 131-159.

Diebold, F.X. and R.S. Mariano, 1995, Comparing predictive accuracy, Journal of Business, Economics and Statistics 13, 253-263.

Fama, E.F. (1970), “Efficient capital markets: a review of theory and empirical work”, Journal of Finance, 25, 383-417.

Fiorentini, G. and E. Sentana, 1998, Conditional means of time series processes and time series processes for conditional means, International Economic Review 39, 1101-1118.

Forbes, W.P., 1996, Picking winners? A survey of the mean reversion and overreaction of stock price literature, Journal of Economic Surveys 10, 123-158.

Fox, R. and M.S. Taqqu, 1986, Large-sample properties of parameter estimates for strongly dependent stationary Gaussian time series, Annals of Statistics 14, 517-532.

Gil-Alana, L.A., 2001, Testing stochastic cycles in macroeconomic time series, Journal of Time Series Analysis 22, 411-430.

Gil-Alana, L.A., 2007, Fractional integration and structural breaks at unknown points in time, forthcoming in Journal of Time Series Analysis.

Gil-Alana, L.A. and P.M. Robinson, 1997, Testing of unit roots and other nonstationary hypotheses in macroeconomic time series, Journal of Econometrics 80, 241-268.

Gil-Alana, L.A. and P.M. Robinson , 2001, Testing seasonal fractional integration in the UK and Japanese consumption and income, Journal of Applied Econometrics 16, 95-114.

Giraitis, L. and P. Leipus, 1995, A generalized fractionally differencing approach in long memory modelling, Lithuanian Mathematical Journal 35, 65-81.

Granger, C.W.J. and Hyung, N., 2004, Occasional structural break and long memory with an application to the S&P 500 absolute stock returns, Journal of Empirical Finance 11, 399-421.

Gray, H.L., Yhang, N. and Woodward, W.A., 1989, On generalized fractional processes, Journal of Time Series Analysis 10, 233-257.

Gray, H.L., Yhang, N. and Woodward, W.A., 1994, On generalized fractional processes. A correction, Journal of Time Series Analysis 15, 561-562.

Hannan, E.J. and Quinn, B.G., 1979, The determination of the order of an autoregression, Journal of the Royal Statistical Society, Series B. 41, 190-195.

Harvey, A., 1985, Trends and cycles in macroeconomic time series, Journal of Business and Economics Statistics 3, 216-227.

Harvey, D.I., S.J. Leybourne and P. Newbold, 1997, Testing the equality of prediction mean squared errors, International Journal of Forecasting 13, 281-291.

Hassler, U. and J. Wolters, 1995, Long memory in inflation rates. International evidence, Journal of Business and Economic Statistics 13, 37-45.

Hosking, J.R.M., 1981, Fractional differencing, Biometrika 68, 165-176.

Hosking, J.R.M., 1984, Modelling persistence in hydrological time series using fractional differencing, Water Resources Research 20, 1898-1908.

King, R.G. and S.T. Rebelo, 1999, Resuscitating real business cycles, in J.B. Taylor and M. Woodford eds., Handbook in Macroeconomics, Vol. 1, 928-1001.

Kwiatkowski, D., P. C. B. Phillips, P. Schmidt, and Y. Shin, 1992, Testing the null hypothesis of stationarity against the alternative of a unit root, Journal of Econometrics 54, 159-178.

Lee, D.K.C. and P.M. Robinson, 1996, Semiparametric exploration of long memory in stock prices, Journal of Statistical Planning and Inference 50, 155-174.

Lildholdt, P., 2002, Sources of seasonal fractional integration in macroeconomic time series, CAF Working Paper Series n.125, University of Aarhus.

Lo, A., 1991, Long term memory in stock prices, Econometrica 59, 1279-1313.

Lo, A, and A.C. MacKinlay, 1988, Stock prices do not follow a random walk. Evidence from a simple specification test, Review of Financial Studies 1, 41-66.

Marinucci, D. and P.M Robinson, 1999, Semiparametric Fractional Cointegration Models, Journal of Statistical Planning and Inference 80, 111-122.

May, C.T., 1999, Non-linear Pricing. Theory and Practise, John Wiley & Sons Inc. New York.

Mayoral, L., 2007, Minimum distance estimation of stationary and nonstationary ARFIMA processes, Econometrics Journal 10, 124-148.

McCracken, M.W., 2000, Robust out-of-sample inference, Journal of Econometrics 99, 195-223.

Montanari, A. Rosso, R. and M.S. Taqqu, 1996, Some long run properties of rainfall records in Italy, Journal of Geophysical Resources_ Atmosphere 101, 431-438.

Montanari, A., Rosso, R. and M.S. Taqqu, 1997, Fractionally differenced ARIMA models applied to hydrological time series. Identification, estimation and simulation, Water Resources Research 33, 1033-1044.

Pesaran, M.H. and S.M Potter, 1997, A floor and ceiling model of US output, Journal of Economics Dynamics and Control 21, 661-695.

Peters, E.E., 1994, Fractal Market Hypothesis, Applying chaos theory to investment and economics, New York, John Wiley and Sons.

Phillips, P.C.B. and P. Perron, 1988, Testing for unit root in time series regression, Biometrika 75, 335-346.

Phillips, P.C.B. and K. Shimotsu, 2005, Exact local Whittle estimation of fractional integration, Annals of Statistics 32, 656-692.

Poterba, J. and L. Summers, 1988, Mean reversion in stock returns. Evidence and implications, Journal of Financial Economics 22, 27-60.

Press, W.H., B.P. Flannery, S.A. Teukolsky and W.T. Vetterling, 1986, Numerical recipes. The art of scientific computing, Cambridge University Press, Cambridge.

Robinson, P.M., 1994, Efficient tests of nonstationary hypotheses, Journal of the American Statistical Association 89, 1420-1437.

Robinson, P.M., 1995, Gaussian semiparametric estimation of long range dependence, Annals of Statistics 23, 1630-1661.

Robinson, P.M., 2001, The memory of stochastic volatility models, Journal of Econometrics 101, 195-218.

Robinson, P.M. and F. Iacone, 2005, Cointegration in fractional systems with deterministic trends, Journal of Econometrics 127, 263-298.

Shea, G., 1991, Uncertainty and implied variance bounds of long memory models of the interest rate term structure, Empirical Economics 16, 287-312.

Smallwood, A. and S.C. Norrbin, 2006, Testing interest rate equalization, Review of International Economics.

Sowell, F., 1992, Maximum likelihood estimation of stationary univariate fractionally integrated time series models, Journal of Econometrics 53, 165-188.

Stock, J.H. and M.W. Watson, 2002, Macroeconomic forecasting using diffusion indexes, Journal of Business and Economic Statistics 20, 147-162.

Szego, G., 1975, Orthogonal polynomials, American Mathematical Society, 4^{th} Edition.

Tanaka, K., 1999, The Nonstationary Fractional Unit Root, Econometric Theory, 15, pp. 549-582.

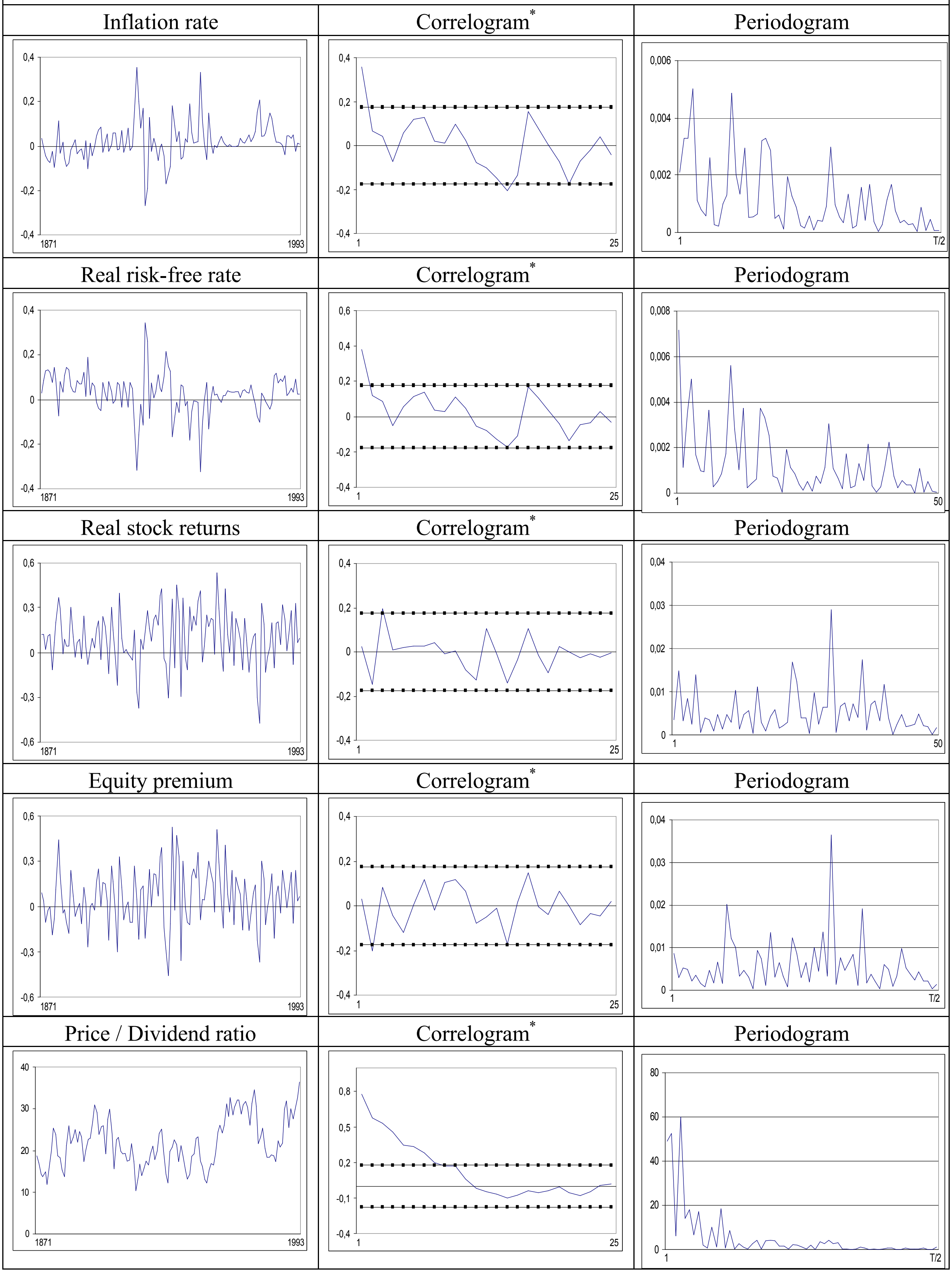
Velasco, C., 1999, Gaussian semiparametric estimation of nonstationary time series, Journal of Time Series Analysis 20, 87-127.

Witt, S.F. and C.A. Witt, 1992, Modelling and forecasting demand in tourism. San Diego, Academic Press.

Woodward, W.A., Q.C. Cheng and H.L. Ray, 1998, A k-factor Gamma long memory model, Journal of Time Series Analysis 19, 485-504.

FIGURE 1

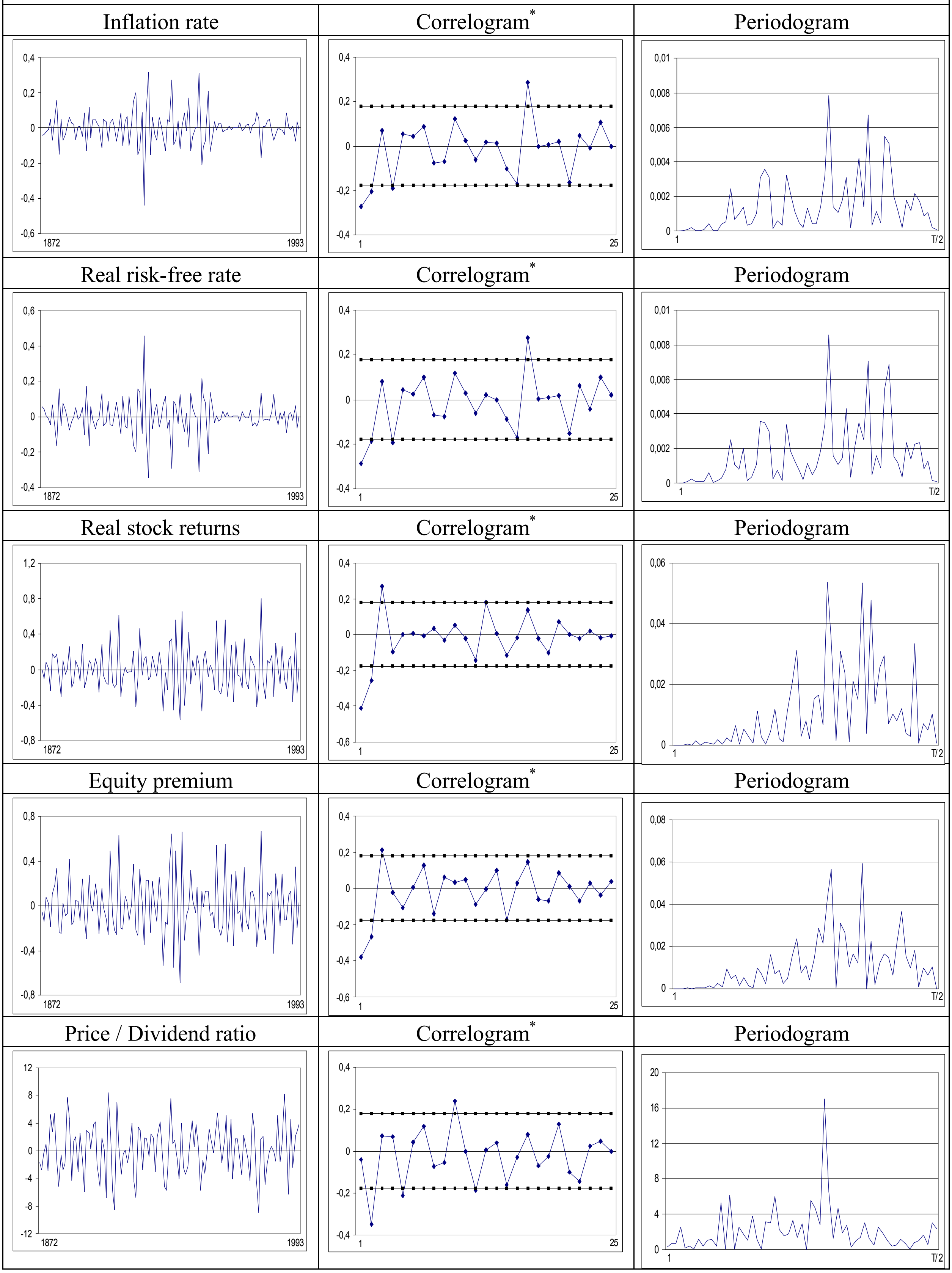
Raw time series, with their corresponding correlograms and periodograms



* The bold lines in the correlograms refers to the Barlett 95% confidence bands

FIGURE 2

First differenced time series, with their corresponding correlograms and periodograms



* The bold lines in the correlograms refers to the Barlett 95% confidence bands

TABLE 1			
Confidence intervals of the non-rejection values of d using $\hat{R}$ in Appendix 1 with $\rho(L; \theta) = (1 - L)^{d+\theta}$ and white noise u_t			
Time Series	No regressors	An intercept	A linear trend
INFLATION RATE	[0.12 (0.25) 0.45]	[0.13 (0.25) 0.46]	[0.07 (0.22) 0.44]
R. RISK-FREE RATE	[0.19 (0.31) 0.49]	[0.17 (0.30) 0.47]	[0.15 (0.29) 0.47]
R. STOCK RETURN	[-0.09 (0.00) 0.14]	[-0.10 (0.00) 0.13]	[-0.10 (0.00) 0.13]
EQUITY PREMIUM	[-0.12 (-0.04) 0.10]	[-0.14 (-0.04) 0.10]	[-0.18 (-0.07) 0.08]
PRICE / DIVIDEND	[0.72 (0.83) 1.02]	[0.58 (0.73) 0.92]	[0.59 (0.73) 0.92]

We test the null hypothesis: $d = d_o$ in the model $(1-L)^d x_t = \varepsilon_t$. In parentheses, the Whittle estimates for d.

TABLE 2			
Confidence intervals of the non-rejection values of d using $\hat{R}$ in Appendix 1 with $\rho(L; \theta) = (1 - L)^{d+\theta}$ and AR(1) u_t			
Time Series	No regressors	An intercept	A linear trend
INFLATION RATE	[-0.13 (-0.07) 0.19]	[-0.18 (-0.08) 0.20]	[-0.44 (-0.18) 0.11]
R. RISK-FREE RATE	[-0.11 (0.04) 0.33]	[-0.08 (0.04) 0.28]	[-0.14 (-0.06) 0.27]
R. STOCK RETURN	[-0.17 (-0.04) 0.20]	[-0.25 (-0.04) 0.18]	[-0.26 (-0.05) 0.18]
EQUITY PREMIUM	[-0.22 (-0.11) 0.00]	[-0.30 (-0.12) 0.00]	[-0.41 (-0.19) -0.04]
PRICE / DIVIDEND	[0.24 (0.72) 0.83]	[0.15 (0.55) 0.58]	[0.13 (0.48) 0.60]

We test the null hypothesis: $d = d_o$ in the model $(1-L)^d x_t = u_t$; $u_t = \tau u_{t-1} + \varepsilon_t$.

TABLE 3						
Rejection frequencies of Robinson's (1994) procedure described in Section 3						
d_1	d_2	T = 120	T = 240	T = 360	T = 480	T = 960
0.7	-0.2	0.851	0.996	1.000	1.000	1.000
0.8	-0.2	0.833	0.995	1.000	1.000	1.000
0.9	-0.2	0.842	0.995	1.000	1.000	1.000
1.0	-0.2	0.859	0.998	1.000	1.000	1.000
1.1	-0.2	0.888	0.999	1.000	1.000	1.000
0.5	-0.1	0.880	1.000	1.000	1.000	1.000
0.6	-0.1	0.691	0.985	1.000	1.000	1.000
0.7	-0.1	0.535	0.916	0.995	0.999	1.000
0.8	-0.1	0.524	0.896	0.991	0.998	1.000
0.9	-0.1	0.578	0.934	0.992	1.000	1.000
1.0	-0.1	0.679	0.985	1.000	1.000	1.000
1.1	-0.1	0.816	1.000	1.000	1.000	1.000
0.5	0	0.693	0.973	0.999	1.000	1.000
0.6	0	0.323	0.717	0.911	0.975	1.000
0.7	0	0.143	0.338	0.550	0.731	0.967
0.8	0	0.160	0.377	0.615	0.769	0.995
0.9	0	0.305	0.723	0.944	0.987	1.000
1.0	0.	0.601	0.974	1.000	1.000	1.000
1.1	0	0.898	1.000	1.000	1.000	1.000
0.4	0.1	0.870	0.995	1.000	1.000	1.000
0.5	0.1	0.558	0.896	0.976	0.998	1.000
0.6	0.1	0.202	0.392	0.578	0.688	0.925
0.7	**0.1**	**0.075**	**0.068**	**0.054**	**0.047**	**0.051**
0.8	0.1	0.207	0.334	0.490	0.588	0.912
0.9	0.1	0.521	0.878	0.977	0.996	1.000
1.0	0.1	0.856	0.996	1.000	1.000	1.000
0.4	0.2	0.857	0.993	1.000	1.000	1.000
0.5	0.2	0.583	0.912	0.985	0.999	1.000
0.6	0.2	0.319	0.598	0.781	0.887	0.998
0.7	0.2	0.331	0.561	0.736	0.843	0.989
0.8	0.2	0.621	0.904	0.976	0.991	1.000
0.9	0.2	0.897	0.996	0.999	1.000	1.000
0.5	0.3	0.746	0.986	1.000	1.000	1.000
0.6	0.3	0.662	0.964	0.998	1.000	1.000
0.7	0.3	0.795	0.973	0.999	1.000	1.000

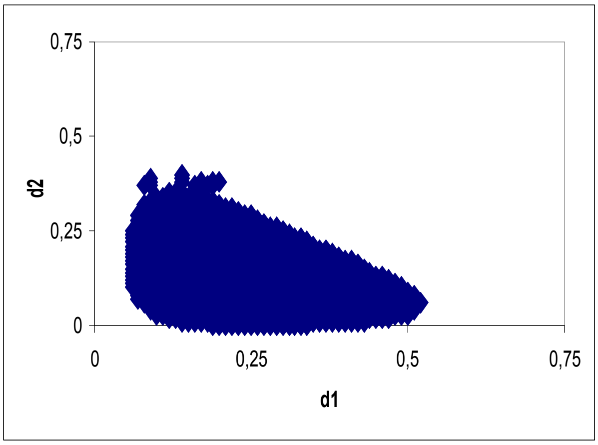
TABLE 4						
Testing H_o (6) in (14), (4) and (8) with $z_t \equiv 1$, $w = w_r$, $r = 6$ and white noise u_t						
d_1	d_2	INFLATION	RISK RATE	STOCK RT	PREMIUM	PRICE / DIV
-0.10	-0.10	39.49	51.69	**4.03**	**4.84**	236.63
-0.10	0.00	36.06	55.05	**3.38***	**0.69***	254.45
-0.10	0.10	36.86	58.98	**4.64***	**0.90***	265.93
-0.10	0.20	37.25	60.01	6.61	**3.03***	272.99
0.00	-0.10	28.25	30.70	**0.14***	**4.35***	170.35
0.00	0.00	16.73	24.09	**0.43***	**0.54***	186.83
0.00	0.10	13.29	23.03	**2.96***	**1.81***	197.19
0.00	0.20	12.66	22.78	6.60	**5.29***	202.97
0.10	-0.10	25.25	22.51	**1.32***	**5.49***	112.99
0.10	0.00	8.42	9.72	**1.95***	**2.74***	125.92
0.10	0.10	**3.04***	6.19	**5.64***	**5.11***	133.05
0.10	0.20	**2.72***	6.26	10.39	9.57	137.62
0.10	0.30	**4.70***	7.72	15.49	14.84	141.08
0.20	-0.10	24.90	20.29	**3.41***	6.91	68.85
0.20	0.00	**5.70***	**4.50***	**5.18***	**5.48***	76.73
0.20	0.10	**0.20***	**0.50***	9.78	8.87	81.48
0.20	0.20	**1.09***	**1.78***	15.19	13.99	83.13
0.20	0.30	**4.84***	**5.32***	20.69	19.63	82.15
0.30	-0.10	25.10	20.09	**5.89***	8.23	38.97
0.30	0.00	**5.65***	**3.62***	8.78	8.19	41.56
0.30	0.10	**0.98***	**0.40***	14.06	12.43	43.27
0.30	0.20	**3.32***	**3.19***	19.81	18.00	43.31
0.30	1.00	26.02	25.29	32.69	34.37	**4.71***
0.40	0.00	6.45	**4.45***	12.23	10.73	19.63
0.40	0.10	**3.30***	**2.70***	17.98	15.68	19.12
0.40	0.70	34.13	23.32	31.40	31.13	**5.73***
0.40	0.80	26.00	25.28	31.98	32.02	**5.08***
0.40	0.90	27.50	26.85	32.64	32.80	**5.11***
0.50	0.00	7.49	**5.84***	15.38	13.12	7.89
0.50	0.10	6.14	**5.83***	21.44	18.62	6.30
0.50	0.20	11.24	11,36	27.34	24.62	**5.81***
0.50	0.30	18.20	18.46	32.72	30.33	**5.86***
0.60	0.00	8.59	7.38	18.23	15..40	**2.70***
0.60	0.10	9.02	9.15	24.48	21.31	**1.00***
0.60	0.20	15.22	15.70	30.31	27.36	**1.25***
0.60	0.30	22.70	23.22	35.51	32.95	**2.59***
0.60	0.40	29.91	30.36	40.08	37.95	**4.70***
0.70	0.00	9.77	9.00	20.82	17.60	**1.22***
0.70	0.10	12.04	12.49	27.15	23.80	**0.04***
0.70	0.20	19.01	19,76	32.86	29.84	**1.26***
0.70	0.30	26.72	27.46	37.85	35.28	**3.77***
0.80	0.00	11.09	10.73	23.20	19.75	**1.72***
0.80	0.10	14.97	15.66	29.54	26.13	**1.39***
0.80	0.20	22.57	23.51	35.10	32.10	**3.57***
0.90	0.00	12.57	12.58	25.41	21.85	**3.19***
0.90	0.10	17.86	18.77	31.70	28.33	**3.82***
1.00	0.00	14.22	14.56	27.49	23.90	**5.05***

The non-rejection values of the null hypothesis at the 5% significance level are in bold and with an asterisk.

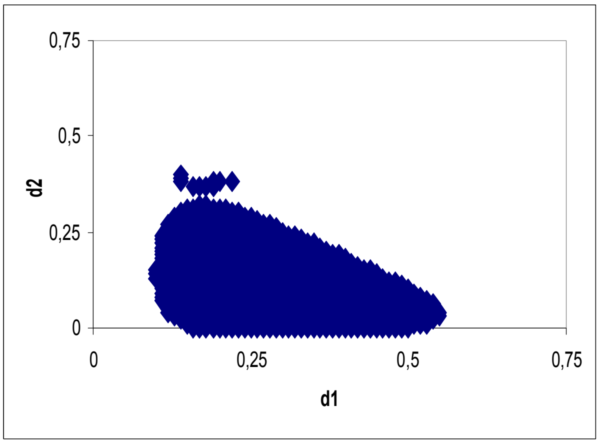
FIGURE 3

Non-rejection values of d_1 and d_2 in (14), (4) and (8) with r = 6 and white noise u_t

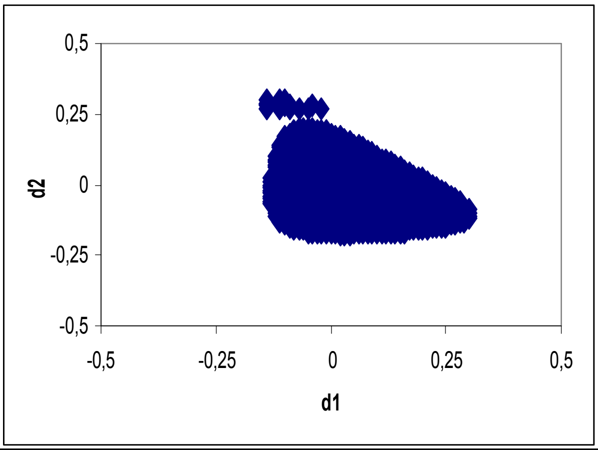
INFLATION RATE



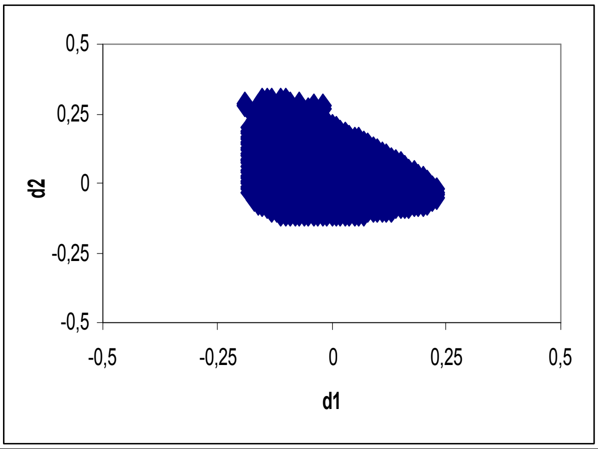
REAL RISK-FREE RATE



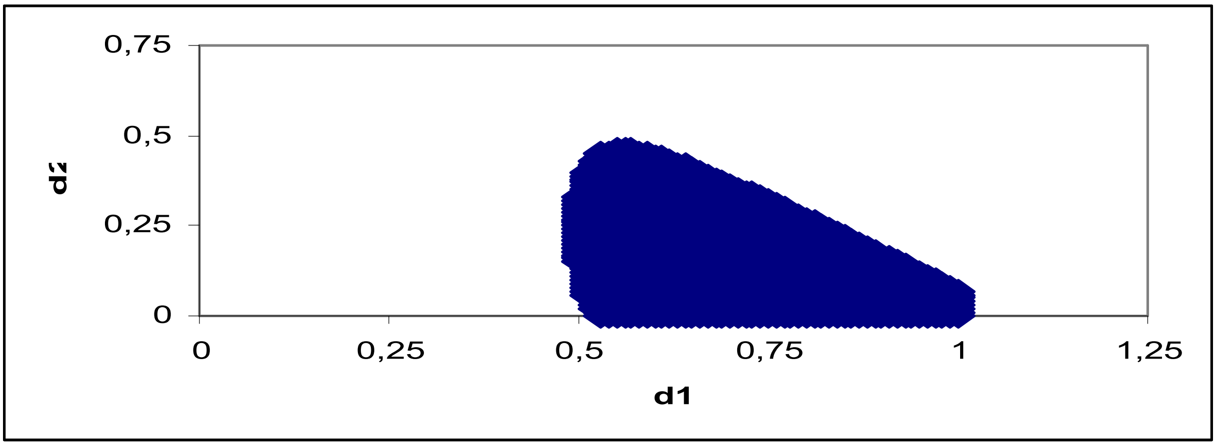
REAL STOCK RETURNS



EQUITY PREMIUM



PRICE / DIVIDEND RATIO



d_1 represents the order of integration at the zero frequency while d_2 is the cyclical one.

TABLE 5			
Selected models for each time series			
Models / Series	Fractional and cyclical differencing (FCD)	Fractional differencing (FD)	Integer differencing (ID)
Inflation rate	$y_t = 0.018 + x_t;$ (0.006) $(1-L)^{0.17}(1-2\cos w_7 L + L^2)^{0.14} x_t = e_t$	$y_t = 0.017 + x_t;$ (0.009) $(1-L)^{0.19} x_t = u_t$ $u_t = 0.21 u_{t-1} - 0.11 u_{t-2} + e_t$	$y_t = 0.020 + x_t;$ (0.009) $x_t = 0.38 x_{t-1} - 0.06 x_{t-2}$ $+ e_t + 0.291 e_{t-1}$
Real risk free rate	$y_t = 0.0348 + x_t;$ (0.015) $(1-L)^{0.25}(1-2\cos w_6 L + L^2)^{0.10} x_t = e_t$	$y_t = 0.021 + x_t;$ (0.011) $(1-L)^{0.04} x_t = u_t$ $u_r = 0.35 u_{t-1} + e_t$	$y_t = 0.016 + x_t;$ (0.007) $x_t = 0.381 x_{t-1} + e_t$
Real stock returns	$y_t = 0.0970 + x_t;$ (0.056) $(1-L)^{-0.05}(1-2\cos w_5 L + L^2)^{0.05} x_t = e_t$	$y_t = 0.0971 + x_t;$ (0.019) $(1-L)^{0.01} x_t = u_t$ $u_r = 0.012 u_{t-1} + e_t$	$y_t = 0.0970 + e_t;$ (0.016)
Equity premium	$y_t = 0.0580 + x_t;$ (0.004) $(1-L)^{-0.06}(1-2\cos w_6 L + L^2)^{0.03} x_t = e_t$	$y_t = 0.0546 + x_t;$ (0.003) $(1-L)^{-0.04} x_t = e_t$	$y_t = 0.0574 + x_t;$ (0.01) $x_t = e_t +$ $0.176 e_{t-1} - 0.239 e_{t-2}$
Price–Dividend ratio	$y_t = 18.811 + x_t;$ (6.679) $(1-L)^{0.68}(1-2\cos w_6 L + L^2)^{0.09} x_t = e_t$	$y_t = 18.762 + x_t;$ (6.123) $(1-L)^{0.73} x_t = e_t$	$(1-L) y_t = 0.163 + x_t;$ (0.018) $x_t = e_t +$ $-0.078 e_{t-1} - 0.340 e_{t-2}$

Standard errors are in parentheses.

TABLE 6

Overall ranking of forecasting performance using different criteria

Series	Model	Theil's U	MAPE	MSE	RMSD	RMSE	MAD
Inflation rate	FCD	2	1	1	1	1	1
	FD	1	2	2	2	2	3
	ID	3	3	3	3	3	2
Real risk free rate	FCD	1	1	1	1	1	1
	FD	3	3	3	3	3	2
	ID	2	2	2	2	2	3
Real stock return	FCD	3	3	3	3	2	3
	FD	2	2	2	2	3	2
	ID	1	1	1	1	1	1
Equity premium	FCD	3	3	3	3	3	3
	FD	1	2	2	2	2	1
	ID	2	1	1	1	1	2
Price – Dividend ratio	FCD	2	1	1	1	1	2
	FD	1	2	2	2	2	1
	ID	3	3	3	3	3	3

FCD stands for Fractional and Cyclical Differentiation, FD for Fractional Differentiation, and ID for Integer Differentiation. Five out-of-sample observations were considered in each case and the ranking was computed on the basis of the average value of the forecasts for each criterion.

TABLE 7			
Selected models for Inflation and Price/Dividend ratio (1871 – 1978)			
	FCD	FD	ID
Inflation	$y_t = 0.018 + x_t$; (0.006) $(1-L)^{0.14}(1-2\cos w_7 L + L^2)^{0.16} x_t = e_t$	$y_t = 0.016 + x_t$; (0.011) $(1-L)^{0.24} x_t = u_t$ $u_t = 0.14 u_{t-1} - 0.12 u_{t-2} + e_t$	$y_t = 0.018 + x_t$; (0.009) $x_t = 0.36 x_{t-1} - 0.07 x_{t-2} + e_t$
Price/ Dividend ratio	$y_t = 18.347 + x_t$; (6.702) $(1-L)^{0.66}(1-2\cos w_6 L + L^2)^{0.16} x_t = e_t$	$y_t = 18.481 + x_t$; (6.505) $(1-L)^{0.81} x_t = u_t$ $u_t = 0.10 u_{t-1} - 0.33 u_{t-2} + e_t$	$(1-L) y_t = 0.151 + x_t$; (0.018) $x_t = e_t + 0.101 e_{t-1}$

TABLE 8						
MSE forecasts for inflation and price/dividend ratio						
a) inflation						
	1 period	3 period	6 period	9 period	12 period	15 period
FCD	1.3732	1.6221	1.5902*	1.6114	1.6110*	1.7071*
FD	1.2165*	1.4093	1.7735	1.6551	1.6895	1.8112
ID	1.3233	1.3921	1.7483	1.6643	1.7420	1.9921
a) price/dividend ratio						
	1 period	3 period	6 period	9 period	12 period	15 period
FCD	2.2819	2.0420	1.9617*	1.8447*	3.3683*	3.9035*
FD	2.3850	2.1614	2.1920	2.9957	4.9017	4.8902
ID	2.3480	1.7070	2.4346	2.1656	4.2935	5.1132