The Evolution of the Phillips Curve: A Modern Time Series Viewpoint

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Abstract

Phillips (1958) original curve involved a nonlinear relationship between inflation and unemployment, estimated by using the United Kingdom data from a few early years (1861-1913), and then visually compared to data for later periods (up to 1957), often getting remarkably good fits. We continue this process by considering how his original results change due to updated theoretic and empirical studies, increased computer power, enlarged the data sets, increased in data frequency and developed time-series econometric models. Using the linear models in annual data in the U.K., the U.S. Australia, Turkey as well as the monthly data U.S., it is basically found that there was little or very weak causation from unemployment to inflation. Rather than using any of the many non-linear models that are now available, we adopt a time-varying parameter linear model as their convenient proxy of non-linear models for testing the non-causality of unemployment on inflation.

Keywords: Phillips Curve, time varying, nonlinearity, forecasting, White theorem
JEL Classification: C5

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[+] We are grateful to Robert Leeson for providing the data set used in Phillips (1958).

1. The Beginnings

In his original paper (1958) Bill Phillips used British data to consider the relationship between wage inflation and unemployment and in the following half century his findings have been extended and re-examined by many authors and in many ways. Our intention is to consider how his original results have changed due to the results of new empirical studies, as computer power has increased, as the data set has enlarged and increased in frequency and as econometric models and tastes have developed. We have attempted to follow the route that Phillips might well have taken, although it is very difficult to appreciate his tastes and preferences as to model specification. We have taken a fairly 'main-line' route using time series models and have attempted to avoid controversy although we realize that this is virtually impossible. We have ignored the macro-theory based developments, including the rational expectations side-track, as we would prefer to rely just on the message that is available in the data when it is viewed carefully.

Over the full fifty years we have focused our attention on the two main variables, inflation here denoted by I_t and unemployment, denoted U_t. Originally I_t consisted of wage inflation but later has moved to price inflation as the economy evolved and trade unions subsided in importance. Inflation and unemployment are probably widely considered to be the two most important economic variables by the majority of the work force of a country and consequently considerable attention is paid to them by the State Government and by the Central Bank. Only interest rates and some financial variables are moving towards a similar level and width of interest. Some other major economic variables, such as production and trade balance, are certainly important but are mainly of concern only to economists.

The original Phillips paper (1958) initially discusses the joint, contemporaneous relationship between I and U, and the paper's title also suggests a joint relationship. It is not until the end of the second page of the paper that it is suggested that the flow of the impact is from U to I. The final paragraph of the first section clearly states that the purpose of the study "is to see whether statistical evidence supports the hypothesis that the rate of change of money wage rates in the UK can be explained by the level of employment and the rate of change of employment…" Thus the relationship to be studied is that of employment explaining inflation.

As was usual at that time, when most macroeconomic data was available only annually, no lags were used in the basic formulation, but only contemporaneous terms were used, however in other early studies by Phillips lagged terms were used. Apart from import prices, no other economic variables were mentioned as possible explanatory variables. To be precise Phillips (1958) writes on page 283 "The purpose of the present study is to see whether statistical evidence supports the hypothesis that the rate of change of money wage rates in the UK can be explained by the level of unemployment and the rate of change of unemployment, *except in, or immediately after, those years in which there was a very rapid rise in import prices."* In virtually all later studies the last part of this statement seems to have been ignored! We have also not investigated the relevance of import prices.

Phillips used British data, which could be hoped to be of fairly high quality, and his series contained almost one hundred terms, which at that time was exceptionally long for a macro series. However the time period used, 1861-1957, was tempestuous, containing at least three major wars as well as several large business cycle swings. On the other hand this high activity level does allow a model to show its ability to be relevant in a wide variety of circumstances.

Phillips also faced a clear shortage of computing power. We understand that the London School of Economics did not have an electronic computer in 1957 when the work was started, and believe that all the calculations would have been carried out on electric calculators.[1]

Phillips paper (1958) is a very nice piece of empirical work, particularly given the computing shortcomings of those times. He obtained a simple curve relating the two variables of interest and found that this curve forecasts fairly well into future decades. The specification was a mixture of the sophisticated, with non-linear explanatory terms being considered, but also with rather simple statements about the quality of the model.

[1] (By Clive W.J. Granger) I clearly remember those days of painful computing, as I completed my Ph.D. at Nottingham in that period. On a personal note I did meet Bill Phillips, but did not know him well. I met him a couple of times at the LSE and acted as an external examiner for him for a Masters exam. We got on fine, although he seemed a little stern. Unfortunately he assumed that I knew a lot more about control theory than I actually did, which made our conversation rather difficult.

No t-values or R-squared statistics are provided, probably because of the computing limitations. The equations include no lag variables, and evaluation was undertaken by forming the model on an early part of the data and comparing the curve so obtained by superimposing it onto later segments of data. The fit was often seen to be surprisingly good but no numerical comparison measure was used.

Just two years later Lipsey (1960) produced a follow-up paper on the same topic and with similar data but using more modern-looking specifications. Lipsey adopted a simpler and more convenient form of non-linearity which proved to be equally successful to that used by Phillips, and R-squared values were now given. However neither Phillips nor Lipsey provided Durbin-Watson statistics, even though during this period. Jim Durbin was their colleague at the LSE. This probably reflects the lack of interest in dynamics used in the specification of econometric models in this era.

During the late 1950's economic models generally were inclined not to use lagged variables in their specifications so the lack of their use by Phillips is not surprising even though he was writing about continuous time error-correction models in his papers on control theory in the same period. We feel that Phillips would have probably been building 'feedback' discrete time models once sufficient computer power became available and as data became more plentiful. This belief is supported by the specification used in Phillips (1959), as discussed below.

2. Consideration of the Work by Phillips and Lipsey from a More Modern Viewpoint

The various models considered by Phillips and Lipsey can be summarized in the form [M]:

$$I_t = a + b_1F_1[U_t] + b_2F_2[U_{t-1}] + b_3F_3\,[U_{t-2}] + k_0F_4[Z_t] + e_t.$$

where I_t is the rate of change of wage rates, U_t is percentage unemployment, and Z_t are some extra explanatory series. Usually k(0)=0. Here the F[.] are various functions, possibly linear, and the b's are coefficients. It should be noted that this equation is both dynamic and non-linear, and if its specification is correct and if b_2 or b_3 are non-zero it also suggests, but does not prove, Granger-causality [later here called just causality] from unemployment to changes in wage rates.

Phillips Model A:

In Phillips' original paper (1958) the curve he considered had a single explanatory term U^c and with no lag so that effectively b_2 and b_3 were taken to be zero. Initially using annual data for the period 1861-1913, the estimates were obtained of $b_1 = 9.638$ and a= -0.9. The estimated value of c was -1.394.[2] The curve fitted for this 1861-1913 period was compared diagrammatically with data from other decades and visually was found to fit adequately well. This included the latest period 1948-1957 but it was shown diagrammatically that if unemployment had been lagged by seven months then the fit would have been excellent.

It should be noted that Model A is not strictly balanced as the variable r can take negative as well as positive values whereas U^c is a 'limited variable' as U is necessarily positive. As the constant a is found to be negative and b_1 is positive, the right-hand side of the equation can give a negative estimate for r but cannot go below a.

Phillips Model B:

In his Melbourne (or Australian) paper,[3] Phillips (1959) used just U to powers -2 and -3, both with lag 3, and the Z_t consisted of the rate of change of export prices [X] with lags 1 and 2 and the rate of change of import prices [IM] with a lag of three. The regression found was

$$I_t = 1.46/U_{t-3}^2 + 0.415/U_{t-3}^3 + 0.15[[X_{t-1} + X_{t-2}]/2] + 0.134\, IM_{t-3} + 2.11$$

This equation is balanced as the last two terms, involving X and IM can be negative.

Phillips Model C:

On the final page of the 1959 article Phillips provides a model of the form

$$I_t = 0.57 I_{t-1} + 0.93/\left(U_{t-2} - 0.26\right) + 2.44 e^{0.02 X_{t-1}} + 0.022 M_{t-1} + 0.295$$

where X and M are respectively the rates of change of export and import prices. He reports that this model fits well, the best of all the models considered, but still not well enough according to his tastes. Although a very small sample is used, annual data over just twelve years, a complicated model is fitted with two explanatory variables as well as

[2] From a comment on page 290 of the paper it seems that Phillips actually fitted the model using least squares except that the constant 'a' was chosen by trial and error.

[3] Chapter 28 of Leeson (2000).

lagged U. This model is also balanced. Values of R-squared are not provided for either equation.

It should be noted that Phillips spends a great deal of time and effort in his papers discussing data sources and problems. Some aspects of this are discussed in an appendix to this paper.

Phillips' Papers 1954 to 1959:

In the period 1954 to 1959 Phillips published five papers, as shown in the list of references and they are republished in Leeson (2000). Only the 1958 paper was empirical in nature, the 'Australian' paper was empirical but not published until much later. The other papers involved economics and control theory and did mention error-correction models. A useful, brief discussion of the control papers has been provided by Pagan (2000).

The Lipsey Models:

In his well known comment on Phillips earlier work Lipsey (1960) considered a model of form [M] but with just the first two powers c_j, one and two. He found that this model provided a very good approximation to Phillips 1958 model. As the Lipsey model is easier to use, especially in a regression framework, it is clearly preferable.

Two simple forms of the achieved Lipsey model are:

[L1] $I_t = -1.42 + 7.06\ U_t^{-1} + 2.31\ U_t^{-2}$, $R^2 = 0.64$

and

[L2] $I_t = -1.52 + 7.60\ U_t^{-1} + 1.61\ U_t^{-2} - 0.023\ \dot{U}_t$, $R^2 = 0.78$.

where $\dot{U}_t$ is the rate of change of unemployment. These models use data for the period 1862 to 1913, with the Bowley data being used for the years 1881-1885. Model L1 is not balanced but model L2 is balanced as it contains a variable that is not limited at zero.

As a final experiment Lipsey includes $\dot{P}_t$, the rate of change of the cost of living index, into the equation and obtains:

[L3] $I_t = -1.21 + 6.45\ U_t^{-1} + 2.26\ U_t^{-2} - 0.019\ \dot{U}_t + 0.02\ \dot{P}_t$, $R^2 = 0.85$.

This equation is balanced.

A difficulty with using these 'dot variables' from a modern viewpoint is that they are defined as $\dot{U}_t = \dfrac{U_{t+1} - U_{t-1}}{2U_t}$, so that the past and future get mixed up. Lipsey states that he did try a form like $\dot{P}_t = \dfrac{P_t - P_{t-1}}{\left(P_t + P_{t-1}\right)/2}$ and found "the results were broadly similar but the correlations slightly lower".

For the years 1923 to 1939 and 1948 to 1957 Lipsey fitted the model:

[L4] $I_t = 0.74 + 0.43\, U_t^{-1} + 11.18\, U_t^{-4} + 0.038\, \dot{U}_t + 0.69\, \dot{P}_t$, $R^2 = 0.91$.

Here $\dot{P}_t$ is the % change in the cost of living wage. It is not surprising to find that this last term has the greatest explanatory power. It should be noted that a quite different specification is used and the parameter values have changed. This can be interpreted as suggesting the necessity of using a time-varying parameter form of model.

The models often contain functions of variables. Although in very general terms the positive function of an I(1) will have essentially the same major properties as an I(1) it is probably best to apply a test. At the end of his paper, on page 31, Lipsey mentions the direction of causation between price changes and wage changes, stating "The analysis so far conducted is not inconsistent with the hypothesis that there is a strong feed-back from price changes to wages. We should test!"

Evaluation:

The attitude towards the evaluation of models has evolved over the years. At the time of the appearance of the Phillips models the emphasis was on how well the model the fitted the data but more recently the emphasis has been on how well the model performs in its planned task, such as forecasting or controlling variables of interest.
Phillips' original nonlinear curve was estimated using some early data and evaluated by showing diametrically that the same curve fitted remarkably well to later periods, although no numerically formed measures were employed.

We superimposed the same curve to later decades using the same coefficients and visually were not found to fit well. If the coefficients of the curve are re-estimated using more recent data the curve fits later data fairly well.

In the present study we consider the purpose of the model to provide forecasts and so evaluations will be made in terms of the relative forecasting abilities of alternative models.

3. Taking a More Recent Time Series Viewpoint.

Current time series analysis will usually start by asking if each individual series is stationary, denoted I(0),or linear 'non-stationary', denoted I(1), where the change of an I(1) series is I(0). It is important to determine the appropriate labels for a pair of series to ensure that an equation in a model is 'balanced', so that the variables on both sides of the equation have the same major properties. The designation is also important when one is trying to avoid spurious regressions, which can occur with a pair of I(1) series. Several tests for I(0) exist. However as these tests may not produce correct results we prefer to build models using both the levels and the differences of all the series that are involved. This will produce several alternative models that can then be compared and evaluated. The models that are considered are:

a) the standard autoregressive model with a pre-selected number of lagged terms
b) a standard bivariate autoregressive model, with the variables to be explained being inflation and unemployment.[4]
c) models could be of the standard ' linear' form with constant coefficients or of the Time-Varying Parameter (denoted TVP) form. As discussed below TVP models are equivalent to non-linear models.
d) TVP and non-linear models. The various Phillips models discussed above are all non-linear and so it should be expected that the present relevant models would also be non-linear but the relevant type of non-linearity could also be expected to change with time. It is thus difficult to specify a relevant form of non-linear model for our explanatory analysis. Fortunately recently Halbert White proved that any non-linear time-series model could be well approximated by a time-varying parameter model, a proof is given in Granger (2007). The TVP can be

[4] If cointegrated variables are involved an error-correction model is used. However, no cases of cointegration were discovered.

found using a standard Kalman-filter program, which is available on several econometric computer packages.

In recent years there has been a change in strategy towards model building. Originally a "best" model was determined using several criteria, fitted to the data and then applied to various practical problems. Currently the alternative strategy being considered is to consider and fit to the data several alternative high quality models, to then evaluate them jointly, particularly by building combined forecasts.

If a test for I(0) is indecisive then two alternative models should be considered, one with I(0) variables and the other with I(1) variables. There is no reason to consider just a single model for purposes of decision making. We call this 'thick modeling' in Granger and Jeon(2004) and this is proving to be a popular approach.

4. Questions of Interest

The questions of interest to Phillips and Lipsey fifty years ago initially can be considered within the context of a linear bivariate vector autoregressive model or an error-correction model but possibly with time-varying parameters and extended by the addition of other explanatory variables. This model is appropriate because of a recent theorem by Halbert White, published in Granger (2007), which showed that any non-linear time series model can be well approximated by a time-varying parameter (denoted TVP) linear model. Usually the variables of interest, inflation and unemployment, will be tested as being either I(1) or I(0). If found to be I(1), then considered in linear and TVP autoregressive models and linear and TVP error-correction models. If a test for I(0) is indecisive then two alternative models should be considered, one with I(0) variables and the other with I(1) variables.

The specific questions considered here will include:-

a) Is unemployment a useful 'causal' variable for inflation? This is investigated in two stages, the first models inflation in terms of lagged inflation, and in the second stage one then adds lagged unemployment to the equation. Evaluation is conducted by noting if the extra information leads to improved forecasts. We use linear autoregressive models, vector autoregressive models and error-correcting models to study what may be called the "causality" of inflation by unemployment.

The main question asks 'what is the appropriate specification of the models linking the variables of interest.' Using standard tests there is some rather weak evidence that inflation is I(1) but that unemployment is usually I(0).

b) The causal question is first asked using linear models and then reconsidered using TVP linear models as a convenient proxy for non-linear models using the result that any non-linear model can be well approximated by a time-varying parameter (TVP) model [See Granger(2007)]. As many alternative forecasting models are being considered evaluation in terms of the success of combinations of forecasts becomes particularly relevant.[5]
c) Initially annual data was used for both the UK and also the USA over several long data spans, starting with Phillips data and ending in 2006. Later some of the results were replicated using monthly data starting after the Second World War. Annual data for Australia and Turkey is also analysed.

5. Outline of Results

In this section we use the abbreviations U_t and I_t for unemployment and inflation and U_{t-1} for U lagged one period.

<u>UK Annual data - Re-analysis of Phillips Data</u>:

The original UK data is annual and goes from 1861 to 1957. Data before 1914 was used to form a model, choosing the 'best' in sample amongst the variables considered. Thus a 'univariate model' would consider a few lags of I as explanatory variables whereas a 'bivariate' model considers lags of both I_t and U_t. If there is causality from unemployment to inflation then the second equation should forecast better.

a) We plotted the original Phillips curve on later data (years 1861-2006), using his estimated coefficients but obtained visibly poor fits.
b) Used the same specification but with re-estimated coefficients and superimposed on data, again produced a visibly poor fit. We might conclude the specific form of

[5] A recent useful discussion is provided by Timmermann(2006).

model proposed by Phillips is no longer as successful as he found on early British data.

c) We next asked if U_t can be seen to cause I_t within a set of standard linear and TVP models. The results are summarised in Table 2. Values are shown for three periods 1914-1957, 1914-2006 and 1958-2006. Variables are considered in undifferenced form, where denoted "level' or differenced. It is seen that in all periods the addition of the unemployment variable does not improve the forecasts, in terms of a reduced Mean Squared Forecast Error (MSFE), except for a very slight change in the last period considered. Thus there is no evidence of unemployment causing inflation in these linear models.

d) Comparing MSFE values between the linear and the TVP models it is seen that the "non-linear proxies" are always better, shown in Table 3. Thus Phillips' use of a non-linear form is justified in other periods but not his actual formulation.

e) Within the group of TVP models there is clear superiority for the bivariate models using differences in unemployment, so that there is evidence for unemployment causing inflation within this class of models.

We would initially conclude from UK data the original Phillips proposition that there is nonlinear causal relationship from unemployment to inflation, as lagged unemployment is included in the estimated model. However, in each of the two later sub-periods the reverse is found and there is evidence of causality.

There is no point in asking if one model is 'significantly worse' than another because this is not a question that now arises in practice. Rather what is important is what weight does the method get in the best combination? Some exploratory results are shown in Table 4, just for the period 1958-2008. The left column is for inflation forecasts, with the top half summarising the MSFE for individual methods and the lower half showing these values for various combinations. It is seen that the simple equal weighted combination of the TVP and bivariate model performs somewhat better in forecasts than all the alternatives.

f) Does Inflation Cause Unemployment?

To further consider the methodology being employed here we also investigated a 'reverse Phillips curve' by asking if inflation causes unemployment. The results are shown in

Tables 2 and 3. It is found that in each of the 'out-of-sample' periods 1914-2006 and 1914-1957 bivariate models are the best but could be linear or TVP. However, in the more recent period 1958-2006 the univariate models were best, either TVP or linear. Thus there seems to have been evidence of causality from inflation to unemployment in the earlier periods but not the recent one and there is no evidence for non-linearity (or TVP) being required. But, using a combination of TVP and linear does seem to do slightly superior to all the alternatives, which is just the well-known 'portfolio effect' showing up.

Annual US Data

Table 5 shows the corresponding results for the US annual data over the period 1946 to 2003. There is little, clear and consistent evidence of causality from any of linear or TVP models. The TVP models forecast better than the linear in most cases.

For Unemployment TVP models are usually superior to the linear models and the bivariate forms generally have lower MSFE values than the univariate, suggesting that there is clearer causality from inflation to unemployment in all periods.

Monthly US Data

A few experiments were performed with monthly US data. For two longer periods [January 1971 – December 2007, and January 1971 to December 1989] and for Inflation the TVP bivariate model was usually the best, suggesting again that there could be a non-linear causality of inflation by unemployment in theses periods. However, for the later period January 1990 –December 2007 a univariate (no causal) TVP model was slightly better than the best causal TVP model which was better than all alternatives.

Annual Australian Data

Annual data was considered from Australia for the years 1956-2006, to expand the group of economies considered. The results for forecasting inflation showed that when using a linear model the bivariate forms did not perform better than the univariate model, but the TVP models were always superior to the linear models and the bivariate forms were always the best, suggesting that inflation can be explained from unemployment with a non-linear model. To forecast unemployment the best model was linear and involved just past U.

Annual Turkish data.

Our final example uses annual data for the period 1956 – 2006. Turkey provides an interesting example as it is a fairly advanced economy with an interesting inflation history, being around 10% in the 1970's but up the 40% range from 1970 to 2000 with occasional peaks up to 80%. However the inflation rate has declined to 6% or so in recent times. During this period unemployment rate was mostly around 7-8% but with the occasional peak around 10%.

When trying to forecast inflation the TVP causal model appears to be clearly superior. The results suggest that unemployment can be very well forecast using a univariate TVP models that is just lagged U. with no need for lagged I.

Overall Summary of Results

a) In all cases it is found that the TVP formulation is superior to the linear, supporting Phillips' use of non-linear model forms but not the particular type used by him and by Lipsey.
b) For the TVP models it is usual to find that "causation from unemployment to inflation', so that models that used unemployment generally forecast inflation better than those that did not. However, the strength of this result declined in the more recent periods.
c) Concerning the "reverse Phillips curve" there was no evidence found for inflation causing unemployment in the Australian or Turkish data. But, there was evidence of inflation causing unemployment in early periods for both the US and Britain but not in recent periods. This does not seem to be a very reliable relationship.

6. Conclusions

We have discovered many features of the Phillips curve surviving in four economies and through many changes in the economies. It seems that the basic relationship considered by Phillips continues in a non-linear form but with changing coefficients which we have captured using time varying parameters. The causation is basically one way, from unemployment to inflation and not in the reverse direction.

It would be natural to consider other explanatory series such as import prices. It would have been fascinating to know what Bill Phillips would have produced using the data that is now available, modern computers and more modern techniques.

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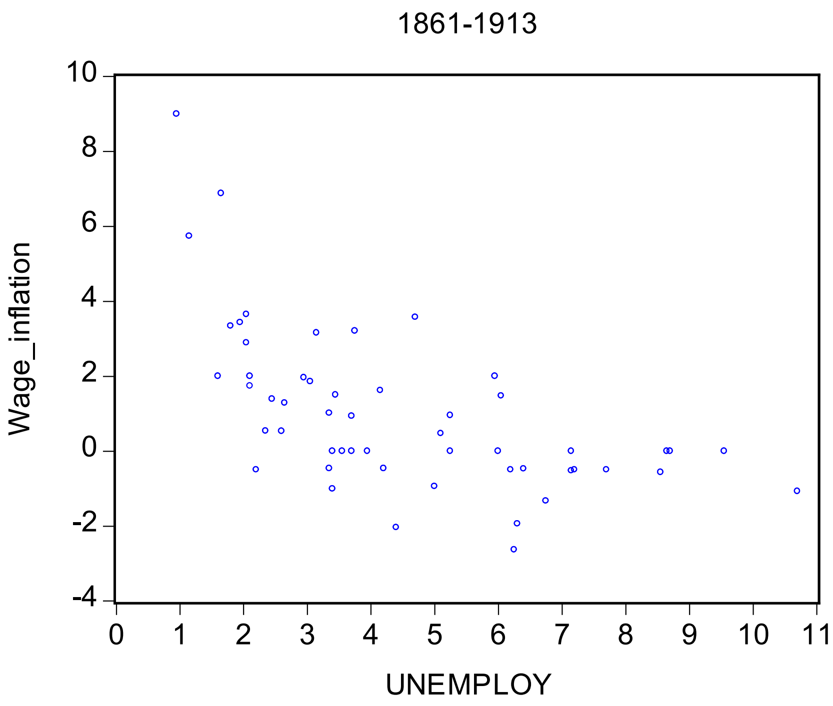
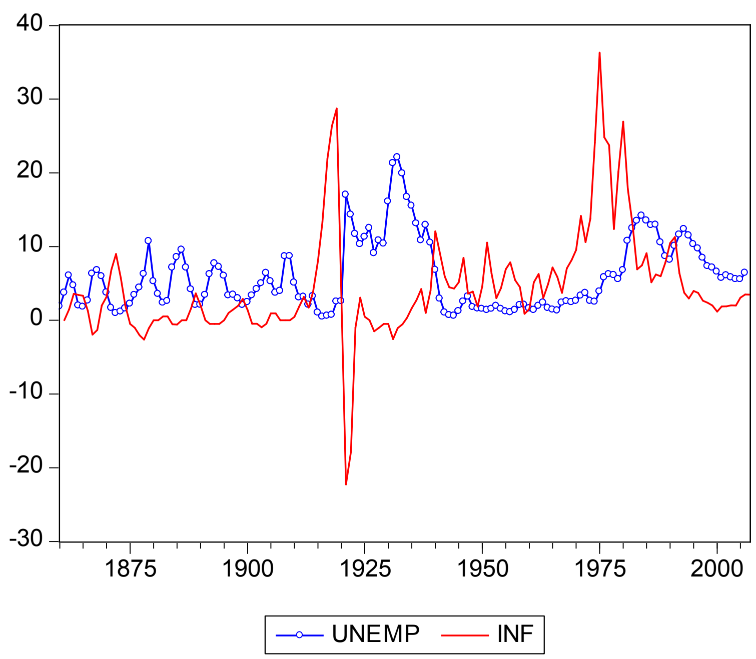
Figure 1: The Phillips Curve Data for estimation

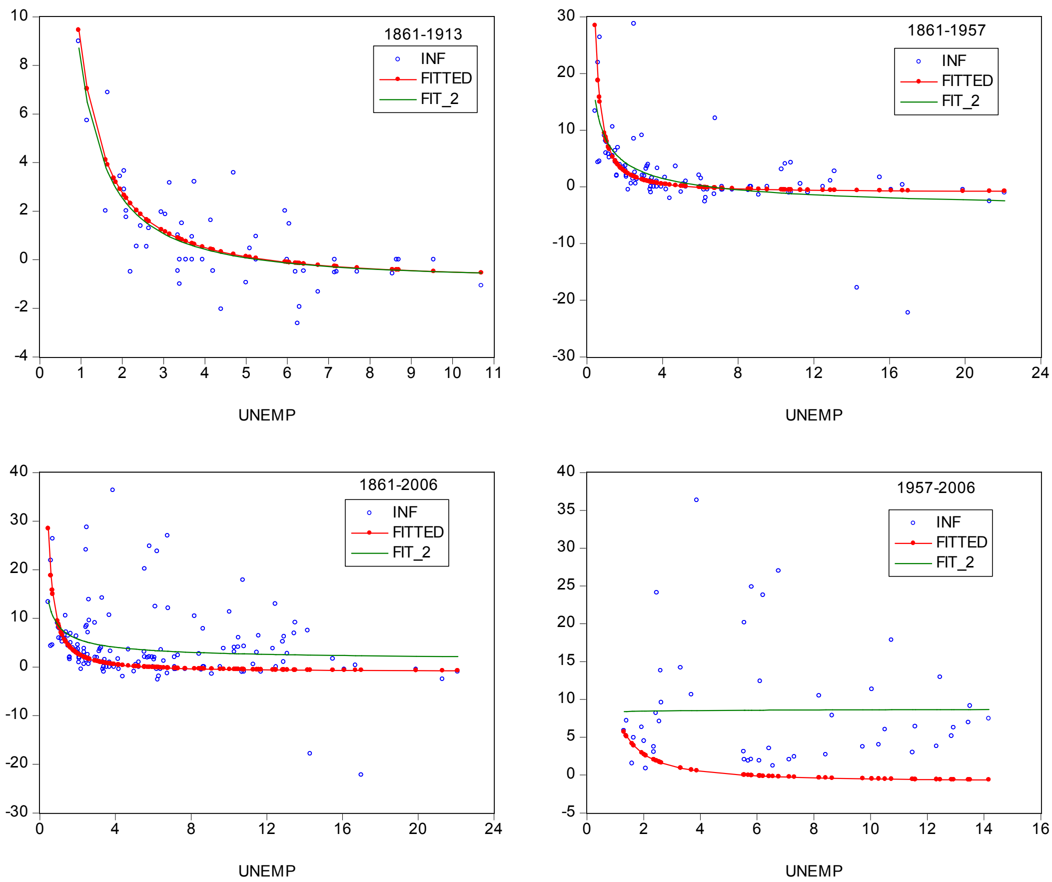


UK extended Figures: Time Series Plots



Note: Nonlinear relationship that Phillips (1958) used is not considered for the time dimension. The causality and nonlinearity are tested using the time domain.

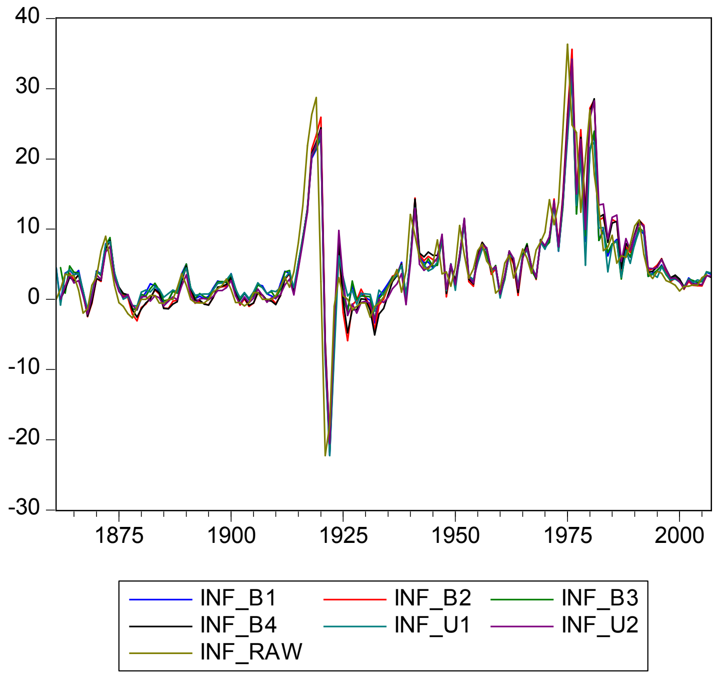
Figure 2. UK extended Figures: Nonlinear Relationship with fitted lines



Note: Fitted – based on Phillips Curve with the same model specification and the same coefficients and Fit_2 – based on Phillips Curve with the same model specification but estimating coefficients

Figure 3: Forecasting inflation and unemployment in TVP

(i) Forecasting the level of inflation in the time-varying settings



Inf_B1 = bivarate forecasting inflation with time-varying model
(inflation = level, unemployment = level)
Inf_B2 = bivarate forecasting inflation with time-varying model
(inflation = difference, unemployment = level)
Inf_B3 = bivarate forecasting inflation with time-varying model
(inflation = level, unemployment = difference)
Inf_B4 = bivarate forecasting inflation with time-varying model
(inflation = difference, unemployment = difference)
Inf_U1 = bivarate forecasting inflation with time-varying model
(inflation = level)
Inf_U2 = bivarate forecasting inflation with time-varying model
(inflation = difference)
Inf_raw = raw data on inflation

(ii) Forecasting the level of UK unemployment

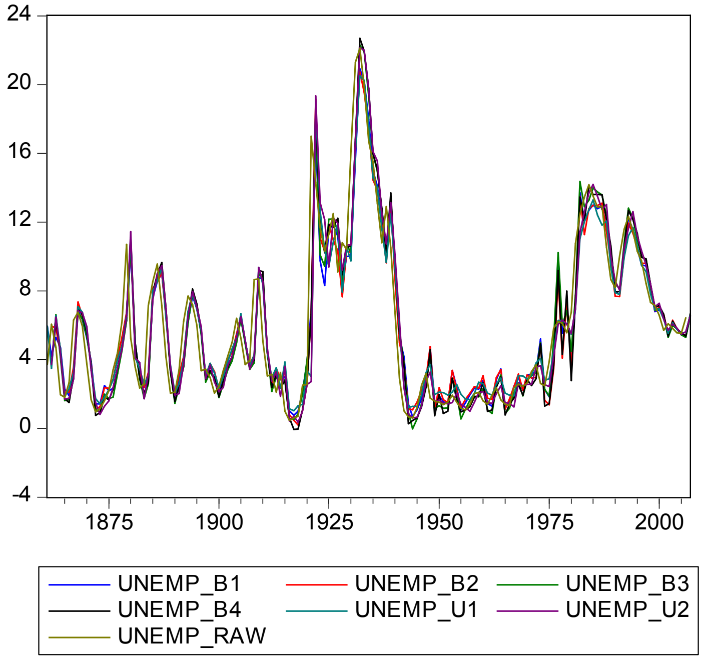
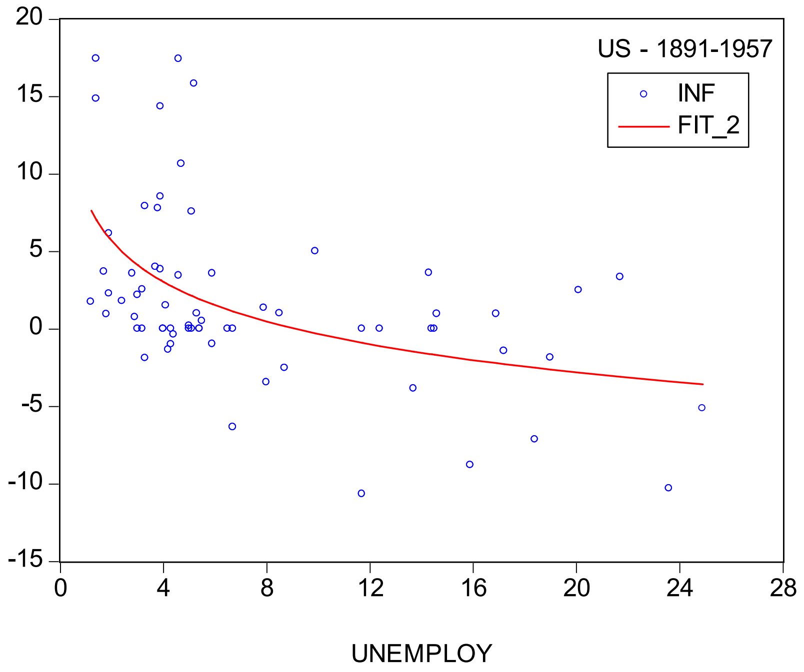
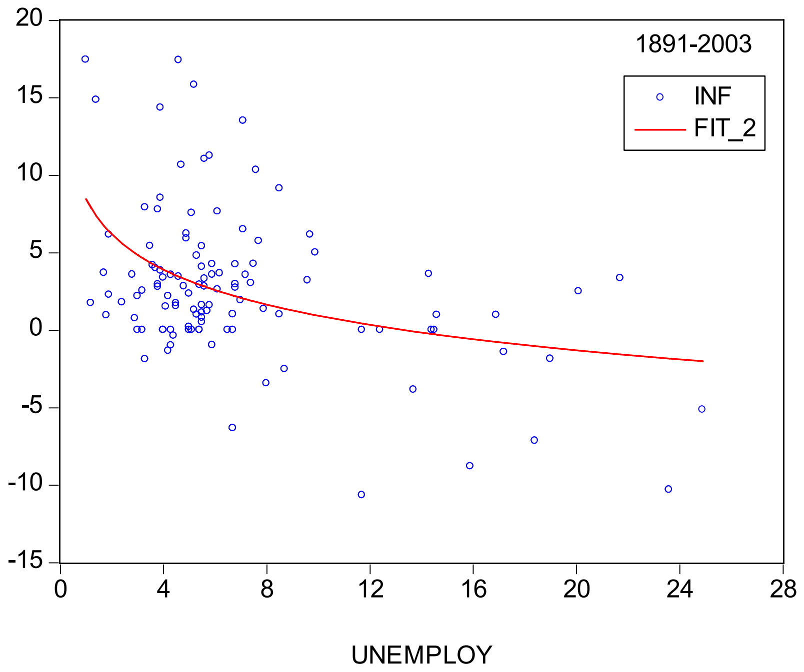


Figure 4: US extended Figures: Nonlinear Relationship with fitted lines





Note: Fit_2 – based on Phillips Curve with the same model specification but estimating coefficients

Figure 5: Time Series Plot – US (1891-2003)

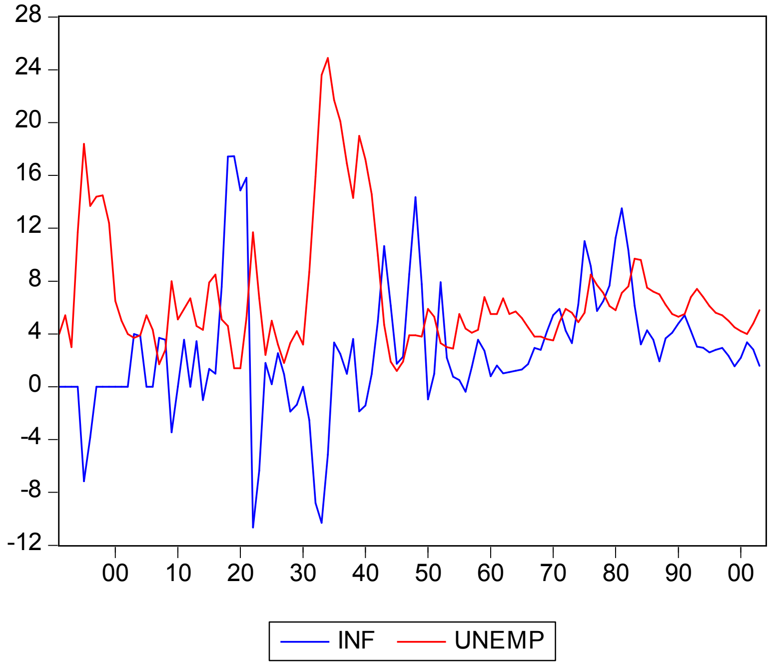


Table 1: Phillips Curve estimation using four aggregated observations

	Equations	coefficients (t-values, p-values)			R^2	loglik	AIC	BIC	DW
case 1	$y + a = b \cdot x^c$	-0.252	16.031	-2.895	0.997	3.404	-0.202	-0.662	3.012
		0.855	4.544	-4.668					
		0.550	0.138	0.134					
case 2	$y + 0.9 = b \cdot x^c$	**0.9**	11.466	-1.617	0.978	-0.532	1.266	0.959	2.382
			7.575	-7.658					
			0.017	0.017					
case 3	$\log_{10}(y + a) = \log_{10} b + c \cdot \log_{10} x$	0.319	1.031	-1.850	na	6.731	-1.865	-2.326	3.193
		0.422	6.086	-2.075					
		0.746	0.104	0.286					
case 4	$\log_{10}(y + 0.9) = \log_{10} b + c \cdot \log_{10} x$	**0.9**	0.984	-1.394	0.966	6.607	-2.304	-2.610	2.705
			11.347	-7.579					
			0.008	0.017					

Estimation using averaged observations:

Interval	Wage	Unemployment
0-2	5.0585	1.5167
2-3	1.5472	2.3500
3-4	0.8482	3.4833
4-5	0.3466	4.4900
5-7	-0.1817	5.9545
7-11	-0.3539	8.3722

Table 2: MSFE – linear AR(2) models; forecasting with differenced UK data

Forecasting series	Independent variables	Out-of-sample Period	MSFE under Expanded	MSFE under Rolling	Out of sample observations
Inflation	Univariate		28.41	**27.51**	
	bivariate Levels		43.10	46.26	
	bivariate Difference	1914-2006	68.66	67.92	93
	Univariate		31.60	32.28	
	bivariate Levels		64.36	66.27	
	bivariate Difference	1914-1957	117.78	118.31	44
	Univariate		25.55	23.22	
	bivariate Levels		24.01	28.28	
	bivariate Difference	1958-2006	24.56	22.68	49
unemployment	univariate		5.30	**5.32**	
	bivariate Levels		8.90	14.84	
	bivariate difference	1914-2006	3.78	4.35	93
	univariate		10.10	10.23	
	bivariate Levels		17.83	17.57	
	bivariate difference	1914-1957	6.12	6.24	44
	univariate		1.00	0.92	
	bivariate Levels		0.88	12.38	
	bivariate difference	1958-2006	1.68	2.65	49

The "Expanded" means that the equations are iteratively estimated. That is, we estimate the equation for 1861-1913 and then forecast for 1914. Then we estimate it on 1861-1914 with forecast for 1915. We proceed this until estimating 1861-2005 with forecasting for 2006.

The "Rolling" means that we initially estimate the equation between 1861-1913, then with 1862-1914, and then 1863-1915. That is, we keep the data length of Phillips original estimation periods.

Table 3: MSFE – time varying models in UK data

MSFE comparisons: Inflation

forecasting inflation / Out-of-sampling period

		inflation	unemploy	1914-2006	1914-1957	1958-2006
TVP	bivariate	level	Level	**22.7**	**26.5**	**19.4**
		difference	Level	**24.1**	**29.7**	**19.1**
		level	difference	**22.4**	**25.8**	**19.4**
		difference	difference	**23.4**	**28.3**	**19.1**
	univariate	level		23.3	25.6	21.2
		difference		23.8	27.6	20.3
Linear	univariate	level		35.5	43.9	27.9
	Bivariate	level	difference	144.5	236.8	61.5
Linear	univariate	Difference		28.4	31.6	25.6
	Bivariate	Difference	difference	68.7	117.8	24.6
no of obs				93	44	49

MSFE comparisons: Unemployment

forecasting Unemployment / Out-of-sampling period

		Inflation	unemploy	1914-2006	1914-1957	1958-2006
TVP	bivariate	Level	Level	**3.6**	**5.9**	1.6
		difference	Level	**3.7**	**6.0**	1.7
		Level	difference	**3.7**	**6.3**	1.5
		difference	difference	**3.9**	**6.1**	1.9
	univariate	Level		4.5	8.3	**1.0**
		difference		4.5	8.5	**1.0**
Linear	univariate		level	24.5	27.2	22.1
	Bivariate	difference	level	35.6	37.8	33.6
Linear	univariate		Difference	5.3	10.1	1.0
	Bivariate	difference	Difference	3.8	6.1	1.7
no of obs				93	44	49

Linear – expanded ending point

Note: Causality in the time-varying settings; inflation helps in forecasting unemployment in 1914-1957 and in overall periods (against what Phillips did), but not in the 1958-2006.

Table 4: MSFE under forecasting combination in UK data

MSFE				Inflation	unemployment
		inflation	unemploy	1958-2008	1958-2008
TVP	bivariate	level	level	19.4	1.6
		difference	level	19.1	1.7
		level	difference	19.4	1.5
		difference	difference	19.1	1.9
	univariate	level		21.2	1.0
		difference		20.3	1.0
linear	univariate	level		64.6	29.8
	bivariate	level	difference	68.5	83.1
combination equal weight	TVP -bivariate			**18.5**	**1.5**
	TVP -unvariate			19.8	**0.9**
	linear			57.2	50.8
	all in TVP, linear			22.2	6.7
combination regression weight	TVP -bivariate			25.7	2.8
	TVP -unvariate			22.8	1.1
	linear			33.7	4.1
	all in TVP, linear			30.6	3.9
no of obs				49	

Table 5: US annual results – MSFE

				forecasting inflation			forecasting unemployment		
		Inflation	unemploy	1914-2003	1914-1957	1958-2003	1914-2003	1914-1957	1958-2003
TVP	bivariate	Level	level	13.47	**24.47**	2.94	4.04	**7.43**	**0.80**
		difference	level	14.12	**25.77**	2.97	3.98	**7.26**	**0.84**
		Level	difference	13.44	**24.42**	2.94	4.36	**8.02**	**0.85**
		difference	difference	14.31	**26.32**	2.82	4.41	**8.14**	**0.84**
	univariate	Level		14.22	26.03	2.93	4.17	7.63	0.86
		difference		15.34	28.47	2.78	4.57	8.33	0.97
Linear	univariate	difference		19.73	37.56	2.69	5.91	10.90	1.14
	bivariate	difference	level	111.33	212.88	14.19	12.12	20.69	3.93
(rolling)	bivariate	difference	difference	26.97	50.23	4.73	8.97	16.49	1.78
no of obs				90	44	46	90	44	46