

THRESHOLD AUTOREGRESSIVE MODELS FOR TESTING ASYMMETRIC ROOTS: EXTENSIONS AND EMPIRICAL EVIDENCE FROM G7 COUNTRIES REAL INTEREST RATES

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Abstract

This paper proposes testing procedures for unit roots in a two-state regime switching threshold model by taking into account of one-sided alternatives. Motivated by the fact that many economic and financial time series go through various phases of business cycles and consequently their movements around equilibriums can exhibit “deepness” and “steepness” asymmetries, Enders and Granger (1998) extended the simple linear model to a two-state regime switching threshold autoregressive model. There are two roots to capture the adjustment process in this model, one governing the economic downturns and other economic upturns. This model was further extended by Caner and Hansen (2001) by allowing deterministic trends, the autoregressive root and short-term dynamics to switch between the two-regimes. Since the threshold parameter is unidentified under the null and the alternative hypothesis is one-sided, in order to establish stationarity, the null hypothesis of both roots being zero is tested against both being negative. In this paper, we propose Sup of the one-sided Wald test developed by Wolak (1987) and one-sided score test by Silvapulle and Silvapulle (1995), and use simulation methods to compute critical values of the tests. Application of one-sided tests shows that G7 real interest rates are stationary, while conventional tests found them to be non-stationary.

JEL classifications: C12, E32, E44

Keywords: Asymmetric roots; threshold autoregressive models; nuisance parameters; real interest rates; one-sided alternatives

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Introduction

There has been a growing discontent among applied researchers with the standard linear ARMA framework invariably used to test for a unit root. A theoretical prediction of stationarity of many time series in several areas of economics, for example, real interest rates in macro-finance, is confused in practice by the persistent failure to reject the unit root null hypothesis by ADF-type tests. These traditional tests that assume linear adjustments to the long run equilibrium can be biased towards not rejecting the null hypothesis of unit root in cases where the series is in fact stationary but demonstrates asymmetric adjustments. Several studies indicated that the standard unit root tests using linear models can be misspecified and have low power when the adjustment process is nonlinear, and the model, for example, switches between two-regimes, one corresponds to economic downturn and other economic upturn. See Pippenger and Goering (1993), and Balke and Fomby (1997) for details. To accommodate this behaviour, Enders and Granger (1998) and Caner and Hansen (2001) specified threshold autoregressive models and propose tests for unit roots.

In this paper, we want to investigate the presence of unit roots in G7 real interest rate series. To do this, we take some important issues related to real interest rate movements into consideration and extend the threshold unit root tests in order to improve their powers to reject the unit root null hypothesis. These issues are: (i) the two roots are negative under the stationary alternative; (ii) real interest rates possess a first-order moving average term MA(1), and hence, as Ng and Perron, (2001) argued, the traditional model selection criteria such as AIC or BIC may fail to select the appropriate lag structure; and (iii) over the years, G7 countries might have made significant changes to the conduct of their monetary policies, which cause sudden or gradual structural breaks in the real interest rate time series. The aim is to extend the tests proposed by Enders and Granger (1998) accommodating the above issues and the fact that the threshold parameter vanishes under the null hypothesis of unit roots and propose new tests based on one-sided tests developed by Wolak (1987) and Silvapulle and Silvapulle (1995). The performance of these new tests for a fully flexible TAR model will then be assessed.

Enders and Granger (1998) studied threshold and momentum threshold models and proposed tests for the null of unit roots against stationary roots and for the null of symmetric adjustment (equal roots) against asymmetric adjustment process. In the threshold models studied by Enders and Granger, only the autoregressive root is allowed to switch between the two regimes. They proposed conventional F-test for the null of unit roots against stationary alternative under which

both roots are negative. Further, an F test is proposed for the null of equal roots. In this paper, we propose two improvements: one is to incorporate the one-sided nature of the alternative hypothesis, and the other is to take account of the fact that the threshold parameter disappears under the null hypotheses, creating a nuisance parameter problem.

Caner and Hansen (2001) studied TAR models by allowing the deterministic terms, autoregressive root and the short-run dynamics to switch between the two states. They proposed Sup Wald test for the null of two roots, one governs the upper regime and the other the lower regime, against stationary alternative that both roots are negative by accommodating the nuisance parameter problem. Further, they proposed some tests to take account of one-sided alternatives, and these are the sums of t-tests. Since the alternative hypothesis is a one-sided multi-parameter one, this paper adapts one-sided tests developed in the literature to unit root testing problem. The main objectives of this paper are to (i) propose Supremum one-sided F, Wald and Score tests for testing non-stationary against stationary hypotheses in TAR models studied by Enders and Granger (1998) by taking account of both nuisance parameter problem and one-sided alternative hypothesis together; (ii) extend them to a fully flexible TAR model of Caner and Hansen (2001); (iii) use modified AIC (MAIC) for model selection, particularly for selecting the optimum number of lag dynamics; and (iv) establish the stationarity property of G7 interest rates using the existing tests and proposed Sup one-sided tests. Monthly G7 real interest rates for the period 1971M1 to 2006M9 are used in this investigation, and our analysis finds all G7 series to be stationary, and the threshold parameter estimates to be significantly different from zero.

The rest of the paper proceeds as follows: Section 2 outlines the econometric methodologies, and proposes some tests for testing the unit root hypothesis against stationary hypothesis. Section 3 describes the G7 real interest rate data and assesses their time series properties. Section 4 conducts the empirical analysis of real interest rates and reports the results. Some concluding remarks are made in section 5.

2. Methodologies

In this section, we discuss various model specifications including a simple linear model used in the application of ADF test, a two-state regime switching threshold autoregressive (TAR) model with only the autoregressive parameter switching, and a TAR with deterministic terms, the autoregressive root and short run dynamics switching. Further, we specify the hypotheses for testing nonstationarity against stationarity, and adjustment is symmetric against asymmetric.

2.1 Model specifications and hypotheses for nonstationarity testing

Consider the following a simple linear autoregressive model:

$$\Delta y_t = m + dt + \rho y_{t-1} + \sum_{i=1}^{i=q} b_i \Delta y_{t-i} + e_t \quad (1)$$

where y_t is the time series, μ and t are deterministic terms drift and time trend respectively, and

Δy_{t-i} , $i = 1, \dots, q$ are short run dynamics and ρ is the root, known as the adjustment parameter. It is

well-known that the null hypothesis $H_{01}: \rho = 0$ is tested against $H_{11}: \rho < 0$ using the non-standard t

test. The lag dynamics are chosen using model selection criteria MAIC, AIC and BIC. The MAIC is recently proposed by Ng and Perron (2001), which is defined as follows:

$$MAIC(k) = \ln(\hat{S}_k^2) + \frac{2(t_T(k) + k)}{T - k_{\max}}$$

where $t_T(k) = (\hat{S}_k^2)^{-1} \hat{r}^2 \sum_{t=k_{\max}}^T \tilde{y}_{t-1}^2$ and $\hat{S}_k^2 = (T - k_{\max})^{-1} \sum_{t=k_{\max}+1}^T \hat{e}_{tk}^2$. \tilde{y}_t is GLS detrended series. \hat{r} is the autoregression coefficient and \hat{e}_t is the error from the OLS equation (1). Clearly, the MAIC utilises a data dependent penalty function with the basic notion to select some lag order k in the interval between 0 and a pre-defined value k_{\max} , where the upper bound k_{\max} satisfies $k_{\max} = o(T)$. In a recent study, however, Peron and Qu (2007) suggested an improvement to the MAIC proposed by Ng-Perron (2001) that has the exact size and power. They argued that using GLS detrending while constructing the MAIC can make the process to be I(2). This shortcoming can be overcome by using the OLS detrended series while constructing the information criteria. Therefore, we use OLS detrended data series for computing the MAIC for the threshold models studied in this paper.

The model (1) is specified under the assumption that all the model parameters are constant and the same across the various regimes, for example, across the various policy regimes. Several empirical studies observed that many economic and financial variables go through various phases of business cycles, especially economic upturns and downturns. It is well-known that these variables behave differently during economic upturns and downturns, and therefore, they can be modeled with asymmetric adjustments. Enders and Granger (1998) developed unit root tests in asymmetric models, namely threshold autoregressive (TAR) model. In this model, the response of Δy_t to y_{t-1} depends on whether y_{t-1} is above or below the threshold parameter τ . Then, the above equation (1) can be extended as follows:

$$\Delta y_t = \begin{cases} r_1 y_{t-1} + \sum_{i=1}^{i=q} b_i \Delta y_{t-i} + e_t & \text{if } \Delta y_{t-1} \geq \tau \\ r_2 y_{t-1} + \sum_{i=1}^{i=q} b_i \Delta y_{t-i} + e_t & \text{if } \Delta y_{t-1} < \tau \end{cases} \quad (2)$$

where Δy_{t-1} is a transition variable and threshold parameter is τ , ρ_1 and ρ_2 are adjustment coefficients

with ρ_1 governing the economic upturn, while ρ_2 governing the economic downturn. Further, in this model, the short run dynamics are not assumed to switch between the two regimes. Note that the

deterministic trends are removed from the series y_t . The model (2) is known as momentum threshold autoregressive model and it captures the steepest asymmetry of the business cycle, and when the

transition variable Δy_{t-1} in (2) is replaced with y_{t-1} the model is known as the threshold autoregressive

model and it captures the deepest asymmetry of the business cycle. Here onwards, we will not make this difference in the terminology and call both types as TAR models.

To establish the stationarity of y_t series, the null hypothesis,

$$H_{02} : r_1 = r_2 = 0 \text{ against the alternative } H_{12} : r_1 < 0 \text{ and } r_2 < 0 \quad (3)$$

is tested using the F test, which is non-standard under the null hypothesis. We denote this statistic by F_1 . In Enders and Granger (1998), the H_{02} was tested against $H_{12}^* : r_1 \neq 0 \text{ and / or } r_2 \neq 0$ using the conventional F test, which has a non-standard distribution under the null of nonstationarity. When the unit root null H_{02} is rejected against H_{12}^* or H_{12} , the presence of the asymmetric roots is tested by testing the null hypothesis,

$$H_{03} : r_1 = r_2 \text{ against the alternative } H_{13} : r_1 \neq r_2 \quad (4)$$

using the F test. We denote this statistics by F_2 .

To estimate the above model (2), we define a Heaviside indicator function I_t as,

$$I_t = \begin{cases} 1 & \text{if } \Delta y_{t-1} \geq t \\ 0 & \text{if } \Delta y_{t-1} < t \end{cases}$$

and the model is reformulated as follows:

$$\Delta y_t = I_t r_1 y_{t-1} + (1 - I_t) r_2 y_{t-1} + \sum_{i=1}^{i=p} b_i \Delta y_{t-i} + e_t \quad (5)$$

Now, we consider a fully flexible TAR model given as,

$$\Delta y_t = \begin{cases} m_1 + d_1 t + r_1 y_{t-1} + \sum_{i=1}^{i=p} b_{1i} \Delta y_t + e_t & \text{if } \Delta y_{t-d} \geq 0 \\ m_2 + d_2 t + r_2 y_{t-1} + \sum_{i=1}^{i=p} b_{2i} \Delta y_t + e_t & \text{if } \Delta y_{t-d} < 0 \end{cases} \quad (6)$$

where the deterministic terms, autoregressive root and the short run dynamics switch between the two regimes. The parameter d is known as the delay parameter. Testing the hypotheses H_{02} against H_{12} , and H_{03} against H_{13} are of interest here as well. To test H_{02} against H_{12} in (6), Caner and Hansen (2001) developed Sup Wald test.

2.2 Sup F tests for hypotheses H_{02} against H_{12} , and H_{03} against H_{13} in models (5) and (6)

Enders and Granger (1998) assumed that in the (detrended) series modelled as (5), only the autoregressive root switches between the two regimes and not the short run dynamics. The F-test proposed by Enders and Granger is the two-sided although the alternative hypothesis H_{12} is one-sided. In this paper, we construct one-sided Sup F for testing the unit root hypothesis H_{02} against H_{12} . Further, the performance of these tests, when structural breaks are present, is not yet investigated. The latter issue will be very briefly addressed in this paper. One of the objectives of this paper is to apply the proposed tests to G7 real interest rates and many of them appear to have a one-point break in the series. To incorporate the presence of structural break in the model, y_t is regressed on the suitable set of deterministic variables including the structural break, such that, $y_t = \alpha x_t + \beta_t$, where x_t represents a set of deterministic components that may include drift, trend and structural break, α is the set of corresponding coefficients. Assuming the break-points are unknown, to identify a possible break point, we used the endogenous procedure suggested by Zivot and Andrews (1992). The residual series $\{\beta_t\}$ is then used in the specification of TAR models such as (5) and (6) while performing the unit root test.

The problem associated with testing the null hypothesis H_{02} is that the threshold parameter disappears under the null hypothesis, resulting in nuisance parameter problem. In such cases, the standard asymptotic theory is not appropriate since it requires a consistent estimate of the parameters. In this paper, we adopted Davies's approach (1977, 1987) to identify the threshold parameter and construct an appropriate test known as Sup test. This methodology exploits the fact that the model is identified when the threshold variable is known. Let Ψ be the set of potential

values for the threshold parameter τ . To compute the Sup test, we choose the potential values of τ selected from transition variables such as y_{t-1} and Δy_{t-1} . Ψ is the set of values of sorted y_{t-1} and trimmed 15% from the top and bottom. $F(\Psi)$ is the set of F_1 statistics corresponding to all the values in the set Ψ , and the largest F_1 statistic is the required test statistic. That is, $\text{Sup}_{t \in \Psi} F(y)$ is the required statistic, which we denote by $\text{Sup } F_1$. Similarly, to test the null hypothesis H_{03} of symmetric adjustment against the alternative H_{13} of asymmetric adjustments, the $\text{Sup } F_2$ statistic can be constructed. We conducted a simulation study and tabulated the critical values for $\text{Sup } F_1$ and $\text{Sup } F_2$ and reported them in table 2 and 3 respectively.

For selecting the optimum lag length of short run dynamics, the modified AIC (MAIC) developed by Ng and Perron (2001) is used. See section 2.1 for details.

2.3 Testing for stationarity with inequality constraint and partial unit roots

Pertrucelli and Woolford (1984) show that the necessary and sufficient conditions for the stationarity of the series is $\rho_1 < 0$, $\rho_2 < 0$ and $(1 + \rho_1)(1 + \rho_2) < 1$ for any value of the threshold parameter τ . As has been argued before, Enders and Granger (1998) tested the null hypothesis of unit roots, $H_{02}: \rho_1 = \rho_2 = 0$, against the alternative $H_{12}: \rho_1 < 0$ and $\rho_2 < 0$ administering the conventional F test. However, this F test reject the null against $H_{12}^*: r_1 \neq 0$ and / or $r_2 \neq 0$, and clearly, this is not the preferred stationary alternative H_{12} . In order to construct a more powerful and appropriate test against the stationary alternative, $H_{12}: \rho_1 < 0$ and $\rho_2 < 0$, Caner and Hansen (2001) defined a set of possible alternatives to the unit root hypothesis and proposed some tests. We outline these tests briefly here and propose some new tests based on one-sided tests developed by Wolak (1987) and Silvapulle and Silvapulle (1995).

Caner and Hansen (2001) suggested a set of alternatives to the nonstationarity null hypothesis. These are: (a) unrestricted alternative hypothesis $H_{12}^*: r_1 \neq 0$ and / or $r_2 \neq 0$; (b) restricted stationary alternative of $H_{12}: \rho_1 < 0$ and $\rho_2 < 0$; (c) partial unit root alternatives that $H_{14}: \rho_1 < 0$ and $\rho_2 = 0$ and $H_{15}: \rho_1 = 0$ and $\rho_2 < 0$. They suggested testing these restrictions using Wald tests the general expression of which is given as:

$$W = \frac{(\hat{r}_i - r_0)^2}{\text{Var}(\hat{r}_i)} = t^2 \quad (7)$$

where \hat{r}_i is the parameter estimate obtained under unrestricted hypothesis H_{12}^* , $i = 1, 2$ and r_0 is the hypothesised restriction, which is zero in our case. In general, W has a standard chi-squared distribution. Using the above framework, a set of two-sided and one-sided Wald tests can be constructed that focus on the two regime threshold model and enable researchers to differentiate among the hypotheses (a), (b) and (c) listed above. The specific test for nonstationarity against the alternative (a) is the two-sided Wald test from (6):

$$R_{2T} = t_1^2 + t_2^2 \quad (8)$$

where t_1 and t_2 are the standard t ratios corresponding to the OLS estimates of the autoregressive coefficients in the upper and lower regimes respectively. On the other hand, the test for nonstationarity against H_{12} given in (b) is the one-sided Wald test of the following form:

$$R_{1T} = t_1^2 1_{(\hat{r}_1 < 0)} + t_2^2 1_{(\hat{r}_2 < 0)} \quad (9)$$

R_{1T} will have power against the one-sided alternatives H_{12} , H_{14} and H_{15} and significant test statistics justify the rejection of the unit root hypothesis. However, undesirable characteristics of this test is that it is unable to differentiate among the alternatives H_{12} , H_{14} and H_{15} , where H_{12} in fact corresponds to the stationarity case and H_{14} and H_{15} to the partial unit root case. As stated by Caner and Hansen (2001), it would be of significant interest to applied researchers to be able to distinguish between the alternatives H_{14} and H_{15} . Towards this end, the individual t statistics, t_1 and t_2 can be used. If only one of $-t_1$ or $-t_2$ is statistically significant this would deduce to the partial unit root case of H_{14} or H_{15} , and thereby, it would be somehow possible to discriminate among the alternative hypotheses H_{12}^* , H_{12} , H_{14} and H_{15} .

The test statistics stated above are continuous function of the t ratios t_1 and t_2 . The test statistics are normalised so that the null of unit roots would be rejected for large values of the test statistics. However, if there are no threshold effects, the asymptotic distribution of each of the above four statistics is found to be data dependent. Caner and Hansen ascertained that asymptotic bounds, free of nuisance parameters other than the trimming range, can be found. Consequently, the critical values and corresponding p-values are tabulated in table III in Caner and Hansen (2001, pp. 1570). In the presence of threshold effects, the asymptotic distribution of each of the four statistics in the identified case are the same as for the ADF test for which the critical values are tabulated by

Dickey-Fuller, which provide a conservative bound for the t_1 and t_2 tests. Further, Caner and Hansen suggested that using bootstrap distribution may improve inference in finite sample. See Caner and Hansen (2001, pp. 1573) for details.

One-sided F and Wald tests

Now, we exploit the fact that the alternative hypothesis H_{12} is one-sided multiparameter alternative hypothesis and construct F and Wald test statistics as outlined in Wolak (1987). For testing for inequality constraint, the p-value of the LR statistics can be computed as:

$$\Pr(LR \geq c) = \sum_{k=1}^P \Pr(F_{k,T-K} \geq c/k) \times w(P, k, \Phi), \quad \Pr(LR = 0) = w(P, 0, \Phi) \quad (10)$$

$$\Phi = R(X' \Lambda^{-1} X)^{-1} R'$$

where, X is a $(T \times K)$ matrix of rank K where T is the number of observation and K is the number of coefficients; R is a $(P \times K)$ matrix of rank P , where P is the number of restrictions. The general expression of inequality is $R\beta \leq r$ where r is a known $(P \times 1)$ vector. K is the number of restrictions, $\sigma^2 \Lambda$ is the covariance matrix of the error where we assume Λ is an identity matrix, c is the calculated test statistic and w is the weight appropriate for the given P . For $P=2$: $w(2,0, \Phi) = \frac{1}{2} p^{-1} \arccos(r_{12})$, $w(2,1, \Phi) = \frac{1}{2}$ and $w(2,2, \Phi) = \frac{1}{2} - \frac{1}{2} p^{-1} \arccos(r_{12})$. r_{12} is the correlation coefficient associated with the (2×2) covariance matrix Φ . See Wolak (1987) for details.

A Score Test for One-Sided Alternatives

We will explain how a one-sided score statistic can be constructed. A one-sided test is useful when the parameter space under the alternative hypothesis can be restricted using prior knowledge or otherwise. An application of the two-sided statistic for such a testing problem can result in model misspecification and subsequently, misleading inferences. Silvapulle and Silvapulle (1995) have developed a procedure whereby a one-sided score statistic can be constructed from its two-sided version and have shown that the one-sided statistic has an asymptotically weighted sum of chi-squared distributions, known as chi-bar squared distribution, under the null hypothesis. We briefly outline this procedure in order to construct a one-sided score statistic from its two-sided version proposed in the previous section. To explain this test we re-state the hypothesis is

interpreted coordinate-wise. We derive the score test as $S_r = [\partial L(g) / \partial r_1, \partial L(g) / \partial r_2]$, where $L(\cdot)$ is the log likelihood function of models (5) or (6). Their details will be given in the appendix.

Assuming a vector $r = (\rho_1, \rho_2)$, we derived the score vector and the test statistics for testing $H_{02}: r = \mathbf{0}$ against $H_{12}: r < \mathbf{0}$ as $\partial L(0, \mathcal{H}) / \partial r$ and $T_0 = T^{-1} s_r' i_{rr}^{-1} s_r$ where, T is the sample size and $r = (\rho_1, \rho_2)$ respectively. Now, using the result that $T^{-1/2} s_r \sim N(i_{rr}^{-1} r, i_{rr})$ under H_{12} for small ρ and following Silvapulle and Silvapulle (1995), we define the score test statistic for H_{02} against H_{12} as

$$T_s = [U i_{rr}^{-1} U' - \inf\{(U-d) i_{rr}^{-1} (U-r) : r < \mathbf{0}\}] \quad (11)$$

where $U = T^{-1/2} i_{rr}^{-1} s_r$. Silvapulle and Silvapulle (1995) have shown that under H_{02} , T_s has a chi-bar squared distribution. The p-value for rejecting H_0 can be computed as

$$\Pr(T_s \geq c) = \sum_{i=1}^3 w_i \Pr(C_i^2 \geq c), \quad (12)$$

The weights are computed using the formulas given above. In order to compute the one-sided T_s statistic what is required is only the two-sided statistic T_0 . Once T_0 has been computed, then $\inf\{\cdot\}$ in (11) can be computed using a quadratic program [see for example, QPROG and NCONF in IMSL or CML in GAUSS].

3. Data series and their time series properties

The motivation for testing for unit root in the real interest rate stems from Fisher's hypothesis which requires the real interest rate be stationary in the long run. The real interest rate is defined as the difference between the nominal interest rate and inflation. For Canada, France, the UK and the USA, 3 months Treasury Bill-rates are used, whereas for Germany, Italy and Japan, the money market rate is used as a proxy for the nominal interest rate. For Canada, Germany, Italy, the UK and the US data is from 1971M1 to 2006M9, whereas for France is from 1971M1 to 2004M10 and for Japan 1971M1 to 2002M9. The annualized monthly inflation rate series is constructed as $1200 \times \log(CPI_t - CPI_{t-1})$. All the data series are collected from IFS (September 2007) Database.

The G7 real interest rate series are plotted in figure 1, and they exhibit the presence of a drift, but no trend was apparent. Therefore we include only the drift in the models for all series. Since it is well-known that real interest rates possess MA(1) innovations, to find out the size and sign of the MA(1) coefficient, we estimated a simple ARMA(1,1) model for each series. The results reported

in the first two rows of table 1 show the presence of significant autoregressive roots, ranging from 0.64 (for Germany) to 0.99 (for Canada, France and the UK). Further, the MA coefficients are significant for all the countries, except for Japan, and are negative, ranging from -0.28 (for Germany) to -0.91 (Canada and the UK). Contrary to what was observed for other countries, the autoregressive root is rather small and negative and the MA coefficient is large and positive (0.65) for Japan. These findings imply that the application of standard unit roots tests to real interest rates may entail some problems as pointed out by DeJong et al. (1992), Schwert (1989) and Ng and Perron (2001). One such problem is the need for large number of lag dynamics to whiten the MA(1) noise term, which many model selection criteria fail to deliver. However, we employ MAIC for this purpose.

We applied the ADF, PP and Ng-Perron tests using information criteria such as AIC and BIC and MAIC for selecting the lag length of short run dynamics. The results are reported in the lower panel of table 1. The PP test rejects the null hypothesis of non-stationarity for all series at the 1% significance level. The ADF and Ng-Perron tests produced nearly consistent results, except for a few cases. When MAIC is used for selecting the lag length, the ADF test fail to reject the unit root hypothesis, whereas when AIC is used the null hypothesis for Japan was rejected at 5%. On the contrary, when BIC is used, the null hypothesis is rejected for Canada, Germany and Japan at the at the 5% level. The standard Ng-Perron test that use MAIC as information criteria doesn't reject the unit root hypothesis for any of the countries. However, the unit root hypothesis is rejected for Canada at 1% and Japan at 5% levels when BIC is used, and for Japan at 5% level when AIC is used. It is worth noting that, as argued by Ng and Perron (2001), the BIC chose smaller k for all the countries, except for Japan, that those chosen by AIC and MAIC. For example, for Canada, MAIC and AIC select the lag of 15, whereas BIC chose only 3. It is interesting to note that for Japan, BIC select a larger k than MAIC. Further, the AIC chose a larger k than MAIC for France, Japan and the UK, whereas same k for the rest of the countries.

Further, we investigated if G7 real interest rates experienced any structural shift during the time period covered by our study. In the 1970s and 1980s, the world experienced substantial oil price shocks and this was a cause of structural shift in many economic time series (Perron, 1989). In the period of December 1973 to January 1974, the dollar price of oil increased by over 250%. The average oil price jumped by 160% in the year 1974 to 1975. Again, in the period of November 1978 to June 1979, the price increase was about 160%. In November 1980, the price hike was roughly 180% from the previous year (see Webber, 2006 for details). The developed countries

adopted drastic measures to contain inflation and to minimise adverse impacts of these shocks on their economies. These oil price increases have had significant impacts on the inflation rates in many countries, which in turn influence the real interest rates. As such, it is of interest to investigate whether the real interest rate series have significant structural breaks, if so, the timing of the breaks. To examine this, using the recursive procedure demonstrated by Zivot and Andrews (1992) we tested for breaks in the G7 real interest rates. The results show the presence of statistically significant structural break in all the real interest rates and the timing of the breaks largely coincide with the period of oil price shocks. For example, for Canada, Japan and the UK, the structural breaks are in 1975M8, 1974M6 and 1975M12 respectively; corresponding to the first oil price shock. For France, Germany, Italy and the US, on the other hand, the breaks are in 1980M2, 1979M7, 1980M4 and 1980M4 respectively, with breaks coinciding the period of second oil price shock. We incorporated these structural breaks in the model and filtered real interest rate series are used in testing for unit roots and asymmetric adjustment process.

4. Empirical Analysis of unit roots and asymmetric roots

In this section, we apply the tests proposed by Enders and Granger (1998) and Caner and Hansen (2001) for establishing stationary properties of G7 real interest rates. Further, we apply the new tests proposed in this paper and compare the results with those of existing tests.

4.1 Enders-Granger threshold models and unit root and asymmetric root tests

We applied EG's TAR and M-TAR models and the F , namely F_1 and F_2 statistics for testing unit roots and asymmetric roots in G7 real interest rate series. In this analysis the threshold parameter is assumed to be zero. Appropriate Heaviside indicator functions were defined and the model (2) was estimated. The augmented models were estimated with the maximum lag length defined as $k_{\max} = \text{int}(12(T/100)^{1/4})$ and the optimal lag length was chosen by MAIC. In the TAR model with transition variable y_{t-1} , the F_1 statistic and the critical values reported in EG, reject the null hypothesis of $H_{02}: \rho_1 = \rho_2 = 0$ against the $H_{12}^*: r_1 \neq 0 \text{ and / or } r_2 \neq 0$ at 5% significant level for Japan, but not rejected for other six countries, implying that these six real interest rate series are non-stationary. The M-TAR model with transition variable Δy_{t-1} was estimated for each series and the null hypothesis of unit roots for Japan is rejected at the 5% level, where as for France at the 10% level. Given the real interest rate for Japan can be stationary, we proceeded to test for the null hypothesis of symmetric adjustments against the asymmetric adjustments in the series using the F_2

test. The results show that the null of symmetric adjustment is rejected for both rates at the 1% level of significance, implying that these real interest rates exhibit asymmetric adjustments.¹

4.2 Enders-Granger TAR and M-TAR models and Sup F_1 and Sup F_2 tests for unit roots and asymmetric roots

We recall that the threshold parameter becomes unidentified under the null hypotheses H_{02} and H_{03} , and therefore we proposed Sup F_1 and Sup F_2 tests. These tests are applied for testing for the presence of unit roots and then for asymmetric roots in the G7 real interest rates. All the rates were first filtered with intercept and one-time structural breaks identified in the real interest rates, and the model (5) is estimated with optimum number of short run dynamic selected by MAIC. Under this model specification, only the autoregressive coefficients are allowed to switch between the two regimes but not the short run dynamics. We simulated the critical values for the test statistics and found that, as expected, the critical values are marginally bigger than those for models without such breaks in the series. Therefore, for drawing the inferences the critical values generated in this paper and reported in table 2 are used. The results of the Sup F_1 and Sup F_2 test statistics for unit roots in TAR and MTAR models, with transition variables y_{t-1} and Δy_{t-1} respectively, are reported in table 4. The delay parameter is assumed to be one in this analysis.

We find that the Sup tests produced somewhat different results. For TAR model, now the nonstationarity hypothesis is rejected for all countries at the 5% level. Clearly, taking the unidentified problem associated with the threshold parameter value τ into account in constructing the test statistics has improved the performance of these Sup F_1 and Sup F_2 tests. Further, we find that the values of the threshold parameter that produce the required Sup tests vary from -2.25 (for France) to 6.84 (for Japan). Note that we used the trimmed bound of [.15, .85] in our analysis. Given that nonstationary hypothesis is rejected, we proceed to test the series for the presence of asymmetric roots using the Sup F_2 test. The critical values used for testing this hypothesis are tabulated in table 3. The null of symmetric adjustment is rejected only for Germany and Japan at the 5% level of significance. Moreover, when Sup tests are applied for M-TAR specification, the unit root hypothesis is rejected for France, Germany, Japan and the USA at the 5% level, and evidence of asymmetric roots was found for France and Japan real interest rate series.

¹ The results of TAR and M-TAR models are not provided in this paper; these are available from the authors on request.

4.3 Caner-Hansen TAR model and Sup Wald test for unit roots

We first study the tests Sup Wald test developed by Caner and Hansen (2001) and then apply the Sup of one-sided versions of Wald and Score tests proposed in section 2.3. We recall the Caner-Hansen TAR model in that the deterministic terms, autoregressive root and the short run dynamics are allowed to switch between the two regimes. Further, the transition variable is Dy_{t-d} , where d is the delay parameter which needs to be estimated along with other parameters in the Caner-Hansen TAR model. The real interest rates filtered only for one-point breaks are used in the models, and four lagged dynamics are included in the both regimes. The null H_{02} of unit roots is tested, and if rejected, then series is tested for the null of H_{12} symmetric adjustments. The critical values tabulated in Caner and Hansen (2001) are used to make inferences, and the results are reported in table 5². The application of Caner-Hansen TAR specification and Sup Wald test shows evidence of strong rejection of the unit root hypothesis H_{02} against $H_{12}^* : r_1 \neq 0 \text{ and / or } r_2 \neq 0$ for all countries. When the series are tested for the null H_{03} symmetric adjustments ($r_1 = r_2$) against H_{13} asymmetric adjustments ($r_1 \neq r_2$), the results reveal that real interest rates of Germany, the UK and the US exhibit asymmetric adjustments. It is interesting to notice that for Germany and the US the attractor is stronger for negative changes in the real interest rate, whereas the attractor is stronger for positive changes for the UK. That is, for the former countries, the adjustment to equilibrium is faster when the process is in the upper regime and for the latter the adjustment is faster when it is in the lower regime.

4.4 Caner-Hansen TAR model and one-sided Sup Wald test and one-sided Sup Score test for unit roots

We have so far tested the null hypothesis of unit roots $H_{02} : r_1 = r_2 = 0$ against the alternative hypothesis $H_{12}^* : r_1 \neq 0 \text{ and / or } r_2 \neq 0$. Clearly, this alternative contains many sub hypotheses, namely a stationary alternative $H_{12} : r_1 < 0 \text{ and } r_2 < 0$ and partial unit root alternatives that $H_{14} : r_1 < 0 \text{ and } r_2 = 0$ and $H_{15} : r_1 = 0 \text{ and } r_2 < 0$. Based on the alternatives H_{14} and H_{15} , Caner and Hansen proposed threshold root test statistics R_{1T} , R_{2T} , t_1 and t_2 . Further, in this paper we proposed Sup one-sided Wald and Score tests against the stationary alternative $H_{12} : r_1 < 0 \text{ and } r_2 < 0$. These tests were applied to G7 real interest rates to test for the unit root null $H_{02} : r_1 = r_2 = 0$ against the

² For TAR, the autoregressive coefficients are positive which makes these estimations unrealistic. Results are not reported in this paper, however, can be obtained from the authors by request.

stationary alternative H_{12} : $r_1 < 0$ and $r_2 < 0$. The p-values of Sup one-sided tests can be computed using the formula (12). The results of R_{1T} , R_{2T} , t_1 and t_2 and Sup one-sided statistics are reported in table 6. The critical values for R_{2T} , R_{1T} , t_1 and t_2 are given in table III, Caner and Hansen (2001), The results indicate that R_{1T} for all the countries are significant at the 1% level of significance. For all G7 countries, we find the estimates of ρ_1 and ρ_2 to be negative and hence by construction R_{2T} and R_{1T} statistics are identical. Considering the individual negative t ratios, $-t_1$ and $-t_2$, we infer with strong evidence that the unit root null hypothesis H_{02} can be strongly rejected for six countries, except for France, in favour of $\rho_1 < 0$. The t_2 statistics lead to rejecting the unit root hypothesis against the alternative of $\rho_2 < 0$ at the 1% level for six countries and at the 5% level for Canada.

5. Conclusion

This paper proposes testing procedures for unit roots in a two-state regime switching threshold model by taking into account of the one-sided alternative that roots are negative. Motivated by the fact that many economic and financial time series go through various phases of business cycles and consequently their movements around equilibriums can exhibit “deepness” and “steepness” asymmetries, Enders and Granger (1998) extended the simple linear model to a two-state regime switching threshold autoregressive model. There are two roots to capture the adjustment process in this model, one governing the economic downturns and other economic upturns. Enders-Granger’s TAR model was further extended by Caner and Hansen (2001) by allowing deterministic trends, the autoregressive root and short-term dynamics to switch between the two-regimes. Since the threshold parameter is unidentified under the null, in order to establish the stationarity in the threshold models, the null hypothesis of both roots being zero is tested using Sup F and Sup Wald tests.

This paper investigates the stationary properties of G7 real interest rates. We find that all series possess an MA(1) innovation, and therefore, a modified AIC is used for selecting the lag length of short run dynamics. Further, we find one-point structural breaks in the real interest rate series. In this paper, we propose Sup of the one-sided Wald test proposed by Wolak (1987) and one-sided Score test by Silvapulle and Silvapulle (1995) to test the unit root hypothesis in the threshold model against the stationary alternative that both roots are negative. We use simulation methods to compute the critical values of the proposed Sup one-sided tests. Application of one-sided tests

shows that all G7 real interest rates are stationary, while the conventional tests found them to be non-stationary.

Figure 1: Real Interest rates for G7 countries

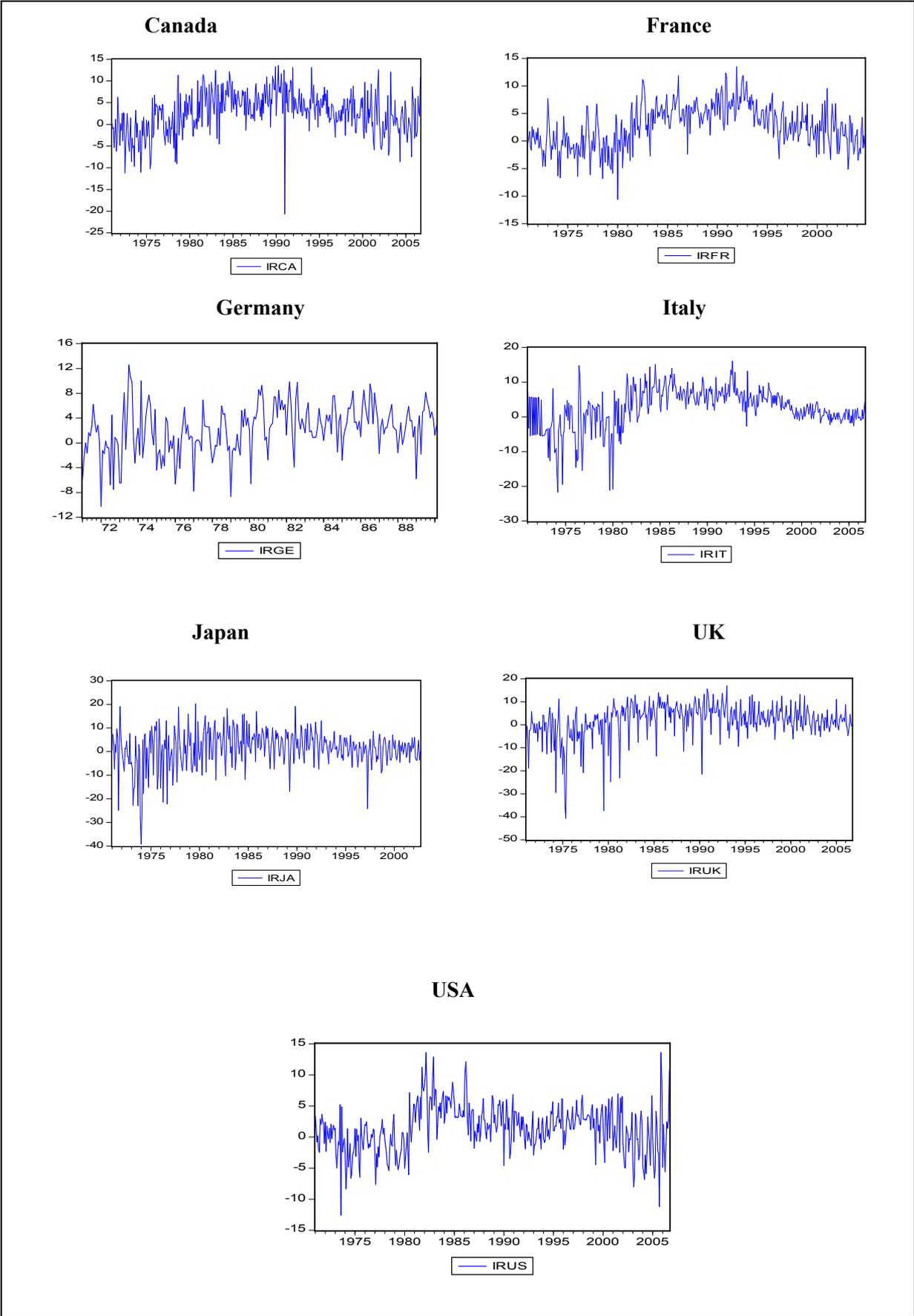


Table 1: Time Series Properties of the Real Interest Rate Series for G7 Countries

		Canada	France	Germany	Italy	Japan	UK	USA
AR(α)		0.99	0.99	0.64	0.97	-0.40	0.99	0.97
MA(θ)		-0.91	-0.84	-0.28	-0.77	0.65	-0.91	-0.82
BIC	k	3	11	0	12	13	12	10
	ADF	-5.24 ^a	-1.49	-9.69 ^a	-2.06	-2.83 ^c	-2.30	-2.24
	Za	-14.77 ^a	-13.05 ^a	-20.27 ^a	-11.39 ^a	-38.35 ^a	-32.22 ^a	-16.68 ^a
	MZa	-26.13 ^a	-1.95	-0.12	-3.49	-5.39 ^c	-1.63	-1.70
AIC	k	15	12	11	15	15	13	11
	ADF	-2.17	-1.57	-1.92	-1.77	-3.13 ^b	-2.50	-1.98
	Za	-33.49 ^a	-12.22 ^a	-20.27 ^a	-12.71 ^a	-31.10 ^a	-32.22 ^a	-16.68 ^a
	MZa	-1.21	-2.31	-0.12	-2.60	-8.65 ^b	-1.63	-1.70
MAIC	k	15	11	11	15	11	12	11
	ADF	-2.17	-1.49	-1.92	-1.77	-2.21	-2.30	-1.98
	Za	-33.49 ^a	-13.05 ^a	-20.27 ^a	-12.71 ^a	-55.60 ^a	-32.22 ^a	-16.68 ^a
	MZa	-1.21	-1.95	-0.12	-2.60	-2.45	-1.63	-1.70
BK	k	13	6	6	11	9	8	11
	Za	-18.08 ^a	-10.14 ^a	-9.95 ^a	-11.89 ^a	-16.48 ^a	-14.75 ^a	-13.23 ^a

Notes: (1) All the series are considered with a constant. (2) Za, MZa and BK denote test statistics from Phillips-Perron test, Ng-Perron test and Phillips Perron test using Bartlett-Kernel, respectively. (3) For PP and Ng-Perron tests using AIC, BIC or MAIC, GLS detrending and AR spectral density are used. (4) a, b, and c refer to significance at 1%, 5% and 10% level, respectively. (5) Newly-West bandwidth is followed when using Bartlett Kernel (BK) spectral density at frequency zero, as presented in the last row.

Table 2: The Critical Values for Rejecting the Null Hypothesis of a Unit Root in Consistent TAR and CM-TAR Models

Sample size	Probability of a smaller value					
	CTAR			CM-TAR		
	F ₁ Statistics			F ₁ Statistics		
	Panel A			Panel B		
	90%	95%	99%	90%	95%	99%
For the series with drift						
50	4.74	5.72	7.99	5.62	6.70	9.25
100	4.79	5.70	7.85	5.71	6.75	9.07
250	4.83	5.72	7.66	5.75	6.75	8.84
For the series with drift and trend						
		Panel C			Panel D	
50	6.22	7.42	10.3	7.16	8.38	11.1
100	6.14	7.21	9.54	7.19	8.34	10.8
250	6.11	7.13	9.37	7.25	8.37	10.7

Note: To see the difference we included structural break at some points and increased sample size for the CMTAR model and simulated the critical values. The critical values for T=429 and break point, bp=0.10 are 5.74, 6.82 and 9.35; and for T=429 and bp=0.30 are 7.49, 8.94 and 12.8 at 10%, 5% and 1%, respectively.

Table 3: The Critical Values for Rejecting the Null Hypothesis of Symmetric Adjustment in Consistent TAR and CM-TAR Models

Sample size	Probability of a smaller value					
	CTAR: F ₂ Statistics			CM-TAR: F ₂ Statistics		
	Panel A			Panel B		
	90%	95%	99%	90%	95%	99%
For the series with drift						
50	3.26	4.25	6.63	4.67	5.92	8.85
100	3.51	4.52	6.92	4.87	6.09	8.91
250	3.60	4.63	7.06	4.94	6.13	8.84
For the series with drift and trend						
50	3.29	4.27	6.68	4.65	5.91	8.92
100	3.52	4.51	6.90	4.85	6.06	9.00
250	3.61	4.62	7.04	4.94	6.14	8.93

Table 4: Results of modified Asymmetric Unit Root Tests for G7 Real Interest Rates using with Restricted Dynamic in Two Regimes

Country	l	t	MAIC	ρ_1		ρ_2		F_1	F_2	lag
				coefficient	t-stat	coefficient	t-stat			
CTAR										
Canada	1975M8	-3.69	2.69	-0.26	-2.70	-0.13	-1.42	9.26 ^a	1.88	15
France	1980M2	-2.25	1.99	-0.06	-0.77	-0.18	-2.17	5.83 ^b	2.21	17
Germany	1979M7	-2.93	2.54	-0.28	-2.30	-0.49	-3.90	8.64 ^a	5.39 ^b	12
Italy	1980M4	-4.09	2.78	-0.12	-1.61	-0.23	-2.74	7.13 ^b	1.56	15
Japan	1974M6	6.84	3.71	-0.61	-4.00	-0.39	-3.02	11.21 ^a	4.67 ^b	13
UK	1975M12	3.43	3.39	-0.28	-2.74	-0.21	-2.27	9.78 ^a	0.69	12
US	1980M4	-3.25	2.23	-0.17	-2.04	-0.22	-2.41	7.90 ^a	0.23	11
CM-TAR										
Canada	1975M8	3.24	2.68	-0.31	-2.82	-0.15	-1.68	4.06	2.93	17
France	1980M2	0.60	1.94	0.02	0.24	-0.21	-2.70	8.74 ^b	8.25 ^b	17
Germany	1979M7	-2.48	2.53	-0.26	-2.21	-0.49	-3.82	55.47 ^a	6.10	11
Italy	1980M4	-3.97	2.81	-0.13	-1.88	-0.30	-3.00	8.28 ^b	3.48	15
Japan	1974M6	-1.19	3.70	-0.59	-4.25	-0.32	-2.37	13.57 ^a	9.55 ^a	13
UK	1975M12	5.82	3.39	-0.39	-3.03	-0.23	-2.54	5.72	2.11	13
US	1980M4	-0.37	2.24	-0.22	-2.54	-0.17	-1.93	7.94 ^b	0.36	11

Notes: (1) a, b, and c indicate significance at 1%, 5% and 10% level, respectively.

(2) Model includes a drift only.

(3) Critical values for testing $H_0: \rho_1 = \rho_2 = 0$ are from table 2. For the CTAR model the values are: 7.66, 5.72 & 4.83 at 1%, 5% and 10% level of significance, respectively. For the CMTAR critical values are 8.84, 6.75 and 5.75 at 1%, 5% and 10% level of significance, respectively.

(4) Critical values for the null hypothesis $H_0: \rho_1 = \rho_2$ are from table 3. For the CTAR models these are 7.06, 4.63 and 3.60 at 1%, 5% and 10% levels, respectively. For the CMTAR critical values are 8.93, 6.14 and 4.94 at 1%, 5% and 10% level of significance, respectively.

Table 5: Results of Asymmetric Unit Root tests for G7 Real Interest Rates using M-TAR Model with Unrestricted Dynamics in Two Regimes with Structural Break

Country	t_D	Coefficients when $\Delta y_{t-1} \geq t_\Delta$						Coefficients when $\Delta y_{t-1} < t_\Delta$						F ₁ stat	F ₂ stat
		c	y_{t-1}	Dy _{t-1}	Dy _{t-2}	Dy _{t-3}	Dy _{t-4}	c	y_{t-1}	Dy _{t-1}	Dy _{t-2}	Dy _{t-3}	Dy _{t-4}	H₀: $\rho_1=\rho_2=0$	H ₀ : $\rho_1=\rho_2$
Canada	0.68	-0.62	-0.51	-0.47	-0.42	-0.33	-0.13	-0.13	-0.29	-0.33	-0.34	-0.21	0.00	18.48 ^a	4.55
(t-stat)		-1.34	-5.78	-5.05	-5.22	-4.49	-2.07	-0.36	-3.52	-3.28	-4.32	2.78	0.07		
France	2.09	-0.37	-0.19	-0.10	-0.06	-0.05	-0.19	0.02	-0.37	-0.22	-0.35	0.05	-0.14	16.04 ^a	2.49
(t-stat)		-0.88	-1.83	-0.85	0.59	-0.47	-2.03	0.12	-5.63	-2.63	-5.24	0.88	-2.43		
Germany	-1.70	-0.03	-0.62	-0.03	0.01	0.06	-0.01	0.34	-0.89	-0.09	-0.05	0.00	-0.08	34.59 ^a	5.59 ^c
(t-stat)		-0.09	-6.02	-0.23	0.17	0.81	-0.10	0.73	-7.84	-0.86	-0.55	0.02	-1.21		
Italy	0.74	0.30	-0.43	-0.31	-0.18	-0.21	-0.19	-0.65	-0.26	-0.38	-0.14	-0.12	0.05	18.47 ^a	3.11
(t-stat)		0.73	-5.38	-3.50	-2.30	-2.72	-2.71	-2.03	-3.90	-4.44	-2.05	-1.82	0.77		
Japan	-3.63	-0.08	-0.83	-0.05	-0.20	-0.16	-0.07	0.38	-0.65	0.04	-0.25	-0.07	-0.29	26.56 ^a	2.34
(t-stat)		-0.14	-6.99	-0.46	-2.35	-2.21	-1.11	0.38	-5.44	0.36	-2.45	-0.78	-3.92		
UK	5.74	2.42	-0.85	-0.01	-0.08	-0.17	-0.11	-0.31	-0.41	-0.35	-0.15	-0.10	-0.11	25.45 ^a	6.29 ^b
(t-stat)		1.90	-5.39	-0.06	-0.71	-1.61	-1.09	-0.81	-5.01	-3.84	-2.19	-1.63	-2.05		
US	-3.28	0.02	-0.32	-0.26	-0.25	-0.15	-0.18	-1.77	-0.63	-0.21	-0.24	-0.25	-0.05	20.68 ^a	7.44 ^b
(t-stat)		0.12	-4.59	-3.25	-3.64	-2.48	-3.22	-2.94	-5.59	-2.20	-2.62	-2.88	-0.68		

Notes: (1) Relevant t-statistics are reported below the respective coefficients for each country; (2) Critical values for testing the null hypothesis $H_0: \rho_1 = \rho_2 = 0$ are from table 2. These are 8.84, 6.75 and 5.75 at 1%, 5% and 10% level of significance, respectively. (3) Critical values for testing $H_0: \rho_1 = \rho_2$ are from table 3 which are 8.93, 6.14 and 4.94 at 1%, 5% and 10% level of significance, respectively.

Table 6: Results for the unit root null hypothesis against stationarity and partial unit roots

Country	$-t_1$	$-t_2$	$R_{1T} = t_1^2 1_{(\hat{r}_1 < 0)} + t_2^2 1_{(\hat{r}_2 < 0)}$	$R_{2T} = t_1^2 + t_2^2$
Canada	5.78 ^a	3.52 ^b	45.80 ^a	45.80 ^a
France	1.83	5.63 ^a	35.05 ^a	35.05 ^a
Germany	6.02 ^a	7.84 ^a	97.71 ^a	97.71 ^a
Italy	5.38 ^a	3.92 ^a	44.15 ^a	44.15 ^a
Japan	6.99 ^a	5.44 ^a	78.45 ^a	78.45 ^a
UK	5.39 ^a	5.01 ^a	54.15 ^a	54.15 ^a
US	4.59 ^a	5.59 ^a	52.32 ^a	52.32 ^a

Note: a and b denote significance at 1% and 5% level of significance, respectively. Critical values for R_{2T} , R_{1T} , t_1 and t_2 are from Caner and Hansen (2001), table III. For R_{2T} appropriate critical values are 11.31, 13.24 and 17.50 at 10%, 5% and 1% level, respectively, whereas for R_{1T} appropriate critical values are 10.84, 12.75 and 16.97 at 10%, 5% and 1% level. For t_1 (also for t_2) the values are 2.97, 3.26 and 3.82, respectively. The asymptotic bounds for t_1 and t_2 are the same under symmetric trimming. We used the trimming bound of [.15, .85]. For all G7 countries, the estimates of ρ_1 and ρ_2 are negative and hence R_{2T} and R_{1T} statistics are identical.

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