

# Money, Capital and Unemployment\*

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## Abstract

We study how monetary policy affects the relationship between trading frictions, such as unemployment, and the accumulation of capital. In the model presented here, firms and households meet sequentially on the labour and the goods markets, both of which being frictional, then on the capital market, which is Walrasian. Firms borrow money from households to buy capital, then hire labour, pay wages, dividends and interests. Firms that could not find a match on either the goods or labour market exit the economy. Households earn wages by working for firms, interests by lending to firms, and dividends through firms ownership. In the model, monetary policy impacts on firms and households' expected payoffs on the labour and goods markets, which ultimately reflects on capital demand behaviour by firms and capital supply decisions by households. Once calibrated to the US economy for the 1949-2003 period, the model is able to replicate medium-term business cycles, defined as the slow-moving oscillations around the balanced growth path. Especially, the model predicts that sustained periods of increasing inflation translate into rising unemployment, and a capital stock per worker above the trend, as observed in the data

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# 1 Introduction

We study how monetary policy affects the relationship between trading frictions, such as unemployment, and the accumulation of capital. In the model, monetary policy impacts on firms and households' expected payoffs on the labour and goods markets, both of which being frictional, which ultimately reflects on capital demand behaviour by firms and capital supply decisions by households. We calibrate the model the US economy starting 1949 ending 2003 and target slow-moving oscillations around trend, defined as medium-term business cycles by Blanchard (1997) and Cormin and Gertler (2006). Our benchmark will be the stock of capital per effective worker.

Trading frictions have attracted a lot of attention since the seminal work of Stigler (1961). The role of frictions in the formation and destruction of matches in the labour market is now well understood (Pissarides, 2000; Mortensen, 2003). More frictions on the job market decreases labour market efficiency and increases unemployment. Modelling trading frictions on the goods market has made it possible to price money positively without the help of ad-hoc assumptions (Shi, 1997; Lagos and Wright, 2005). It has also helped clarify the effect of inflation on trade (see, e.g., Berentsen, Rocheteau and Shi, 2007). In all these models trading frictions tend to reduce the volume of exchange, either through the quantity traded (intensive margin) or through the number of trades (extensive margin).

Though important, we believe that this *lower-trade effect* of frictions is only part of the story. If firms have to borrow money to buy the capital they need, and if frictions make it uncertain whether they will find a worker or a customer, then capital decisions by firms become risky ones. What if a firm did not get any customer? What if a firm could not even find a worker? Incorporating a capital decision into a model with frictions (in both the labor market and the goods market) generates a risk of default for firms with three important effects for the economy: First, unmatched firms will disappear from the economy; second, workers matched with firms that could not find a customer on the goods market will not be paid; finally, the meeting probabilities on the goods and labour markets will adjust endogenously reflecting entry

decisions and forced exit for bankrupted firms.

In this paper we define trading frictions as features of the economy that make it non-Walrasian, such as the difficulty to find a suitable trading partner, asymmetric information or coordination failures. We will concentrate on the first one, and model it as an endogenous probability of not being able to trade on our two frictional markets: the labour market and the goods market. Inflation will impact both the intensive margin (wages and output) and the extensive margin (meeting probabilities) on these two markets. Since capital supply decisions by households and capital demand by firms are based on the expected payoff on these two markets, we have a simple channel through which monetary policy impact on the accumulation of capital. Once calibrated to the US economy using historical inflation rates from 1949 to 2003, the model is able to replicate the joint co-movement of unemployment and capital per effective worker that follows sustained periods of increasing or decreasing inflation. Specifically, periods of high inflation translate into higher unemployment and an above trend stock of capital per worker, and vice versa. This suggests that the slow-moving oscillations around balanced growth, defined as medium-term business cycles by Blanchard (1997) and Corbin and Gertler (2006), are primarily a monetary phenomenon.

There exists several models that combine frictions on more than one market, or try to link frictions on one market to the operating of other Walrasian markets. For instance, Pissarides (2000) studies an extension of his textbook model of frictional unemployment in which firms have to decide for their level of capital. Neither trade on the goods market nor money are explicitly modelled, however, and capital decisions are taken once the firm has hired a worker eliminating the risk of default (Pissarides, 2000, p. 24). Shi (1999), Aruoba and Wright (2003), and Aruoba, Waller and Wright (2007) examine how frictions on the goods market impact on capital decisions. However, they do not distinguish between firms and households so that any non-used capital can be consumed in the end by the agent, also eliminating the risk of default. Finally, Berentsen, Menzio and Wright (2007) study the interaction between labour market frictions, goods market frictions, and monetary policy. Although there is no capital in their model, there is the risk a firm cannot pay wages if unmatched with a customer. They eliminate

this risk by opening a second-chance Walrasian market on which firms sell leftover output. By contrast, in this paper, the consequences of default for households and markets are explicitly modelled.

The paper is organized as follows. Section 2 describes the environment and the agents' decision problems. In Section 3 we characterize the equilibrium. Section 4 describes some medium-term business cycle features that are relevant for our purpose. The model is calibrated in Section 5 where we compare its predictions with the data. The last section concludes.

## 2 The Model

The economy is populated with two types of agents, *households* and *firms*. We index them by  $h$  and  $f$ , respectively. The measure of households is one, and the measure of firms is arbitrarily large, although not all firms will be active at any point in time. Households work for firms, lend money to firms, consume and save. Firms produce using labour and capital. Firms borrow money from households to build capital, hire workers to whom they pay wages. In addition, firms pay interests (on loans) and dividends to households.

Time is discrete and the horizon is infinite. Each period is divided into three subperiods, say morning, afternoon and night, during which the market structure and economic activity differ. Agents discount between periods at rate  $\beta \in (0, 1)$ , but not between subperiods within a period. As shown in Figure 1, we introduce three value functions for the three markets,  $U^j$ ,  $V^j$  and  $W^j$ , respectively, with  $j = h, f$ .

Money is intrinsically useless, perfectly divisible and storable. The gross growth rate of the money supply at date  $t$  is  $\gamma_t$ ; that is,  $M_{t+1} = \gamma_t M_t$ , where  $M_t$  is the quantity of money per household at the beginning of period  $t$ . New money is injected by lump-sum transfers to households in the last subperiod. In what follows, we look at a representative period  $t$  and work backwards from the last to the first subperiod.

*The last subperiod:* Several activities take place in the last subperiod. Households are

endowed with a vector of endowments  $\vec{x}$ , which they can consume in quantity  $x$ . These endowments, however, can also be sold to firms as a necessary input (raw materials) to produce the capital firms need. Firms are the only agents with the ability to turn endowments into capital. Without loss of generality, we assume that capital is produced by firms according to a linear technology

$$k = \vec{x} - x,$$

where  $\vec{x} - x$  are leftover endowments. These leftover endowments are sold to firms on a Walrasian market to operate at the end of this last subperiod. In this paper, we take it seriously the idea that capital is transformed from raw materials, and capital is essentially a real good. We adopt this approach to guarantee that capital will not compete with fiat money as medium of exchange.

Prior to this market for endowments, there is a financial market on which firms (who have just paid the wages, interests and dividends to households), can borrow money from households, money they will need in the coming market to buy endowments from households to build capital. While firms decide how much to borrow, households decide how much of their wealth to lend to firms and how much to keep in the form of money. This supply of loanable funds coming from households and the demand for loanable funds coming from firms equalize on the Walrasian financial market determining the real interest rate in the economy. Moreover, all incomes due by firms to households (wages, interests and dividends) are paid at the opening of this last subperiod.

A key feature of the model is that some firms will go bankrupt and hence will not be able to meet their payment commitments. Basically, a firm will be able to pay out wages and interests on loans if and only if the firm is matched on the job market and successfully trades on the goods market, an event that happens with probability  $\psi$  to be defined below. In other words, any firm that is either unmatched on the job market or unable to trade on the goods market will have no revenues, and will have to go bankrupt. This generates a flow of firms out of the economy: a portion  $1 - \psi$  of firms exit the economy every period. We call  $\psi$  the *survival*

*rate* for firms. This also generates a risk of default, which enters the budget constraint of the representative household. Apparently, bankruptcy of firms is welfare costly in that it destroys the resources (labor, capital) that firms borrowed from households.

*The second subperiod:* Generally speaking, there are three pricing alternatives that could be used to model decentralized goods markets: bargaining, competitive search (directed search), and competitive pricing (Walrasian price taking). This paper uses competitive pricing for the following reason: when price is determined by bargaining or competitive search, firm's capital stock enters the maximizing objective function as a state variable. As a result, not only existing firms' and new firms' demand for capital for the next period will be different, but prices and quantities traded will also be different across different markets. This tends to produce a non-degenerate distribution of capital stock. With competitive pricing, however, firms and households take price of goods as given. In this case, existing firms' and new firms' demand for capital as well as supply of goods will be identical. This makes the model analytically tractable.<sup>1</sup>

The idea of introducing perfectly competitive markets into search models can go back at least to Lucas and Prescott (1974) model of unemployment. Following the same idea, we capture search-type frictions by assuming that, although there is a perfectly competitive market in the second subperiod, not all agents get in. Specifically, we let the measures of households and firms that get in to the goods market be a function of the measures that want to get in,  $M^h = M^h(1, 1 - u)$  and  $M^f = M^f(1, 1 - u)$ , which implies the probabilities of getting into the market are  $\alpha^h = M^h(1, 1 - u)$  and  $\alpha^f = M^f(1, 1 - u)/(1 - u)$ , where  $u$  is the measure of unemployed households. Once successfully getting in to the goods market, households and firms trade competitively at a price of  $\phi$ . As a result of the trade, the firm uses capital and labor to produce  $q^f$ , the household consumes  $q^h$ , and money changes hands from the household to the firm. The competitive goods market closes when the second subperiod ends.

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<sup>1</sup>An alternative way to avoid the distribution of capital is to assume capital completely depreciate between two periods. With this assumption, the distribution of capital is degenerate under all three pricing mechanisms.

*The first subperiod:* The labor market that opens in the first subperiod is a standard Mortensen-Pissarides labor market. In that market, existing jobs are destroyed at an exogenous rate  $\delta$ . New firms enter the market at a cost and post vacancies, and are matched bilaterally with unemployed households at random. The probability for a household to meet a firm is  $\lambda^h = L(u, v)/u$ , where  $v$  is the measure of vacancies posted by firms. As is standard, the matching function  $L$  has constant return to scales, and thus  $\lambda^h = L(1, v/u)$ . Likewise, the probability for a firm to meet a household is  $\lambda^f = L(u/v, 1)$ .

When a firm and a worker meet, they bargain over the wage and sign a contract. This contract stipulates that (1) the firm will use the worker's labour force if the firm successfully enters the goods market. The nominal wage  $w$  is determined via Nash bargaining between the firm and the household. Although wages are determined in the first subperiod, but they are not actually paid until the last subperiod; (2) if the firm fails to enter the goods market, then no production will take place. In this case, the firm will go bankrupt, and no wage will be paid.

We want to emphasize that unemployment exists in the economy because during the matching process some vacancies will not find any worker (an event that happens with endogenous probability  $1 - \lambda^f$ ), and some of the existing jobs will be destroyed at the exogenous rate  $\delta$ . Both events create unemployment.

## 2.1 Households

As we mentioned earlier, a household will receive wage payment from a firm if and only if, first, the household is employed by the firm, and, second, the firm has successfully traded on the goods market and thus has revenues to honour its payment commitments. As in the standard Mortensen-Pissarides model, we let  $e$  denote *employment status*:  $e = 1$  indicates that a household is matched with a firm in the labor market;  $e = 0$  indicates otherwise. In addition, we let  $s$  denote *trading status* of that the household has signed a wage contract with:  $s = 1$  indicates the firm has successfully traded on the previous goods market;  $s = 0$  indicates otherwise.

Let  $W_{e,s}^h$  denote the household's expected payoff from entering the last subperiod with  $m$  units of money and  $a$  units of financial assets. A representative household chooses consumption of good  $x$ , lends  $\hat{a}$  to the firms in the financial market, and brings money balances  $\hat{m}$  into the next period, to solve

$$W_{e,s}^h(m, a) = \max_{x, \hat{m}, \hat{a}} \left\{ x + \beta \hat{U}_s^h(\hat{m}, \hat{a}) \right\} \quad (1)$$

$$s.t. \hat{m} + \hat{a} + px = p\bar{x} + \Delta + \psi(1+r)a + esw + e(1-s)b + m + \tau - T,$$

where  $\bar{x}$  is the household's endowment,  $\Delta$  is dividend income,  $p$  is the price of  $x$ ,  $r$  is the real interest rate,  $b$  is unemployment insurance benefit,  $\tau$  is the lump-sum transfer from the central bank, and  $T$  is a lump-sum tax. For money to grow at a constant rate  $\gamma$ , the lump-sum transfer must satisfy  $\tau = (\gamma - 1)M$ . For notational ease we use a hat over a variable to denote the value of the variable in the next period.

Whether it signed a contract or not in the previous job market, a household still has resources via his endowments, interests on loans to firms and dividends paid by firms. Whether it signed a contract or not, a household need to decide how much money to bring along for shopping in the decentralized goods market and how much to lend to firms. It is clear from the household's budget constraint that the default risk of firms affect households' wealth in two ways: first, the household will have wage income if and only if it is employed ( $e = 1$ ) and the firm that the household has signed a contract with has successfully traded on the previous goods market ( $s = 1$ ). Second, due to the default risk, the rate of return to lend to firms is  $\psi(1+r)$ .

As explained in Lagos and Wright (2005), the quasi-linear utility function implies that the optimal choice of  $(\hat{m}, \hat{a})$  is independent of  $(m, a)$ , and the distribution of  $(\hat{m}, \hat{a})$  is degenerate at the beginning of the following period.

Let  $U_1^h(m, a)$  be the value function for an employed household entering the first subperiod, and  $U_0^h(m, a)$  be the value function of an unemployed household. Thus,

$$\begin{aligned} U_1^h(m, a) &= \delta V_0^h(m, a) + (1 - \delta) V_1^h(m, a), \text{ and} \\ U_0^h(m, a) &= \lambda^h V_1^h(m, a) + (1 - \lambda^h) V_0^h(m, a), \end{aligned}$$



where  $\delta$  is the rate at which current jobs are destroyed, and  $\lambda^h$  is the probability for an unemployment household to find a job. Note that by contrast to firms, separated or unmatched households do not exit the economy. They simply proceed to the goods market knowing that they will not receive any salary in the last subperiod.

An unemployed household entering the goods market with money holding  $m$  and financial assets  $a$  has expected lifetime utility

$$V_0^h(m, a) = \alpha^h \left[ u(q^h) + W_{0,0}^h(m - \phi q^h, a) \right] + (1 - \alpha^h) W_{0,0}^h(m, a),$$

where the utility function  $u$  is concave, with  $u(0) = 0$ , and  $u'(0) = \infty$ . With probability  $\alpha^h$ , the household successfully enters the goods market and trades competitively with firms at a nominal price  $\phi$ . As a result, the household consumes  $q^h$ .

In contrast, for an employed household, with probability  $1 - \alpha^f$  the firm that the household has signed a contract with will go bankrupt, and so the household will not receive any salary in the last subperiod. Thus,

$$\begin{aligned} V_1^h(m, a) &= \alpha^h \left[ u(q^h) + \alpha^f W_{1,1}^h(m - \phi q^h, a) + (1 - \alpha^f) W_{1,0}^h(m - \phi q^h, a) \right] \\ &\quad + (1 - \alpha^h) \left[ \alpha^f W_{1,1}^h(m, a) + (1 - \alpha^f) W_{1,0}^h(m, a) \right]. \end{aligned}$$

## 2.2 Firms

We call a firm that has successfully entered the goods market (and thus has traded with households) in the second subperiod a *surviving firm*. A surviving firm enters the last subperiod with cash receipts  $m$  and capital stock  $k$ . In the last subperiod, existing firms adjust their capital stocks (repays old loans, borrow new loans), pays wages, real interests on previous period capital, and dividends to households. Apparently, firms do not need money in either the first or the second subperiods. Thus, the existing firm's problem is

$$W_1^f(m, k, k_{-1}) = \max_{\hat{k}} \frac{m}{p} - \frac{w}{p} - (1 + r) [k - (1 - \rho)k_{-1}] \frac{p-1}{p} + \beta \hat{U}_1^f(\hat{k}),$$

where  $\hat{k}$  is the firm's demand for capital for the next period, and  $\rho \in (0, 1)$  is the rate at which capital depreciates. The firm's previous-period capital stock  $k_{-1}$  enters the objective function

as a state variable, and it is because the funds that an existing firm borrowed amounted to  $[k - (1 - \rho)k_{-1}]p_{-1}$ . We deflate nominal terms by current price level  $p$ .

We assume free entry of firms: firms considering participating to the economy (and then known as new firms) decides whether to pay a real cost of  $l$  to enter the labor market with a vacancy that might match with an household. New firms that decide to enter the labor market then choose next period capital stock to maximize their expected profits. Even though there is no reason for existing and new firms to choose the same level of capital, we can show that they do so and we simply note this level  $\hat{k}$ . Thus,

$$W_n^f = \max \left\{ 0, -l + \max_{\hat{k}} \beta \hat{U}_n^f(\hat{k}) \right\}.$$

On the labour market existing firms are separated from their workers at rate  $\delta$  and new firms are matched with workers at rate  $\lambda^f$  so that

$$U_1^f(k) = (1 - \delta) V_1^f(k), \text{ and}$$

$$U_n^f(k) = \lambda^f V_n^f(k).$$

Finally, a firm successfully enters the goods market with probability  $\alpha^f$ . As a result, the firm produces  $q^f$  and pays variable costs  $c(q^f)$  so that

$$V_1^f(k) = \alpha^f \left[ -c(q^f) + W_1^f(\phi q^f, k, k_{-1}) \right], \text{ and}$$

$$V_n^f(k) = \alpha^f \left[ -c(q^f) + W_1^f(\phi q^f, k, 0) \right].$$

The cost function represents to cost of moving from one structure of production to another. We assume the cost function  $c$  satisfies the usual assumptions,  $c(0) = c'(0) = 0$ ,  $c'(q) > 0$ , and  $c''(q) \geq 0$ . The cost function here captures the realistic feature that other than the cost of labor and capital, firms incur some variable costs in production, and these costs are proportional to the firm's output. We interpret these cost as a proxy for the putty-clay structure of the economy. The cost of adjusting capital varies over time, from the costs of adjusting to a new capital stock.

Note that while existing firms (that survive the goods market) only need to repay top-up capital stock  $k - (1 - \rho)k_{-1}$ , new firms will repay the full amount of capital stock. Apparently, dividends across these two types of firms will be different.

We can now compute the *survival rate* for firms. The measures of surviving firms and firms that borrowed money on the financial market are  $M^f$  and  $v + M_{-1}^f$ , respectively. Thus, we define the survival rate for firms as follows:

$$\psi = \frac{M^f}{v + M_{-1}^f}. \quad (2)$$

### 3 Equilibrium

Before proceeding to characterize an equilibrium, we introduce some convenient mathematical features of the model. First, due to the linearity of the household's value function  $W_{e,s}^h(m, a)$ , we could reduce the three value functions ( $U^j$ ,  $V^j$  and  $W^j$ ) into one Bellman equation.

$$W_{1,1}^h(m, a) = \beta \left\{ \delta \hat{W}_{0,0}^h(0, 0) + (1 - \delta) \hat{\alpha}^f \hat{W}_{1,1}^h(0, 0) + (1 - \delta)(1 - \hat{\alpha}^f) \hat{W}_{1,0}^h(0, 0) \right\} + \frac{I_1}{p} \\ + \frac{m + \psi(1+r)a}{p} + \max_{\hat{m}, \hat{a}} \left\{ \beta \hat{\alpha}^h [u(\hat{q}^h) - \frac{\hat{\phi} \hat{q}^h}{\hat{p}}] - \hat{m} \left( \frac{1}{p} - \frac{\beta}{\hat{p}} \right) - \hat{a} \left[ \frac{1}{p} - \frac{\beta \hat{\psi}(1+\hat{r})}{\hat{p}} \right] \right\},$$

$$W_{1,0}^h(m, a) = \beta \left\{ (1 - \hat{\lambda}^h) \hat{W}_{0,0}^h(0, 0) + \hat{\lambda}^h \hat{\alpha}^f \hat{W}_{1,1}^h(0, 0) + \hat{\lambda}^h (1 - \hat{\alpha}^f) \hat{W}_{1,0}^h(0, 0) \right\} + \frac{I_0}{p} \\ + \frac{m + \psi(1+r)a}{p} + \max_{\hat{m}, \hat{a}} \left\{ \beta \hat{\alpha}^h [u(\hat{q}^h) - \frac{\hat{\phi} \hat{q}^h}{\hat{p}}] - \hat{m} \left( \frac{1}{p} - \frac{\beta}{\hat{p}} \right) - \hat{a} \left[ \frac{1}{p} - \frac{\beta \hat{\psi}(1+\hat{r})}{\hat{p}} \right] \right\}, \text{ and}$$

$$W_{0,0}^h(m, a) = \beta \left\{ (1 - \hat{\lambda}^h) \hat{W}_{0,0}^h(0, 0) + \hat{\lambda}^h \hat{\alpha}^f \hat{W}_{1,1}^h(0, 0) + \hat{\lambda}^h (1 - \hat{\alpha}^f) \hat{W}_{1,0}^h(0, 0) \right\} + \frac{I_0}{p} \\ + \frac{m + (1+r)a}{p} + \max_{\hat{m}, \hat{a}} \left\{ \beta \hat{\alpha}^h [u(\hat{q}^h) - \frac{\hat{\phi} \hat{q}^h}{\hat{p}}] - \hat{m} \left( \frac{1}{p} - \frac{\beta}{\hat{p}} \right) - \hat{a} \left[ \frac{1}{p} - \frac{\beta(1+\hat{r})}{\hat{p}} \right] \right\},$$

where  $I_1 = p\bar{x} + \tau + \Delta + w - T$ , and  $I_0 = p\bar{x} + \tau + \Delta + b - T$ .

It is straightforward to show that the demand for  $q$  are identical across households, regardless of their employment status. The first-order conditions are

$$\hat{m} : \beta \hat{\alpha}^h [u'(\hat{q}^h) - \frac{\hat{\phi}}{\hat{p}} \hat{q}^{h'}(\hat{m})] = \frac{1}{p} - \frac{\beta}{\hat{p}}, \text{ and} \quad (3)$$

$$\hat{a} : \beta \hat{\alpha}^h [u'(\hat{q}^h) - \frac{\hat{\phi}}{\hat{p}}] \hat{q}^{h'}(\hat{a}) = \frac{1}{p} - \frac{\beta \hat{\psi}(1 + \hat{r})}{\hat{p}}. \quad (4)$$

In these expressions,  $\hat{q}^{h'}(\hat{m})$  reveals the relationship between households' money holding and the amount of goods purchased, and  $\hat{q}^{h'}(\hat{a})$  reveals the marginal effect of households' supply of loanable funds on  $\hat{q}^h$ . Once we specify how  $q^h$  is determined in equilibrium, we can substitute for these derivatives in (3) and (4).

Second, with repeated substitution, the existing firm's maximization problem can be simplified into the following program:

$$V_1^f(k) = \alpha^f \left\{ \frac{\phi q^f}{p} - \frac{w}{p} - c(q^f) - (1+r)[k - (1-\rho)k_{-1}] \frac{p-1}{p} \right\} + \beta \alpha^f (1-\delta) \max_{\hat{k}} \hat{V}_1^f(\hat{k}).$$

The first-order condition with respect to  $\hat{k}$  is

$$\left[ \frac{\hat{\phi}}{\hat{p}} - c'(\hat{q}^f) \right] \hat{q}^{f'}(\hat{k}) = (1 + \hat{r}) \frac{p}{\hat{p}}, \quad (5)$$

Condition (5) is a standard neoclassical capital demand function of firms: it equates the marginal benefit to the marginal cost of acquiring capital. As we mentioned earlier, a convenient feature of the model is that new firms' demand for capital are identical to existing firms', and, therefore, condition (5) also defines new firms' demand for capital.

We now characterize equilibrium. Here is an outline of what will follow. We begin by specifying how nominal wages ( $w$ ), quantity of goods supplied ( $q^f$ ), and consumption of goods ( $q^h$ ) are determined. We then use some properties of these solutions to simplify the households and firms' maximization problem. In particular, we derive the money demand function and conditions with respect to the demand and supply of capital. These three conditions together with the goods market clearing condition, the credit market clearing condition, the wage equation, the steady state condition for unemployment (the Beveridge curve), and the free-entry condition in the labor market define a steady-state monetary equilibrium.

### 3.1 The Generalized Nash Problem in the Labor Market

As we said, we assume in the labor market the nominal wage  $w$  solves the generalized Nash problem, with bargaining power for the households given by  $\eta$  and threat points given by

continuation values. The household's payoff from being employed is  $V_1^h(m, a)$  and threat points  $V_0^h(m, a)$ . Due to the linearity of  $W^h(m, a)$ , the surplus for household is  $V_1^h(m, a) - V_0^h(m, a) = \alpha^f \frac{w-b}{p} + \alpha^f \beta(1 - \delta - \hat{\lambda}^h)[\hat{\alpha}^f \hat{W}_{1,1}^h(0, 0) + (1 - \hat{\alpha}^f) \hat{W}_{1,0}^h(0, 0) - \hat{W}_{0,0}^h(0, 0)]$ . Similarly, the firm's surplus is  $V_n^f(k) = \alpha^f [\frac{\phi q^f}{p} - \frac{w}{p} - c(q^f) - (1+r)k \frac{p-1}{p}] + \alpha^f \beta(1 - \delta) \max_{\hat{k}} \hat{V}_1^f(\hat{k})$ . Notice that the current-period nominal wages  $w$  do not appear in either  $\hat{\alpha}^f \hat{W}_{1,1}^h(0, 0) + (1 - \hat{\alpha}^f) \hat{W}_{1,0}^h(0, 0) - \hat{W}_{0,0}^h(0, 0)$  or  $\hat{V}_1^f(\hat{k})$ , so we take their values as given at this stage. Let  $A$  and  $B$  denote the equilibrium values of these two terms, respectively. The bargaining problem in the labor market is

$$\max_w \left[ \frac{w-b}{p} + \beta(1 - \delta - \hat{\lambda}^h)A \right]^\eta \left[ \frac{\phi q^f}{p} - \frac{w}{p} - c(q^f) - (1+r)k \frac{p-1}{p} + \beta(1 - \delta)B \right]^{1-\eta}.$$

The solution to this bargaining problem yields

$$\frac{\eta}{1 - \eta} = \frac{\frac{w-b}{p} + \beta(1 - \delta - \hat{\lambda}^h)A}{\frac{\phi q^f}{p} - \frac{w}{p} - c(q^f) - (1+r)k \frac{p-1}{p} + \beta(1 - \delta)B}. \quad (6)$$

We will substitute the steady-state values of  $A$  and  $B$  into this equation after we derive the equilibrium solution.

### 3.2 The Goods Market

We now proceed to specify how equilibrium is determined in the goods market. As households and firms take goods price  $\phi$  parametrically and trade competitively with each other, the goods market clearing condition requires that

$$M^h q^h = M^f q^f, \quad (7)$$

where  $M^h$  and  $M^f$  are respectively the measures of households and firms who successfully enter the goods market.

It is a simple matter to show that due to the positive opportunity cost of holding money, households do not hold "idle" money balances; that is,  $q^h = m/\phi$ . Likewise, firms exhaust their capital stocks. Thus,

$$q^f = f(k, 1) \quad (\text{qf})$$

where it is clear that a representative firm uses the hired worker in combination with  $k$  units of capital to produce output. The production function  $f$  satisfies the usual properties:  $f(k, 0) = f(0, 1) = 0$ ,  $f_1 > 0$ , and  $f_{11} < 0$ .<sup>2</sup>

### 3.3 Equilibrium

We can now use the properties of the goods market solution to simplify the households' and firms' maximization problem. Inserting  $q^h(m) = 1/\phi$ ,  $q^h(a) = f_1(a/p_{-1}, 1)/p_{-1}$  and  $q^f(k) = f_1(k, 1)$  into first-order condition (3), (4) and (5), dropping all time indexes in what follows (as we focus on stationary equilibria in which real allocations are constant), we arrive at

$$\frac{u'(q^h)}{\phi/p} = 1 + \frac{\gamma - \beta}{\beta\alpha^h}, \quad (8)$$

$$\left[ u'(q^h) - \frac{\phi}{p} \right] f_1\left(\frac{a}{p_{-1}}, 1\right) = \frac{\gamma - \beta\psi(1+r)}{\gamma\beta\alpha^h}, \text{ and} \quad (9)$$

$$\left[ \frac{\phi}{p} - c'(q^f) \right] f_1(k, 1) = \frac{1+r}{\gamma}, \quad (10)$$

where  $\gamma$  is the gross growth rate of money supply, and the steady-state survival rate for firms  $\psi$  is given by (2), with  $M^f = M_{-1}^f$ .

Condition (8) is a standard money demand function, as in Lagos and Wright (2005). It equates the marginal benefit and the marginal cost of acquiring money. Condition (9) and (10) defines the supply ( $a/p_{-1}$ ) and demand ( $k$ ) for capital, respectively. Notice this is a system with nine unknowns ( $q^h, q^f, r, k, u, v, m/p, w/p, a/p_{-1}$ ) but only six equations so far (equations (6) to (10)), we need three more equilibrium conditions to close the model.

First, recall that the free-entry condition in the labor market requires that  $\max_{\hat{k}} \beta \hat{U}_n^f(\hat{k}) = l$ , where  $l$  is the real cost that a new firm must incur to enter the labor market. Inserting the steady-state value of  $\hat{U}_n^f(\hat{k})$ , we arrive at

$$l = \frac{\beta\alpha^f\lambda^f}{1 - \beta\alpha^f(1 - \delta)} \left[ \frac{\phi q^f}{p} - \frac{w}{p} - c(q^f) - \frac{1+r}{\gamma}k + \beta\alpha^f(1 - \delta)(1 - \rho)\frac{1+r}{\gamma}k \right]. \quad (11)$$

---

<sup>2</sup>Growth in total factor productivity and in the number household could be introduced easily. For notational convenience we stick to a model without exogenous growth. Both elements are reintroduced in the model in the calibration and simulation part.

Second, the credit market-clearing condition equates the supply of capital ( $a/p_{-1}$ ) to the demand for capital ( $k$ ). Notice that the demand for capital comes from two types of firms: existing and new firms, and the measure of each type of firms is  $M^f$  and  $v$ , respectively. Hence, the market-clearing condition becomes

$$\frac{a}{p_{-1}} = (M^f + v)k. \quad (12)$$

Finally, note that the so-called Beveridge curve (i.e., the steady-state condition for unemployment) will allow us to express  $v$  in terms of the unemployment rate  $u$ . Thus,

$$N(u, v) = (1 - u)\delta. \quad (13)$$

To close this section, recall that so far we have taken the values of  $\hat{\alpha}^f \hat{W}_{1,1}^h(0, 0) + (1 - \hat{\alpha}^f) \hat{W}_{1,0}^h(0, 0) - \hat{W}_{0,0}^h(0, 0)$  and  $\hat{V}_1^f(\hat{k})$  as given. We now solve for the steady-state values of these terms as follows:

$$A \equiv \hat{\alpha}^f \hat{W}_{1,1}^h(0, 0) + (1 - \hat{\alpha}^f) \hat{W}_{1,0}^h(0, 0) - \hat{W}_{0,0}^h(0, 0) = \frac{\alpha^f}{1 - \beta\alpha^f(1 - \delta - \lambda^h)} \frac{w - b}{p}, \text{ and}$$

$$B \equiv \hat{V}_1^f(\hat{k}) = \frac{\alpha^f}{1 - \beta\alpha^f(1 - \delta)} \left[ \frac{\phi q^f}{p} - \frac{w}{p} - c(q^f) - \frac{1 + r}{\gamma} \rho k \right].$$

Substituting the steady-state values of  $A$  and  $B$  into the wage equation (6) and rearrange, we arrive at

$$\left[ \frac{1 - \eta}{1 - \beta\alpha^f(1 - \delta - \lambda^h)} + \frac{\eta}{1 - \beta\alpha^f(1 - \delta)} \right] \frac{w}{p} - \frac{1 - \eta}{1 - \beta\alpha^f(1 - \delta - \lambda^h)} \frac{b}{p} \quad (14)$$

$$= \frac{\eta}{1 - \beta\alpha^f(1 - \delta)} \left[ \frac{\phi q^f}{p} - c(q^f) - \frac{1 + r}{\gamma} k + \beta\alpha^f(1 - \delta)(1 - \rho) \frac{1 + r}{\gamma} k \right].$$

Conditions (8) to (14) fully characterize a stationary monetary equilibrium. The unknowns are:

**Definition:** A stationary monetary equilibrium consists of,

(a) a set of prices  $\{p, r, w\}$

- (b) the household's decisions  $\{x, \hat{m}, \hat{a}\}$   
(c) the firm's decisions  $\{\hat{k}\}$   
(d) the firm's dividend  $\Delta$   
(e) the government's unemployment insurance benefit  $b$ , lump-sum tax  $T$ , and money transfers  $(\gamma - 1)M$

such that,

- (1) given  $\{p, r, w\}$ , the household's optimal plan solves the maximization problem (1),  
(2) given  $r$ , the firm's capital demand solves the firm's profit maximization problem,  
(3) all markets clear,  
(4)  $\Delta$  equals to the firm's net profit,  
(5) real allocations  $\{q, x\}$  are constant over time.  
(6) the government budget constraint is balanced.

To better understand how the equilibrium is determined, manipulations of the equilibrium conditions enable us to reduce the system to a simple two-variable-two-equation  $(k, u)$  system as follows:

$$\frac{\beta\alpha^h u'(\frac{f(k,1)M^f}{M^h})}{\gamma - \beta + \beta\alpha^h} \left\{ (\gamma - \beta)f_1[(v(u) + M^f)k, 1] + \beta\psi f_1(k, 1) \right\} = 1 + \beta\psi f_1(k, 1)c'(f(k, 1)), \text{ and} \quad (15)$$

$$\begin{aligned} & f(k, 1) \frac{u'(\frac{f(k,1)M^f}{M^h})}{1 + \frac{\gamma - \beta}{\beta\alpha^h}} - c(f(k, 1)) - [1 - \beta\alpha^f(1 - \delta)(1 - \rho)]kf'(k, 1) \left[ \frac{u'(\frac{f(k,1)M^f}{M^h})}{1 + \frac{\gamma - \beta}{\beta\alpha^h}} - c'(f(k, 1)) \right] \\ &= \frac{b}{p} + [1 - \beta\alpha^f(1 - \delta) + \eta\beta\alpha^f\lambda^h] \frac{l}{(1 - \eta)\beta\alpha^f\lambda^f}. \end{aligned} \quad (16)$$

Equation (15) equates the marginal benefit to the cost of supplying loanable funds. As it determines households' supply of loanable funds for a given  $u$ , we call it the capital supply curve. We call equation (16) the capital demand curve, as it defines firms' demand for capital for a given  $u$ . The shape of these two curves are derived in section 5.

**Proposition 1** *There always exists a unique steady state monetary equilibrium. A rise in inflation increases unemployment, but the effect on capital stock is ambiguous.*



In the model frictions are represented by various probabilities through which the two types of agents, firms and households, fail to find a trading partner. Being unemployed is one of them. Although the implications of frictions for households can be damageable (no wage if not employed or no purchase on the goods market if not matched with a firm), the consequences for firms are more dramatic: firms simply disappear, implying destruction of the borrowed resources and the end of any employment contract. The frictions for firms are summarized in what we call a survival rate which sums the probability of being successful for a firm on both the labour and the goods market.

## 4 Quantitative Analysis

In the coming two sections we simulate the model and compare the result with US data for the 1949-2003 period. We will look at slow-moving oscillations of time series which occur over a longer time frame than usual business cycles. These slow-moving components are documented in King and Rebello (1999) Figure 1.C for instance. Comin and Gertler (2006) name these slow-moving oscillations medium-run business cycle.

We assume the long-run behaviour of the US economy is represented by the steady state of the Solow model with Harrod-neutral technological progress,  $Y = F(K, AL)$ . The variable  $K$  represents the total stock of capital,  $A$  is total factor productivity and  $L$  is the civilian workforce. The balanced growth path is characterized by a constant stock of capital per effective worker,  $k = K/AL$ , and constant output per effective worker  $Y/AL$ . We note  $\bar{k}$  the constant value for  $k$ . In the RBC literature the equilibrium stock of capital per worker is computed using the Hodrick-Prescott trend of the actual  $k$  time series, and cyclical deviations from the trend correspond to the business cycle. Here we compute  $\bar{k}$  as the mean of the actual  $k$ . We then consider deviations from the HP trend of  $k$  around  $\bar{k}$ . Following Comin and Gertler (2006) we interpret those oscillations as medium run business cycles.

Let us start by describing these time series. The two main variables we are after are unemployment (as a measure of trading frictions) and capital. Since growth in the labour force

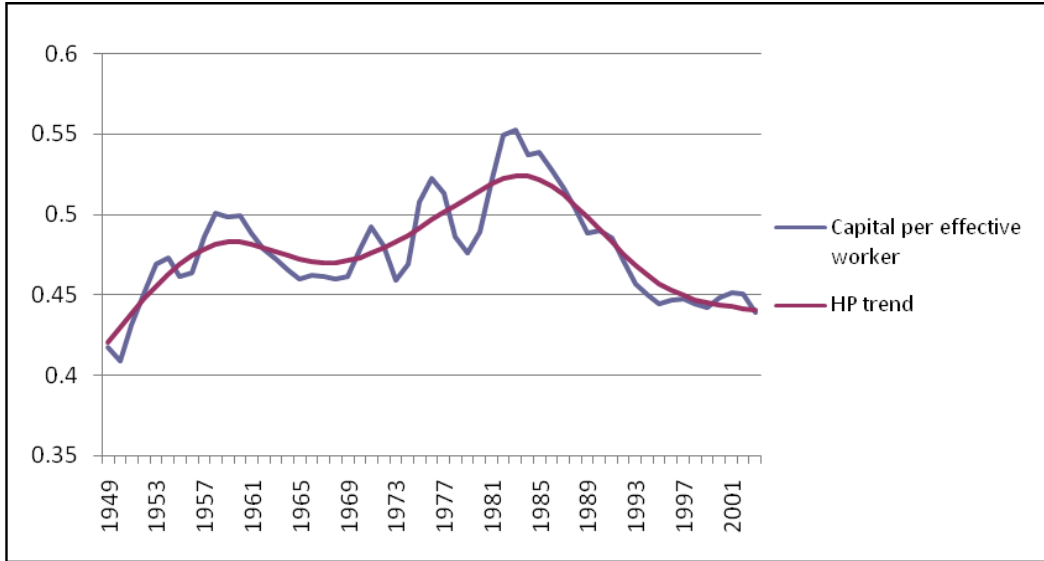


Figure 1: Capital per effective worker

and TFP are not modelled, we need first to stationarize our measure of the capital stock by working with the stock of capital per unit of effective worker, that is  $k = K/AL$ . One problem with this strategy is that both TFP and labour participation are pro-cyclical (see, for instance, King and Rebello 1999). This means that we may see fluctuations in  $k$  that are due to rapid increases in TFP and labour participation for which our model has nothing to say. For instance, at times of booms  $k$  will fall (or not increase as much as usual) due to sharp increase in  $A$  and  $L$ . We control for this by computing a time series of  $k$  in which rather than using observed growth rates of  $A$  and  $L$ , we use the mean of each growth rates over the study period. Hence  $k$  is constructed by dividing each year's actual stock of capital (our data sets are described in the Appendix) by our computed values of  $A$  and  $L$  using these growth rates. This correction is consistent of King and Rebello's investigation result showing that "most of the cyclical variation in total hours worked stems from changes in unemployment (in case one wants to interpret  $L$  as total hours worked).

Figure 1 displays plots of actual data for the stock of capital per effective worker,  $K/AL$ , for the US economy starting 1949 ending 2003 where the solid lines represent the HP trend.

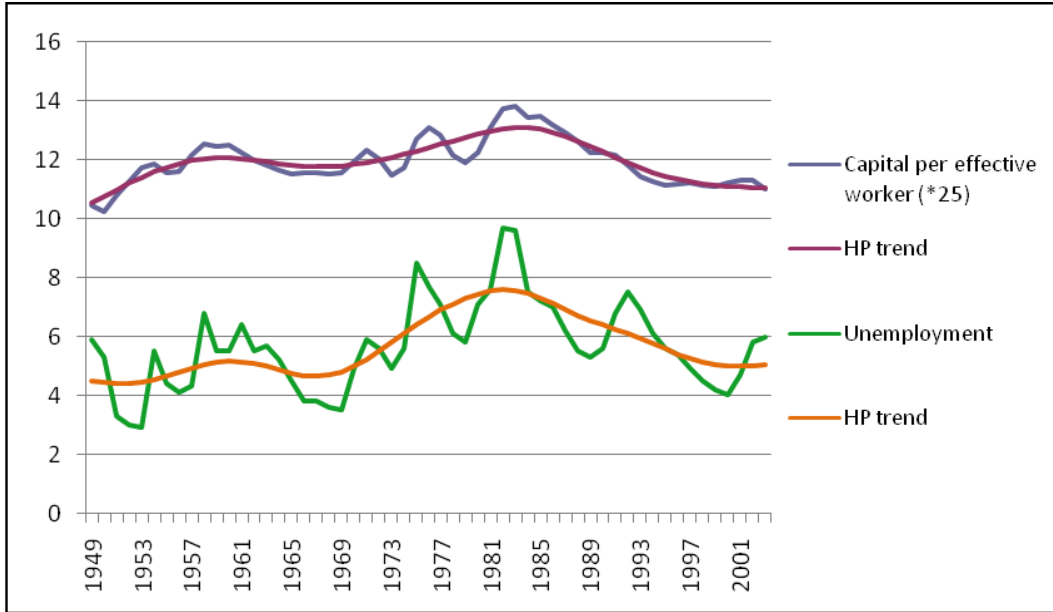


Figure 2: Capital per effective worker and unemployment

The mean and the variance are clearly independent from time, although the covariance is not. Especially there seems to be quite a bit of persistence. This is confirmed by several unit root tests which show that the Null cannot be rejected.<sup>3</sup> Figure 2 represents the same data (which we multiply by 25 for sake of comparison) combined with observed and trended unemployment for the same period. A quick look shows that the two trends seem to be positively correlated. This confirms business cycle investigations by Stock and Watson (1998) and King and Rebello (2000) that the capital stock is acyclical.

Figure 3 represents combinations of the actual and HP-trend values for unemployment  $u$  and our corrected measure of capital per effective worker  $k$ . Finally Figure 4 gives the least-square linear regression of actual corrected  $k$  against actual  $u$ . Both figures confirm our previous observation: there is a clear positive relationship between unemployment and the stock of capital per worker. The US economy overaccumulates capital at times of recessions while the opposite

<sup>3</sup>The augmented Dickey-Fuller cannot reject the Null with a  $p$ -value of 0.8762. The Phillips-Perron test cannot reject the Null with a  $p$ -value of 0.6582. A correlogram of  $k$  and a plot of the actual  $k$  against the corrected  $k$  are given in the Appendix.

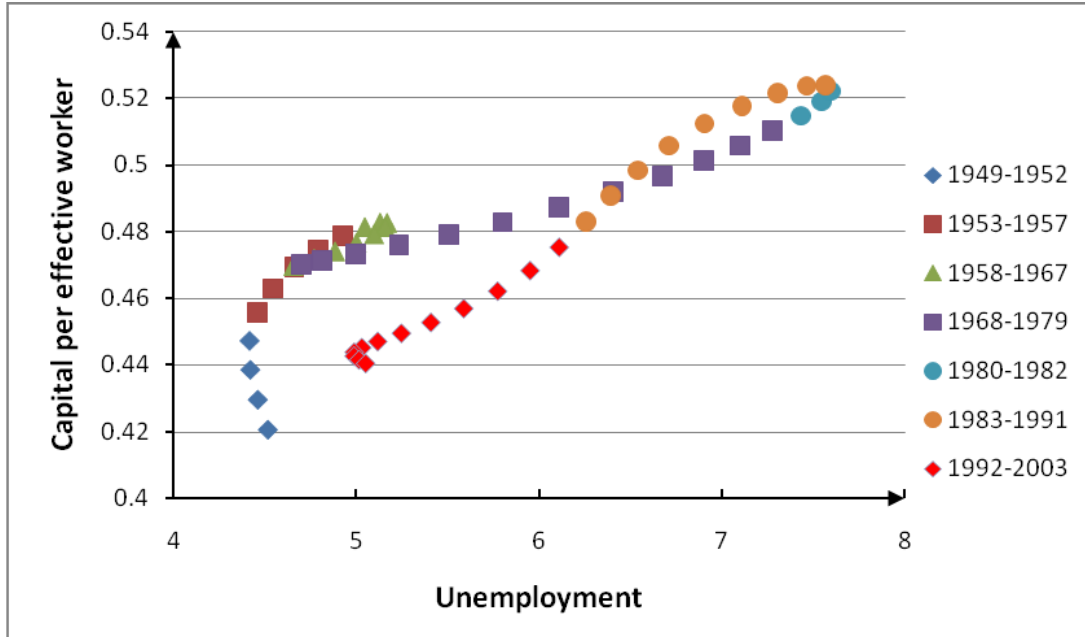
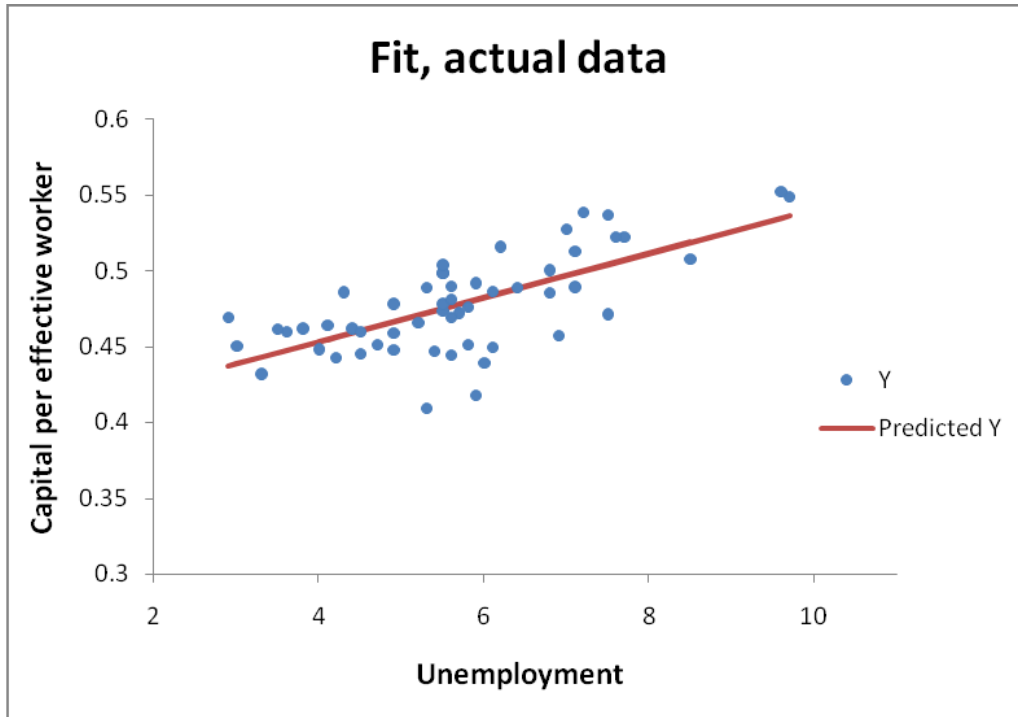


Figure 3: HP trend combinations of unemployment and capital per effective worker

movement is observed at times of economic boom.

## 5 Calibration and Simulation

The two steady state equilibrium relationships between  $u$  and  $k$  are too complex to derive comparative statics results via differentiation, so we directly calibrate the model to post-war US data. Three blocks need to be taken a stand on: the functional form for the utility, variable cost and production functions, the technology parameters and the preferences parameters. In order to facilitate comparison we will use a set of baseline values that are used in dynamic stochastic equilibrium models and real business cycle models, drawing mostly from Cooley and Prescott (1995) and Gomme and Rupert (2007). We adapt the calibration exercise to the question we address and the model we are using, however. The preference parameters, for instance, are obtained by following the procedure in Lucas (2000), Lagos and Wright (2005) and Craig and Rocheteau (2007) of matching the theoretical money demand to the data. Similarly,



the job market separation rate is derived using steady state equations and data from Shimer (2005).

It is probably a good idea at this stage to give a brief summary of the main aspects of the model and the key differences with a textbook DSGE model. First, the capital market is the only competitive market. Both the goods market and the labour market are frictional in the sense that there is a strictly positive and endogenous probability of not finding a trading partner. Second, the number of firms and matching properties are not fixed but respond to changes in the economy, opening the door to the study of booms and recessions through endogenous shifts in meeting probabilities. Third, we are interested in steady state equilibria interpreted as medium-run equilibria of the economy, that is in between the very long run sight of the Solow model and the short-run business analysis of DSGE models (Blanchard, 1997). Fourth, technology is not subject to stochastic shocks and our measure of capital is interpreted as the stock of capital per unit of effective worker. Finally, labour is indivisible as in Hansen (1989)

and household do not value leisure: all household are looking for a full-time job. Our main message is that fluctuations in the stock of capital are responses from households and firms to shifts in the intensity of frictions in the economy, and that changes in monetary policy is the prime responsible for those shifts.

### 5.0.1 The functional forms

The production function is given by  $q = f(k, l) = k^\theta$  if  $l = 1$  and 0 otherwise so that labour is indivisible as in Hansen (1985). The utility function is the constant elasticity substitution (CES) utility function given by  $u(q) = Aq^{1-\alpha}/1 - \alpha$ . The parameters  $A$  and  $\alpha$  are estimated by fitting the money demand from the model to the US data. As first approximation we set  $A = 1$  and  $\alpha = 0.5$ . Finally the variable cost function is taken to be  $c(q) = 0.1q$ .

### 5.0.2 The technology parameters

The technology parameters are  $\rho$ ,  $\theta$ ,  $\delta$  and  $\eta$ . The depreciation rate  $\rho$  is the weighted average of the various types of capital that enter the capital stock (except housing). We set its value for be equal to 10% as in Prescott (1986) which is also consistent with measures of depreciation calculated by the Bureau of Economic Analysis. Since the capital market is competitive, capital is paid its marginal productivity so that  $\theta$  is the capital share. Following Cooley and Prescott (1995) we set the share of capital in production  $\theta$  to 0.36. Because the labour market is not competitive,  $1 - \theta$  will not correspond to the share of labour (see Merz 1995; Andolfatto 1996). We follow Shimer (2005) and set the elasticity of the job matching function  $\eta = 0.72$ . The annual job destruction rate  $\delta$  is constructed in the following way: given the average monthly job finding probability  $\lambda^h$  reported in Shimer (2005), which yields  $\lambda^h \simeq 1$  over the year, and the average unemployment rate over 1949-2003, that is  $\bar{u} = 0.566$ , we compute the corresponding job destruction rate using the functional form of  $\lambda^h$  and the steady state equality on the job market to find  $\delta = 0.06$ .

### 5.0.3 The preference parameters

The preference parameters are  $\beta$ ,  $\alpha$ . The preference parameter  $\beta$  is computed in two ways. A first method sets it equal to the real interest rate computed as the difference between the average nominal interest rate over the study period, 7.2%, and the average inflation rate, 3.9%. This yields an annual discount rate of  $\beta = 0.967$ . Because the model produces a value for the real interest rate, we also provide an estimation using our own model value of the real interest rate, a calibration exercise known as Uzawa discounting.

### 5.1 The steady state $k$ and $u$

Table 1 summarizes the value attributed to the paramters.

$\rho$	$\theta$	$\delta$	$\eta = \sigma$	$\alpha$	$\beta$
0.1	0.36	0.06	0.72	0.5	0.967

Figure 5 plots the capital supply curve, labeled  $K^S$ , and the capital demand curve, labeled  $K^D$ , in  $(k, u)$  space. The capital supply curve correspond to the combinations of  $u$  and  $k$  that satisfy the first-order condition for households with respect to their supply of loanable funds. Prior to any substitution it is given by

$$\begin{aligned}
 & \underbrace{\beta\alpha^h \left[ u'(q^h) - \frac{\phi}{p} \right] f_1\left(\frac{a}{p-1}, 1\right)}_{\text{marginal return on the goods market}} \quad + \quad \underbrace{\frac{\beta\psi(1+r)}{\gamma}}_{\text{marginal return on the capital market}} \\
 = & \underbrace{1}_{\text{marginal cost}}
 \end{aligned}$$

The capital demand curve is given by the combinations of  $u$  and  $k$  that satisfy the first-order condition for firms with respect to their demand for capital. It is given by

$$\underbrace{\frac{\phi}{p} \cdot f_1(k, 1)}_{\text{marginal return}} = \underbrace{\frac{(1+r)}{\gamma}}_{\text{marginal cost}}$$

The capital supply curve is upward sloping. One reason for that is the following: when unemployment increases, there are less active firms around which lowers the expected

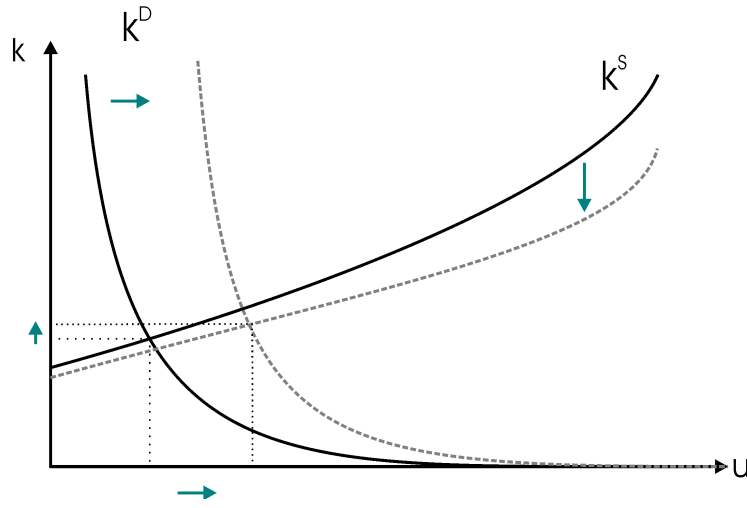


Figure 4: Effect of inflation on the equilibrium

quantity traded on the goods market. As a result agents bring less real balances and save more, increasing the supply of loanable funds. As for the downward sloping capital demand curve, there are two conflicting effects. That higher unemployment is associated with a lower expected traded quantity means that less capital is needed. On the other side, because there are less firms around each firms needs more capital individually. From our simulation it appears that the first effect dominates the second one. If that is true, this creates another incentive for household to increase their supply of funds when unemployment increases by increasing the marginal return on capital.

How does inflation impact our equilibrium relationship? Figure 5 illustrates the impact of increasing inflation from 1% to 15%. A rise in inflation shifts the capital supply curve downwards while the demand curve shifts outward. As a result, unemployment always increases but the overall effect on the equilibrium level of capital in each firm ambiguous. According to our simulation, however, it seems that higher inflation will drive up the stock of capital in each firm.



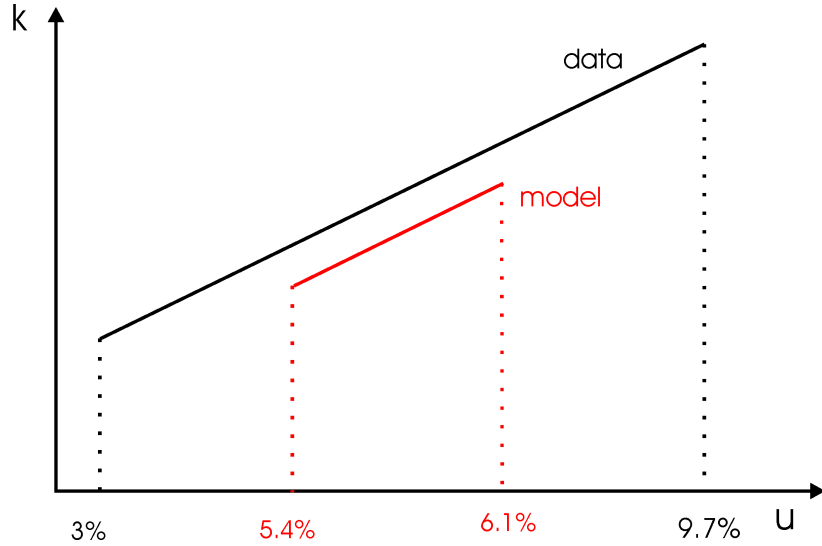


Figure 5: "Stock of capital in each active firm per effective worker", the model and the data

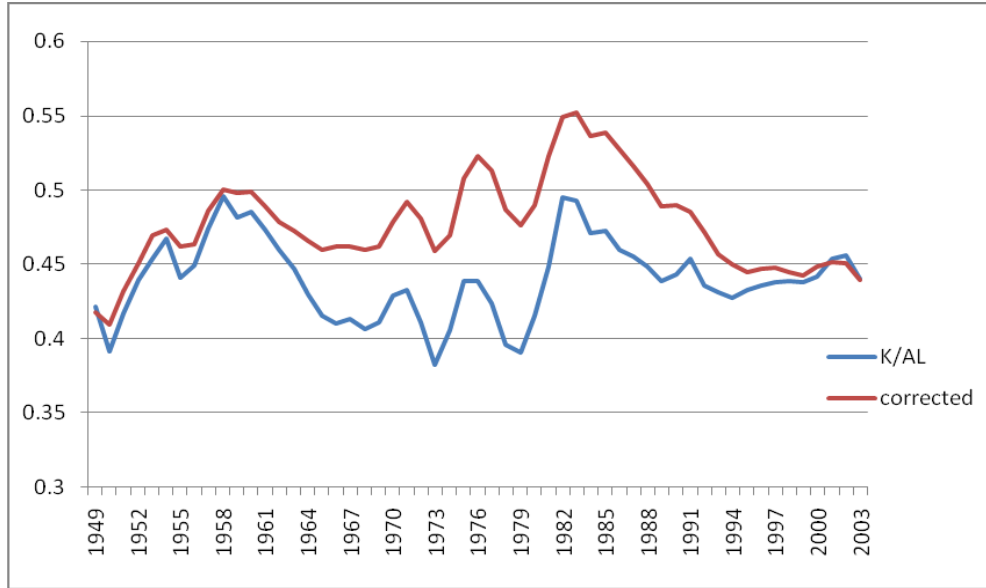
## 5.2 Simulating the US economy using historical inflation rates

In this section we run the model using historical inflation rates for the US economy starting 1949 up to 2003. We report the result on the figure below and contrast it with observed data. The model does replicate the increasing relationship between unemployment and capital per effective worker. However, the predicted magnitude of shifts in the unemployment rate is smaller than that from the data. Note that the stock of capital in Figure 5 is the "stock of capital in each active firm per effective worker", before it enters the labour market. To be consistent with the methodology used in National Accounts, one must keep track of the capital stock abandoned by unsuccessful firms in the previous period (see Appendix A.2).

## 6 Conclusion

What do we learn from this exercise? First, once corrected for the pro-cyclicality of TFP and labour participation, capital per effective worker in the US economy is positively associated with unemployment for the 60 years that have followed the end of World War II. In this paper we propose an explanation that relies on the role played by trading frictions in the capital

supply decision by household and the capital demand decision by firms. These trading frictions, especially unemployment, are endogenous and react to shifts in monetary policy. This creates new channel to explain deviations of the stock of capital from its balanced growth path.



## Appendix

### A1. Data sources

**Capital stock:** Data are "Capital Market: sum of structures and equipment and software", that is the capital stock minus housing and consumer durables. Data are from Gomme and Rupert (2007) method 1 (current cost deflated by consumption deflator). These data are accessible via the Cleveland Fed website.

**Total factor productivity:** Percentage change in Total Factor Productivity calculated by the Bureau of Labour Statistics.

**Labour:** Data are "Civilian Workforce" from the 2007 Economic Report of the President.

### A.2. The steady state value of capital per effective worker in our model

The total stock of capital is made of the stock of capital per firm each period (intensive margin) multiplied by the measure of firms each period (extensive margin), plus non-used capital from firms that have exited the economy. If  $M_{-1}^f$  is the measure of firms that survived the previous period and  $v$  the number of new firms entering the economy, then the capital built

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob	
		1	0.865	0.865	43.418	0.000
		2	0.645	-0.409	68.003	0.000
		3	0.484	0.251	82.151	0.000
		4	0.411	0.075	92.527	0.000
		5	0.366	-0.058	100.94	0.000
		6	0.277	-0.182	105.85	0.000
		7	0.126	-0.176	106.89	0.000
		8	-0.022	-0.013	106.92	0.000
		9	-0.114	-0.028	107.81	0.000
		10	-0.162	-0.080	109.64	0.000
		11	-0.204	-0.050	112.59	0.000
		12	-0.233	0.102	116.54	0.000
		13	-0.243	0.010	120.95	0.000
		14	-0.241	-0.048	125.39	0.000
		15	-0.244	-0.082	130.04	0.000
		16	-0.249	-0.005	135.03	0.000
		17	-0.250	-0.044	140.20	0.000
		18	-0.245	-0.090	145.29	0.000
		19	-0.238	-0.072	150.22	0.000
		20	-0.234	-0.022	155.11	0.000
		21	-0.189	0.227	158.41	0.000
		22	-0.099	0.083	159.35	0.000
		23	-0.012	-0.031	159.36	0.000
		24	0.005	-0.183	159.36	0.000

and ready to be used before the opening of labour market is

$$\left(M_{-1}^f + v\right) K_t$$

The capital left over by firms which disappeared in the previous period is equal to

$$\left(M_{-2}^f + v_{-1} - M_{-1}^f\right) (1 - \rho) K_{t-1}$$

and that of firms who disappeared two periods ago is equal to

$$\left(M_{-3}^f + v_{-2} - M_{-2}^f\right) (1 - 2\rho) K_{t-2}$$

so that the total stock of capital after  $\frac{1}{\rho}$  periods is given by

$$\begin{aligned}
K &= (M_{-1}^f + v) K_t \\
&+ (M_{-2}^f + v_{-1} - M_{-1}^f) (1 - \rho) K_{t-1} \\
&+ (M_{-3}^f + v_{-2} - M_{-2}^f) (1 - 2\rho) K_{t-2} \\
&+ \dots \\
&+ (M_{-3}^f + v_{-2} - M_{-2}^f) \left[ 1 - \left( \frac{1}{\rho} - 1 \right) \rho \right] K_{t - \left( \frac{1}{\rho} - 1 \right)}
\end{aligned}$$

In steady state  $K_t = K_{t-1} = \dots$  and  $M_{-1}^f = M_{-2}^f = \dots$  so that

$$K^* = M^f K + vK \left[ 1 + (1 - \rho) + (1 - 2\rho) + \dots + \left[ 1 - \left( \frac{1}{\rho} - 1 \right) \rho \right] \right]$$

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