A Comparison Of Methods For Spatial-Temporal Forecasting With An Application To Real Estate Prices(*)

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Abstract

The improvements in the reporting and maintenance of data sets containing spatial and temporal domains as well as powerful computers have opened the way for Spatial-Temporal (ST) models and estimation techniques in many disciplines. In this paper we compare a Spatial Temporal Linear Model (STLM) proposed in the literature to forecast real estate prices to a Spatial Errors Model (SEM) cast in state-space form (SSSEM). We explore in detail the incorporation of the time and spatial information in the estimation of the parameters of both models. We derive analytical expressions that show how the spatial and time information are handled in the estimation of the hedonic parameters in each case. The estimates from the STLM and from the Kalman Filter of the SSSEM account for spatial correlation of contemporaneous and past sales, although the relative weighting of information differs. This is not the case for the Kalman smoothed estimates. The fixed time estimates from STLM are expected to be close to the average of the time-varying estimates produced by the Kalman Filter over the same time period. We illustrate both methods with a sample from Brisbane, Australia for the period 1985-2005. We find the estimation of the STLM model is not computationally feasible for samples larger than 9,000. This is a severe draw back considering the usual size of real estate data sets. A comparison of prediction performance indicates the RMSE of the SSSEM based predictions is considerably lower than those obtained from STLM.

JEL Codes: C13, C51, C22
Keywords: spatial-temporal, Kalman filter, real estate

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A Comparison Of Methods For Spatial-Temporal Forecasting With An Application To Real Estate Prices

Introduction

Increased accuracy and improvements in the reporting and maintenance of data sets containing spatial and temporal domains as well as powerful computers have opened the way for Spatial-Temporal (ST) models and estimation techniques in many disciplines. ST methods have clear advantages over purely spatial or time-series methods as they do not require data to be pooled over either domain and thus do not lead a loss of information through aggregation. ST modeling has recently been introduced to real estate market analysis although it has been popular in other disciplines such us environmental sciences.

The nature of real estate market data has led to many difficulties in proposing effective modeling methods over time. Earlier estimation methods such as OLS and Grid Estimators did not account for spatial or temporal factors effectively and required the data to be pooled over space and time. Currently most real estate models have a spatial component in the mean or variance to account for the presence of spatial correlation in the data. However purely spatial models still require data to be aggregated over time and thus also lead to a loss in valuable information. A handful of spatiotemporal models have been introduced recently to real estate market analysis. These are the first attempts to allow for combined analysis of both the spatial and temporal domains in the data.

The temporal domain is one dimensional and contains a natural ordering while the spatial domain is most commonly two dimensional and does not contain a natural ordering. Further, the spatial domain also contains more features than the temporal domain that need to be considered, adding to the complexity of its analysis.

The ST real estate models have developed as extensions of existing spatial models in the literature. One of the pioneering models was the Spatial Temporal Linear Model (STLM) proposed by Pace et al**.** (2000) in the *International Journal of Forecasting*. The filtering of spatial and temporal components is handled through a covariance structure with both spatial and temporal weight matrices. The parameters of the model are constant over time and estimated through a generalized least squares estimator.

This paper seeks to compare both analytically and empirically the STLM model to a Spatial Errors Model (SEM) cast in state-space form, which we will denote by SSSEM. The SSSEM model is a special case of a more general set of models which have become popular in other disciplines known as the Krigged Kalman Filter. In the SSSEM the covariance has a time varying spatial structure and the parameters are time-varying. The SSSEM model can be estimated through classical likelihood methods or Bayesian methods.

Section 2 shows analytically how the time and spatial domain are handled by the STLM and SSSEM, and demonstrate that the processes of information assimilation of each of the models leads to the result that the averages of the time varying coefficients' estimates obtained from the application of the Kalman filter to the SSSEM are comparable to the fixed coefficient estimates obtained via the STLM, although the Kalman smoothed estimates are not. We also show that the SSSEM is better suited to handle large data sets than the STLM. Section 3 illustrates the results using a sample of monthly housing sales for the Brisbane metropolitan area for the period 1985-2005 and presents a comparison of prediction performance. Section 4 concludes.

2. The STLM and SSSEM Models

2.1 Introduction

This section shows how the two models incorporate the spatial and time evolution of sales observations and how the estimation of the hedonic parameters includes this evolution. To ease the presentation we first present a simple case of six houses and three time periods which are design to aid the understanding of the general findings.

Both methods require a measure of distance between houses as well as a time ordering of sales. Figure 1 presents the spatial location of six houses and Table 1 presents the ordering of the sales for these houses.

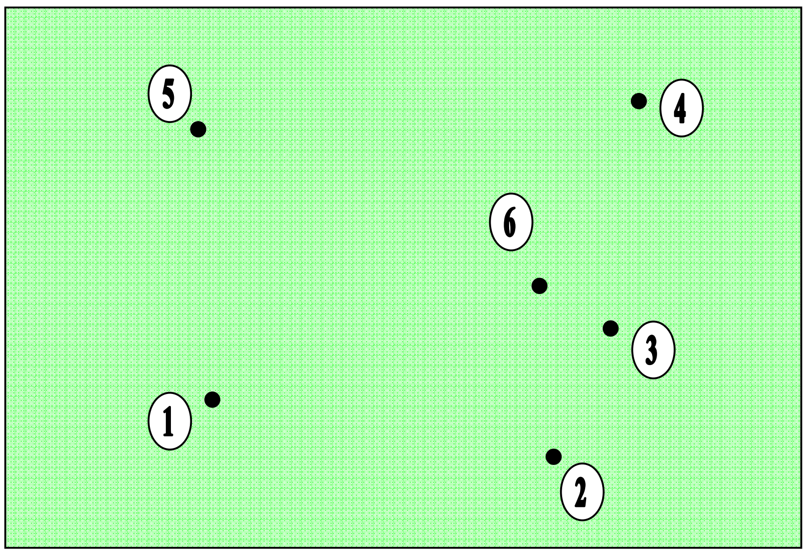

Figure 1. Location of Six Houses

Thus, for instance house three was sold in period one however it was sold after house two, which was also observed in period one. The observations are ordered in time therefore, observation one is the furthest in the past while observation six is the most recent.

Table 1: Time of Sales Ordering

Observation /House	Time of Sale
1	1
2	1.1
3	1.2
4	2.1
5	2.2
6	3

Given the spatial distribution of the observations and the time of sale, the STLM and the SSSEM specifications can be obtained in their general forms.

2.2 Spatial-Temporal Linear Model (STLM) Derivation

The STLM has the following form,

$$Y = X\beta + \varepsilon \tag{1}$$
$$\varepsilon = W\varepsilon + u \tag{2}$$
$$W = \phi_s S + \phi_T T + \phi_{ST} ST + \phi_{TS} TS \tag{3}$$
$$u \sim N(0, \sigma^2 I) \tag{4}$$

where,
N - Number of observations in the sample.
$\underset{N\times 1}{Y}$ - Matrix of the observations of the dependent variable (log of house prices).
$\underset{N\times K}{X}$ - Matrix of the independent variables (hedonics) and a constant term.
$\underset{K\times 1}{\beta}$ - Unknown parameter vector (shadow prices of hedonics).
$\underset{N\times 1}{u}$ - Vector of white noise error terms
$\underset{N\times 1}{\varepsilon}$ - Autocorrelated errors
$\underset{N\times N}{W}$ - Spatial-Temporal weight matrix
$\underset{N\times N}{S}$ - Spatial weight matrix
$\underset{N\times N}{T}$ - Temporal weight matrix
$\phi = [\phi_s, \phi_T, \phi_{ST}, \phi_{TS}]$ - Set of scalar autoregressive parameters.

It is assumed that the observations are ordered chronologically, the oldest observations in *X* are in the first row while the most recent in the nth row. The matrix *W* is partitioned into a spatial component *S* and a temporal component *T*. The spatial weight matrix *S* represents the spatial relations among previous observations; similarly, the temporal weight matrix *T* represents temporal relations among previous observations. The *S* and *T* matrices are weighted in time and space by autoregressive parameters. The matrices act in a similar way to a lag operator in space and time respectively. A lag operator is applied to data with regular periodicity, where as these matrices are the 'lag operators' for data with irregular periodicity, which is one of the features of real estate data. By concentrating only on previous observations the calculations are simplified greatly via the lower triangular structure of both weight matrices due to the chronological ordering. This analysis ignores any contemporary spatial correlation which may be significant. This weakness of the model is discussed by Cressie (1993) in relation to STARMA type models employed in the literature. The significance of contemporary spatial correlation will depend on the time scale that is modeled by the data, for example, if one period is a year then contemporary spatial correlation in house prices would be highly significant as prices of houses sold in the same year would be expected to be highly correlated and thus this modeling structure would be inappropriate. However, if one time period is a day then the contemporary spatial correlation may be omitted without a significant loss in information.

Pace *et al.* (2000) specify the following assumptions for the S and T weight matrices and the autoregressive parameters;

$$S \times [1] = [1]$$
$$T \times [1] = [1]$$
$$j \geq i \Leftrightarrow S_{ij} = 0, T_{ij} = 0$$
$$-1 < \phi < 1$$
$$S_{ij} \geq 0, T_{ij} \geq 0$$

The S and T matrices are assumed to be row stochastic which implies that each row of the matrix will sum to 1. However as by assumption the matrices are lower triangular, the first rows of the matrices consist only of zeros. Here [1] denotes an N by *1* vector of 1s. The matrices are specified as lower triangular as spatial and temporal dependence is only on the previously sold properties therefore, the matrices S and T will only contain non-zero elements below the main diagonal.

The S matrix is obtained as a weighted average of individual neighbour matrices constructed based on a fixed number of spatial neighbours, m_s. We assume m_s is chosen as three to illustrate the construction of S. The first step in deriving the spatial weight matrix involves obtaining the distances between each observation and all the previous observations. As the number of observations, N, increases the number of comparisons required will rise at an increasing rate as the number of comparisons for N observations (total number of houses in the sample) is given by

$$\sum_{i=1}^{N}(i-1) = \frac{1}{2}N(N-1) \qquad (5)$$

As a result, this step may cause computational problems with large data sets.

The distances obtained via a pair-wise comparisons yield the distance matrix D. The units for the distances in matrix D are arbitrary as they are only used to determine the rank of the potential neighbours of each property. The numbers, chosen in (6) for the pair-wise distance comparisons, correspond in relative magnitudes to the spatial locations of the six houses. The matrix is lower triangular as each observation is only compared to previous observations, thus the first observation has no past observations to be compared to.

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 \\ 6 & 2 & 0 & 0 & 0 & 0 \\ 8 & 6 & 4 & 0 & 0 & 0 \\ 4 & 8 & 7 & 6 & 0 & 0 \\ 5 & 2 & 1 & 3 & 6 & 0 \end{bmatrix} \tag{6}$$

To construct *S*, m_s individual neighbour matrices of dimension *N* by *N*, are required. Due to their sparse nature they are less likely to cause computational difficulties. The individual neighbour matrices can be obtained from the distance matrix, *D*. In the simple case presented about, three individual neighbour matrices are computed. S_1 represents the first nearest neighbour matrix. Each row in this matrix contains a 1 for the closest neighbour for the considered observation and all other elements are 0. The smallest element in each row of the *D* matrix implies the minimum distance from the observation corresponding to the row number to the observation corresponding to the column number, therefore they are 'first nearest neighbours' and thus this element is given a value of 1 in the S_1 matrix.

$$S_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{7}$$

Similarly, the second nearest neighbour matrix S_2 is obtained via a similar technique. This matrix contains a 1 for the element in each row with the second smallest distance value. Thus the first two rows of this matrix consist of 0s as the second observation only has one potential neighbour which is the first closest neighbour, and the first observation does not have any neighbours in the past.

$$S_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{8}$$

The final individual neighbour matrix, given our choice of m_s =3, required to obtain the spatial weight matrix is the third nearest neighbour matrix S_3. Similarly to the construction of the previous two spatial matrices, S_3 contains a 1 in each row for the third smallest value in that row in the *D* matrix.

$$S_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{9}$$

The spatial weight matrix, *S*, is derived as a weighted average of the individual neighbour matrices. Pace *et al.* (2000) note that sensitivity analysis of the results the authors obtained via the STLM indicates that changes caused by selecting equal or differentiated weights for the individual neighbour matrices yield minimal changes. An analytical investigation of the effects of various weight specifications for the calculation of the spatial weight matrix is a possible direction of further research however for the purpose of the comparison conducted in this study equal weights are applied.

$$S_{(ij)} = 0 \qquad \text{for } i = 1,\ j = 1 \ldots N \tag{10}$$

$$S_{(ij)} = \sum_{l=1}^{3} S_{l(ij)} \qquad \text{for } i = 2,\ j = 1 \ldots N \tag{11}$$

$$S_{(ij)} = \sum_{l=1}^{3} \frac{1}{2} S_{l(ij)} \qquad \text{for } i = 3,\ j = 1 \ldots N \tag{12}$$

$$S_{(ij)} = \sum_{l=1}^{3} \frac{1}{3} S_{l(ij)} \qquad \text{for } i > 3,\ j = 1 \ldots N \tag{13}$$

In the above expressions $S_{(ij)}$ denote the element of the spatial matrix *S* in row *i* and column *j*. The elements in the first row of all of the individual neighbour matrices are 0, therefore the elements of the first row of the *S* matrix which are a weighted average of the individual matrices values, are also 0. The second observation only has one previously sold neighbour therefore the weight given to this neighbour is 1. The third observation has two previously sold neighbours, and thus assuming equal weights, are given a weight of a half. For the fourth observation and above, each observation will have three previously sold spatial nearest neighbours, therefore the elements of the spatial weight matrix are obtained by giving each respective neighbour the weight of one third. The above calculation may easily be generalized to any m_s as the fixed number of spatial neighbours. When the row number exceeds m_s each spatial neighbour is weighted by $\frac{1}{m_s}$. Similarly to the above expressions, when the row number is less than or equal to m_s the weights are distributed equally. For example for row m_s each neighbour is given a weight of $\frac{1}{m_s - 1}$. Via this specification the obtained spatial weight matrix is row stochastic, as the elements of each row will sum to 1 with the exception of the first row. This is a convenient definition as it has a filtering interpretation. For example, the product SY gives the average log sales price of the m_s nearest neighbours of each property.

The resultant spatial weight matrix is given by (14).

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{bmatrix} \qquad (14)$$

The second component of the weight matrix in the STLM is the temporal weight matrix T. If the STLM is rewritten in a linear regression form, the T matrix only appears as a product with other matrices.

$$\begin{aligned} Y = {} & X\beta - \phi_S SX\beta - \phi_T TX\beta - \phi_{ST} STX\beta - \phi_{TS} TSX\beta \\ & + \phi_s SY + \phi_T TY + \phi_{ST} STY + \phi_{TS} TSY + u \end{aligned} \qquad (15)$$

In practice this implies the T weight matrix does not need to be calculated or stored as the product of T and another variable represents the running average of that variable thus it may be obtained via linear filter routines in programs such as Matlab. This is a practical advantage of the model which is highlighted by Pace *et al.* (2000) as it saves on the storage of a matrix with a large potential number of non-zero elements. However for our purpose the temporal T matrix is calculated explicitly.

The temporal weight matrix is defined in a similar fashion to the spatial weight matrix. Each row of the T matrix contains a non-zero element for m_T closest, previously sold neighbours in time for the observation corresponding to that row, where m_T denotes the number of temporal neighbours selected in the model. As the observations are assumed to be ordered in time from the outset there is no necessity to calculate the distance in time between all previously sold houses or to construct individual neighbour matrices. The closest m_T neighbours to an observation will be the m_T preceding observations in the data set. Similarly to the S matrix, the T matrix is row stochastic therefore the elements of each row with the exception of the first row will sum to 1 and equal weights are given to each temporal neighbour.

For the case in Figure 1 and Table 1 the temporal weight matrix is given by

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{bmatrix} \tag{16}$$

If the STLM is written in linear regression form, the matrix of hedonics *X* as well as the dependent variable *Y* are not only filtered via the *T* and *S* matrices individually but also by their products *TS* and *ST*.

$$ST = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{5}{18} & \frac{5}{18} & \frac{1}{9} & 0 & 0 & 0 \\ \frac{11}{18} & \frac{5}{18} & \frac{1}{9} & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

$$TS = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{6} & 0 & 0 & 0 & 0 \\ \frac{11}{18} & \frac{5}{18} & \frac{1}{9} & 0 & 0 & 0 \\ \frac{7}{18} & \frac{5}{18} & \frac{1}{9} & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

From this analysis it is evident that the STLM takes the spatial and temporal effects via the nearest neighbours approach by considering all the observations in the sample simultaneously. The result of this approach is that all the filtering matrices and therefore the specified weight matrix *W* in (2) are of dimension *N* by *N*. Although many of the matrices are sparse and thus computationally feasible even with a relatively large number of observations, the derivation of the spatial weight matrix requires the construction of a relatively dense distance matrix which may lead to computational difficulties with large data sets.

In order to simplify the estimation of the parameters in this model, Pace *et al* (2000) use the form in (15), which allows the use of Least Squares methods. Several restrictions can be imposed on the structure of the lag parameters $\phi = [\phi_s, \phi_T, \phi_{ST}, \phi_{TS}]$. In addition they show the form of the likelihood to obtain estimates of ϕ. We will not reproduce the details here, except to indicate that the estimator of the hedonic parameters amounts to an Estimated Generalised Least Squares procedure, which we present below as it will be required for the comparison to the SSSEM. Let the covariance matrix of the correlated error term ε be denoted by Ψ.

$$
\begin{aligned}
&Var(\varepsilon) = \Psi \\
&\qquad \varepsilon = W\varepsilon + u \\
&\Rightarrow u = \varepsilon - W\varepsilon \\
&\qquad \Rightarrow u = (I-W)\varepsilon \\
&\Rightarrow \varepsilon = (I-W)^{-1}u
\end{aligned}
$$

$$
\begin{aligned}
Var(\varepsilon) = \Psi = E[\varepsilon\varepsilon'] &\qquad \because E(\varepsilon) = 0 \\
= E[(I-W)^{-1}uu'(I-W)^{-1\prime}] & \\
= (I-W)^{-1}E(uu')(I-W)^{-1\prime} & \\
= \sigma^2 (I-W)^{-1}(I-W)^{-1\prime} &\qquad \because E(uu') = \sigma^2 I \\
= \sigma^2\Omega &\qquad \Omega = (I-W)^{-1}(I-W)^{-1\prime} \qquad (19) \\
\hat{\Omega} = (I-\hat{W})^{-1}(I-\hat{W})^{-1\prime} & \\
\hat{W} = \hat{\phi}_S S + \hat{\phi}_T T + \hat{\phi}_{ST} ST + \hat{\phi}_{TS} TS &
\end{aligned}
$$

$$
\begin{aligned}
\hat{\beta}_{EGLS} &= (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}Y \\
&= (X'[(I-\hat{W})^{-1}(I-\hat{W})^{-1\prime}\ ]^{-1}X)^{-1}X'[(I-\hat{W})^{-1}(I-\hat{W})^{-1\prime}\ ]^{-1}Y \\
&= (X'[(I-[\hat{\phi}_S S + \hat{\phi}_T T + \hat{\phi}_{ST} ST + \hat{\phi}_{TS} TS])^{-1}(I-[\hat{\phi}_S S + \\
&\quad + \hat{\phi}_T T + \hat{\phi}_{ST} ST + \hat{\phi}_{TS} TS])^{-1}{}']^{-1}X)^{-1}X'[(I-[\hat{\phi}_S S + \hat{\phi}_T T + \\
&\quad + \hat{\phi}_{ST} ST + \hat{\phi}_{TS} TS])^{-1}(I-[\hat{\phi}_S S + \hat{\phi}_T T + \hat{\phi}_{ST} ST + \hat{\phi}_{TS} TS])^{-1}{}']^{-1}Y
\end{aligned} \qquad (20)
$$

The circumflex accent on the matrices and parameters in expressions denotes estimates of the respective matrices and parameters. The variance covariance matrix of the coefficients is standard and given by

$$
\begin{aligned}
Var(\hat{\beta}_{EGLS}) &= E[(\hat{\beta}_{EGLS} - \beta)(\hat{\beta}_{EGLS} - \beta)'] \\
&= \sigma^2(X'\hat{\Omega}^{-1}X)^{-1} \\
&= \sigma^2(X'[(I-\hat{W})^{-1}(I-\hat{W})^{-1}{}'\ ]^{-1}X)^{-1} \\
&= \sigma^2(X'[(I-[\hat{\phi}_S S + \hat{\phi}_T T + \hat{\phi}_{ST} ST + \hat{\phi}_{TS} TS])^{-1}(I-[\hat{\phi}_S S + \\
&\quad + \hat{\phi}_T T + \hat{\phi}_{ST} ST + \hat{\phi}_{TS} TS])^{-1}{}']^{-1}X)^{-1}
\end{aligned} \qquad (21)
$$

As the hedonic coefficient estimates and their standard errors are both given as functions of the *S* and *T* weight matrices this implies they will incorporate the information on the spatial and temporal autocorrelations within the sample for each house relative to previously observed houses. Therefore the estimates are based on past information known at the time of sale of each observation.

2.3 State Space Spatial Error Model Derivation

The SSSEM model was proposed by Cominos (2006) and Cominos *et al* (2007) as a suitable alternative to obtain The derivation of the SSSEM is approached in a similar fashion to the STLM. The model is given in a very general form and a step by step procedure is applied to bring the model up to the stage where all the required matrices are specified.

The Spatial Errors Model (SEM) is a commonly used model in Spatial statistics (see Lessage and Pace (2004), Florax and de Graaff (2004)) and it takes the form

$$Y = X\beta + \varepsilon \qquad (22)$$
$$\varepsilon = \rho\Upsilon\varepsilon + u \qquad (23)$$

where,
N - Number of observations in the sample.
$\underset{N\times 1}{Y}$ - Matrix of the observations of the dependent variable (log of house prices).
$\underset{N\times k}{X}$ - Matrix of the independent variables and a constant term.
$\underset{k\times 1}{\beta}$ - Vector of unknown parameters.
$\underset{N\times 1}{\varepsilon}$ - Vector of spatially correlated error terms.
$\underset{N\times 1}{u}$ - Vector of error terms.
$\underset{1\times 1}{\rho}$ - spatial correlation parameter.
$\underset{N\times N}{\Upsilon}$ - Spatial weight matrix.

The starting point of the specification of the SSSEM is given by its general form that allows the N observations to be modeled accounting by their chronological ordering

$$Y_t = X_t\beta_t + \varepsilon_t \qquad (24)$$
$$\varepsilon_t = \rho\Upsilon_t\varepsilon_t + u_t \qquad t = 1...T \qquad (25)$$
$$\beta_t = \beta_{t-1} + \eta_t \qquad (26)$$
$$E(\varepsilon_t) = 0 \qquad Cov(\varepsilon_t\varepsilon_t') = H_t \qquad (27)$$
$$u_t \sim N(0, \sigma_u^2 I_t)$$
$$\eta_t \sim (0, Q), \qquad Q = \sigma_\eta^2 I_k$$
$$E(\varepsilon_t\eta_t^T) = 0$$

$$b_t \sim (\beta_t, P_t) \qquad (28)$$
$$\beta_0 = 0 \qquad P_0 = \kappa I \qquad (29)$$

N_t - Number of observations in period *t*.

$\underset{N_t \times 1}{Y_t}$ - Matrix of the observations of the dependent variable (log of house prices).

$\underset{N_t \times k}{X_t}$ - Matrix of the independent variables and a constant term in period *t*.

$\underset{k \times 1}{\beta_t}$ - Vector of unknown parameters.

$\underset{N_t \times 1}{\varepsilon_t}$ - Vector of spatially correlated error terms.

$\underset{N_t \times 1}{u_t}$ - Vector of error terms.

$\underset{1 \times 1}{\rho}$ - spatial correlation parameter.

$\underset{N_t \times N_t}{\Upsilon_t}$ - Spatial weight matrix.

$\underset{N_t \times 1}{\eta_t}$ - Vector of innovations.

In practice the model parameters $\rho, \sigma_u^2, \ \sigma_\eta^2$ can be estimated via numerical Maximum Likelihood methods or Bayesian estimation. The estimation of these parameters does not play a key role in the comparison of the STLM and the SSSEM therefore their estimation is not considered in this paper (the interested reader is refereed to Harvey (1990)).

The first step is the determination of the spatial weight matrix Υ_t for **every time period**. Although there are alternative methods to determine the distance between neighbours, for our purpose it suffices to say that Υ_t is a row stochastic matrix based on the distance between houses that were sold in *t*. For instance, for the case in Figures 1 and Table 1, three spatial weight matrices are found. In period one houses 1, 2 and 3 were observed, in period two houses 4 and 5 were observed and in period three only house 6 was observed. Each weight matrix takes into account only the houses observed in its respective period therefore W_t is of dimension N_t by N_t as opposed to *N* by *N* as in the STLM specification. Here N_t denotes the number of observations in period *t*.

As the observations are not spatially correlated to themselves the diagonal elements of the weight matrices are zeros similarly to the STLM specification. For example, as houses 1 and 2 are neighbours (refer to Table 1 and Figure 1), houses 1 and 3 are not neighbours, and houses 2 and 3 are neighbours. The weight matrix for the first time period would be,

$$\Upsilon_1 = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix} \tag{30}$$

And similarly for the remaining two time periods we would have

$$\Upsilon_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{31}$$

$$\Upsilon_3 = [0] \tag{32}$$

In practice, when data sets are very large a fixed number of neighbours is used to define the non-zero weights in the Υ_t matrix similar to the case of STLM.

Once the weight matrices for each period have been specified, the hedonic coefficients of the SSSEM may be obtained via algorithms such as the Kalman Filter and Kalman Smoother which are applied to the State Space Form (SSF) of the model. The SSSEM is already specified in its state-space form, where the *measurement equation* is given by (24) and the *transition equation* via (26). Although the error term of the *measurement equation* ε_t is spatially autocorrelated, it is uncorrelated over time and therefore satisfies the assumptions of the Kalman algorithms. The Kalman algorithms require the specification of the variance covariance matrix of the *measurement equation* error term denoted by H_t which can easily be derived,

$$\begin{aligned}
&u_t = \varepsilon_t - \rho\Upsilon_t\varepsilon_t \\
&\Rightarrow u_t = (I - \rho\Upsilon_t)\varepsilon_t \\
&\Rightarrow \varepsilon_t = (I - \rho\Upsilon_t)^{-1}u_t \\
&Var(\varepsilon_t) = E[\varepsilon_t\varepsilon_t'] \qquad \because E(\varepsilon_t) = 0 \\
&\qquad = E[(I - \rho\Upsilon_t)^{-1}u_t u_t'(I - \rho\Upsilon_t)^{-1\,\prime}] \\
&\qquad = (I - \rho\Upsilon_t)^{-1}E(u_t u_t')(I - \rho\Upsilon_t)^{-1\,\prime} \\
&\qquad = \sigma_u^2(I - \rho\Upsilon_t)^{-1}(I - \rho\Upsilon_t)^{-1\,\prime} \\
&\qquad = H_t
\end{aligned} \tag{33}$$

The hedonic coefficients are the state vector in this specification, and thus, SSSEM may report the conditional or unconditional estimates obtained via the Kalman Filter or the smoothed estimates obtained via the Kalman Smoother. To obtain a better understanding of how these filters help the model to incorporate the temporal evolution of house prices, the Kalman algorithms, the Filter and the Smoother, are presented. In the above definitions both error terms, ε_t and η_t are assumed to be normally distributed.

The Kalman Filter can be divided into a set of *prediction equations* and *updating equations* (the reader is refer to Harvey (1990) for detailed explanation of the Kalman Filter algorithm). The resulting estimates of the hedonic parameters obtained from the prediction equations are the conditional estimates, given by

$$b_{t|t-1} = b_{t-1} \tag{34}$$

$$P_{t|t-1} = P_{t-1} + Q = P_{t-1} + \sigma_\eta^2 I \tag{35}$$

Applying the updating equations of the filter we obtain the unconditional estimates

$$\mathrm{b}_t = b_{t|t-1} + P_{t|t-1}X_t{}'F_t^{-1}(Y_t - X_t b_{t|t-1}) \tag{36}$$

$$P_t = P_{t|t-1} - P_{t|t-1} X_t{}' F_t^{-1} X_t P_{t|t-1} \tag{37}$$

$$F_t = X_t P_{t|t-1} X_t{}' + H_t = X_t P_{t|t-1} X_t{}' + \sigma_u^2 (I - \rho \Upsilon_t)^{-1} (I - \rho \Upsilon_t)^{-1\prime} \tag{38}$$

In order to show how equations (35)-(38) incorporate the information over time, we show their expressions for three time periods.

Period 1 conditionals

$$b_{1|0} = b_0$$

$$P_{1|0} = P_0 + \sigma_\eta^2 I$$

Period 1 unconditionals

$$\begin{aligned}
\mathrm{b}_1 &= b_{1|0} + P_{1|0} X_1{}' F_1^{-1} (Y_1 - X_1 b_{1|0}) \\
&= b_0 + P_{1|0} X_1{}' [X_1 (P_0 + \sigma_\eta^2 I) X_1{}' + \\
&\quad + \sigma_u^2 (I - \rho \Upsilon_1)^{-1} (I - \rho \Upsilon_1)^{-1\prime}]^{-1} (Y_1 - X_1 b_0) \\
&= f_{b1}(\Upsilon_1, b_0, P_0, \sigma_\eta^2, \sigma_u^2, \rho, Y_1, X_1)
\end{aligned}$$

$$\begin{aligned}
P_1 &= P_{1|0} - P_{1|0} X_1{}' F_1^{-1} X_1 P_{1|0} \\
&= P_{1|0} - P_{1|0} X_1{}' [X_1 (P_0 + \sigma_\eta^2 I) X_1{}' + \\
&\quad + \sigma_u^2 (I - \rho \Upsilon_1)^{-1} (I - \rho \Upsilon_1)^{-1\prime}]^{-1} X_1 P_{1|0} \\
&= f_{P1}(\Upsilon_1, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1)
\end{aligned}$$

$$F_1 = X_1 P_{1|0} X_1{}' + \sigma_u^2 (I - \rho \Upsilon_1)^{-1} (I - \rho \Upsilon_1)^{-1\prime}$$

In the first period the Kalman Filter estimates for the hedonic coefficients are a function of the spatial relationships of the houses observed in period one, via Υ_1. Therefore this estimate only considers the information available in period one, the spatial relationships and the observations of the dependent and independent variables. With each recursion the estimates of the hedonic coefficients and the covariance matrix become increasingly complex (the expression for the third period coefficient estimate is over a page long) if fully expanded therefore for periods two and three the conditional and unconditional estimates are given as a general function rather than an explicit expression. The full derivation and the explicit functional forms of all the general functions presented in this Section are provided in Appendix 1 of Svetchnikova (2007). The goal of this Section is to analyze where each piece of information is assimilated into the obtained estimates and thus compare this process to that utilized by the STLM.

Period 2 conditionals

$$\begin{aligned}
b_{2|1} &= b_1 \\
&= f_{b21}(\Upsilon_1, b_0, P_0, \sigma_\eta^2, \sigma_u^2, \rho, Y_1, X_1)
\end{aligned}$$

$$P_{2|1} = P_1 + \sigma_\eta^2 I$$
$$= f_{P21}(\Upsilon_1, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1)$$

Period 2 unconditionals

$$\mathrm{b}_2 = b_{2|1} + P_{2|1} X_2{}' F_2^{-1} (Y_2 - X_2 b_{2|1})$$
$$= f_{b21}(\Upsilon_1, \ldots) + f_{P21}(\Upsilon_1, \ldots) X_2{}' f_{F2}^{-1}(\Upsilon_1, \Upsilon_2, \ldots)(Y_2 - X_2 f_{b21}(\Upsilon_1, \ldots))$$
$$= f_{b2}(\Upsilon_1, \Upsilon_2, b_0, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, Y_1, Y_2)$$
$$P_2 = P_{2|1} - P_{2|1} X_2{}' F_2^{-1} X_2 P_{2|1}$$
$$= f_{P21}(\Upsilon_1, \ldots) - f_{P21}(\Upsilon_1, \ldots) X_2{}' f_{F2}^{-1}(\Upsilon_1, \Upsilon_2, \ldots) X_2 f_{P21}(\Upsilon_1, \ldots)$$
$$= f_{P2}(\Upsilon_1, \Upsilon_2, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2)$$
$$F_2 = X_2 P_{2|1} X_2{}' + \sigma_u^2 (I - \rho\Upsilon_2)^{-1} (I - \rho\Upsilon_2)^{-1\prime}$$
$$= f_{F2}(\Upsilon_1, \Upsilon_2, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2)$$

Period 3 conditionals

$$b_{3|2} = b_2$$
$$= f_{b32}(\Upsilon_1, \Upsilon_2, b_0, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, Y_1, Y_2)$$
$$P_{3|2} = P_2 + \sigma_\eta^2 I$$
$$= f_{P32}(\Upsilon_1, \Upsilon_2, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2)$$

Period 3 unconditionals

$$\mathrm{b}_3 = b_{3|2} + P_{3|2} X_3{}' F_3^{-1} (Y_3 - X_3 b_{3|2})$$
$$= f_{b32}(\Upsilon_1, \Upsilon_2 \ldots) + f_{P32}(\Upsilon_1, \Upsilon_2 \ldots) X_3{}' f_{F3}^{-1}(\Upsilon_1, \Upsilon_2, \Upsilon_3 \ldots)(Y_3 - X_3 f_{b32}(\Upsilon_1, \Upsilon_2 \ldots))$$
$$= f_{b3}(\Upsilon_1, \Upsilon_2, \Upsilon_3, b_0, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, X_3, Y_1, Y_2, Y_3)$$
$$P_3 = P_{3|2} - P_{3|2} X_3{}' F_3^{-1} X_3 P_{3|2}$$
$$= f_{P32}(\Upsilon_1, \Upsilon_2 \ldots) - f_{P32}(\Upsilon_1, \Upsilon_2 \ldots) X_3{}' f_{F3}^{-1}(\Upsilon_1, \Upsilon_2, \Upsilon_3 \ldots) X_3 f_{P32}(\Upsilon_1, \Upsilon_2 \ldots)$$
$$= f_{P2}(\Upsilon_1, \Upsilon_2, \Upsilon_3, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, X_3)$$
$$F_3 = X_3 P_{3|2} X_3{}' + \sigma_u^2 (I - \rho\Upsilon_3)^{-1} (I - \rho\Upsilon_3)^{-1\prime}$$
$$= f_{F3}(\Upsilon_1, \Upsilon_2, \Upsilon_3, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, X_3)$$

The conditional estimates in each period take into account the spatial relationships observed in previous periods, the initial conditions on the state vector and the three covariance parameters. For example, in period three the conditional estimates of the hedonic coefficients are based on the information on the houses observed in periods one and two. This is similar to the principle applied to the STLM where the model is designed so that the estimates only consider the spatial relationships between previous observations. However, if the unconditional estimates are used then the *updating equations* ensure that contemporaneous period information is also taken into account.

In the final period recursion of the Kalman Filter the unconditional estimates of the hedonic coefficients and the covariance matrix of these estimates utilize all the information in the sample. Importantly, this information has entered in a sequential form, and the relative importance of new information is added through the matrix F_t.

The initial state (starting values) for the Kalman Filter in the SSSEM model is specified to suit the non-stationary nature of the *transition equation*. If the *transition equation* is stationary and time invariant the starting values for the filter are given implicitly by the model specifications via the unconditional mean and variance of the state vector (hedonic coefficient vector). However, in the case of non-stationarity Harvey (1990) suggests two possible approaches. One approach assumes the initial state vector to be fixed and thus the initial variance is zero. The second approach, chosen by Cominos *et al.* (2007) for the SSSEM specification, is to define the initial state vector as random with a diffuse distribution, defining the initial variance to be very large which reflects the lack of prior knowledge about the state vector. This approach implies that the estimates obtained via the Kalman Filter recursions in the periods near the beginning of the sample may be very inaccurate and very sensitive to different choices of starting values. The initial iterations of the Kalman Filter need a burn-in period to assimilate enough information from the data to yield reliable estimates.

The Kalman Smoother is an algorithm applied to the unconditional estimates obtained via the Kalman Filter and its purpose is to adjust estimates in each period to account for all the information in the sample. For example, the first period unconditional estimate of the hedonic coefficient vector will be updated such that the smoothed estimate will be based on the information in all the periods in the sample. We present below that equations of a fixed interval Kalman Smoother to illustrate the process.

$$\begin{aligned} b_{t|T} &= b_t + P_t*(b_{t+1|T} - b_t) \qquad\qquad t = 1,...,T \\ P_{t|T} &= P_t + P_t*(P_{t+1|T} - P_{t+1|t})P_t*' \\ P_t* &= P_t' P_{t+1|t}{}^{-1} \end{aligned}$$

As the recursions of the Kalman Smoother work backwards, the smoothing algorithm begins with period T-1, as period T estimates are already based on the information of the entire sample and act as the starting values for this algorithm. For the above case of three time periods,

Period 2 Smoother

$$\begin{aligned} P_2^* &= P_2' P_{3|2}{}^{-1} \\ &= [f_{P2}(\Upsilon_1, \Upsilon_2, ...)]' f_{P32}^{-1}(\Upsilon_1, \Upsilon_2, ...) \\ &= f_{P2*}(\Upsilon_1, \Upsilon_2, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2) \end{aligned}$$

$$
\begin{aligned}
b_{2|3} &= b_2 + P_2^*(b_{3|3} - b_2) \\
&= f_{b2}(\Upsilon_1, \Upsilon_2, ...) + f_{P2*}(\Upsilon_1, \Upsilon_2, ...) \{f_{b3}(\Upsilon_1, \Upsilon_2, \Upsilon_3, ...) - f_{b2}(\Upsilon_1, \Upsilon_2, ...)\} \\
&= f_{b23}(\Upsilon_1, \Upsilon_2, \Upsilon_3, b_0, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, X_3, Y_1, Y_2, Y_3)
\end{aligned}
$$

$$
\begin{aligned}
P_{2|3} &= P_2 + P_2^*(P_{3|3} - P_{3|2})P_2^{*\prime} \\
&= f_{P2}(\Upsilon_1, \Upsilon_2, ...) + f_{P2*}(\Upsilon_1, \Upsilon_2, ...)[f_{P3}(\Upsilon_1, \Upsilon_2, \Upsilon_3, ...) - \\
&\quad - f_{P32}(\Upsilon_1, \Upsilon_2, ...)]\,[f_{P2*}(\Upsilon_1, \Upsilon_2, ...)]' \\
&= f_{P23}(\Upsilon_1, \Upsilon_2, \Upsilon_3, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, X_3)
\end{aligned}
$$

The smoothed coefficients () and the smoothed covariance matrix () are updated such that they now make use of all the information in the sample. The smoothed estimates and covariance matrix for period two now contain the information of all the spatial weight matrices, Υ_1, Υ_2 and Υ_3. However, in reality economic agents in period two do not possess the information contained in Υ_3 therefore the smoothed estimate for period two does not accurately reflect the information of economic agents in period two. Similarly, the first period estimates are smoothed to include the information of the entire sample.

Period 1 Smoothed

$$
\begin{aligned}
b_{1|3} &= b_1 + P_1^*(b_{2|3} - b_1) \\
&= f_{b1}(\Upsilon_1, ...) + f_{P1*}(\Upsilon_1, ...)[f_{b23}(\Upsilon_1, \Upsilon_2, \Upsilon_3, ...) - f_{b1}(\Upsilon_1, ...)] \\
&= f_{b13}(\Upsilon_1, \Upsilon_2, \Upsilon_3, b_0, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, X_3, Y_1, Y_2, Y_3) \\
P_{1|3} &= P_1 + P_1^*(P_{2|3} - P_{2|1})P_1^{*\prime} \\
&= f_{P1}(\Upsilon_1, ...) + f_{P1*}(\Upsilon_1, ...)[f_{P23}(\Upsilon_1, \Upsilon_2, \Upsilon_3, ...) - f_{P21}(\Upsilon_1, ...)][f_{P1*}(\Upsilon_1, ...)]' \\
&= f_{P13}(\Upsilon_1, \Upsilon_2, \Upsilon_3, b_0, P_0, \sigma_\eta^2, \sigma_u^2, \rho, X_1, X_2, X_3)
\end{aligned}
$$

Although theoretically the choice of the smoothed coefficient for the SSSEM specification is inconsistent in a forecasting sense, the impact on the results in practice may not be very significant. The difference between the smoothed and the Kalman Filter estimates (conditional and unconditional) of the coefficient vector and its covariance matrix is expected to be greatest near the beginning of the sample. The adjustment is scaled by a function of the covariance matrix of the respective period, which also implies the adjustment is greatest at the beginning of the sample period as the initial state vector is assumed to have a diffuse distribution.

3. Empirical Results

3.1 Introduction

To complement the analytical discussion, the State Space Spatial Error Model (SSSEM) and the Spatial Temporal Linear Model (STLM) are applied to a real estate data set from the Brisbane Metropolitan area. The data are discussed next.

3.2 Model and Data

The model and data used in this study are that from Cominos (2006) and Cominos *et al* (2007). The hedonics included in the model are determined by data availability. This data set contains the house sale price, date of sale (month,year), size of the lot (Area), number of Bedrooms, number of bathrooms, number of car parking spaces (includes carport and lock up garage, denoted by CarLug), and the number and street address which were geocoded into longitude and latitude coordinates.

Cominos (2006) discusses in detail the problem of the reliability of real estate data. As the data from more recent years tend to be more reliable due to improvements in the collection and maintenance of data sets, for the purpose of the empirical comparison of the STLM and the SSSEM the data previous to the period 1985-2005 have been excluded. This period provides more than enough data to conduct the comparison as the quantity of data rises at an increasing rate with time. The upward trend in the quantity of data is illustrated for the period 1985-2000 in Figure 2.

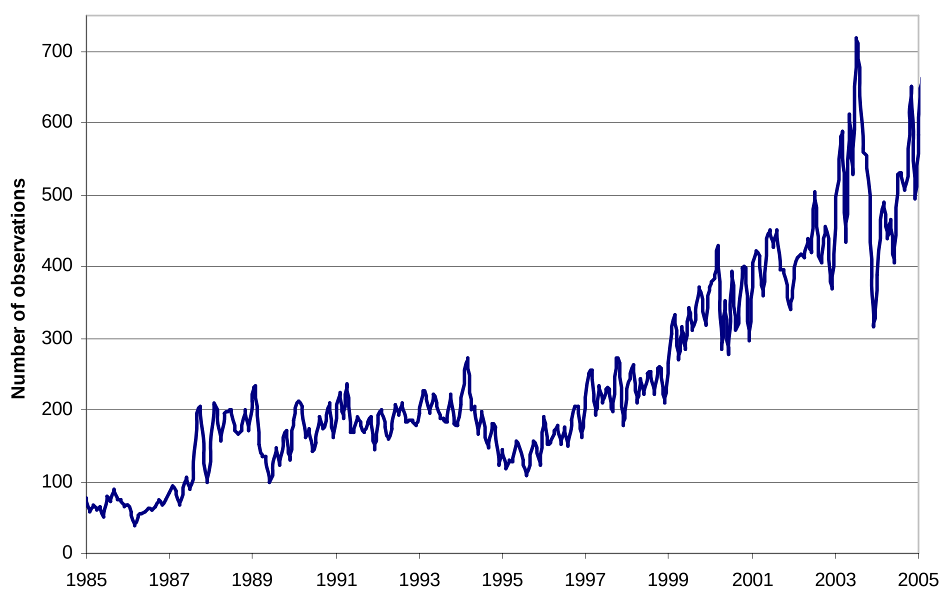

Figure 2: Number of observations per Month

The original data set by Cominos (2006) included houses with acreage. As the aim of models such as the STLM and the SSSEM is to model a 'typical' house, these houses were filtered out from the data for the purpose of this study. By modeling a 'typical' house the STLM and the SSSEM minimize the problem of heterogeneity in the hedonic residuals. Neither of the modeling specifications correct for heterogeneity which is inherent in the real estate market as it is not homogeneous by nature, thus the houses in the data set should be as homogenous as possible while allowing for enough variation in the hedonics to give reliable estimates of the hedonic coefficients. As a result houses with a number of CarLug, bedrooms and bathrooms larger than six were excluded in this study. The distributions and summary statistics for the prices as well as hedonic variables are illustrated next.

The hedonic variables included in the provided data set have been chosen for a number of reasons. The information about these hedonics has been more commonly available for most properties and similar variables are often used in the literature. The distribution and the summary statistics for the area in square meters, the number of bedrooms, the number of bathrooms and the number of car spaces and lock up garages are presented in Figure 3 to Figure 9.

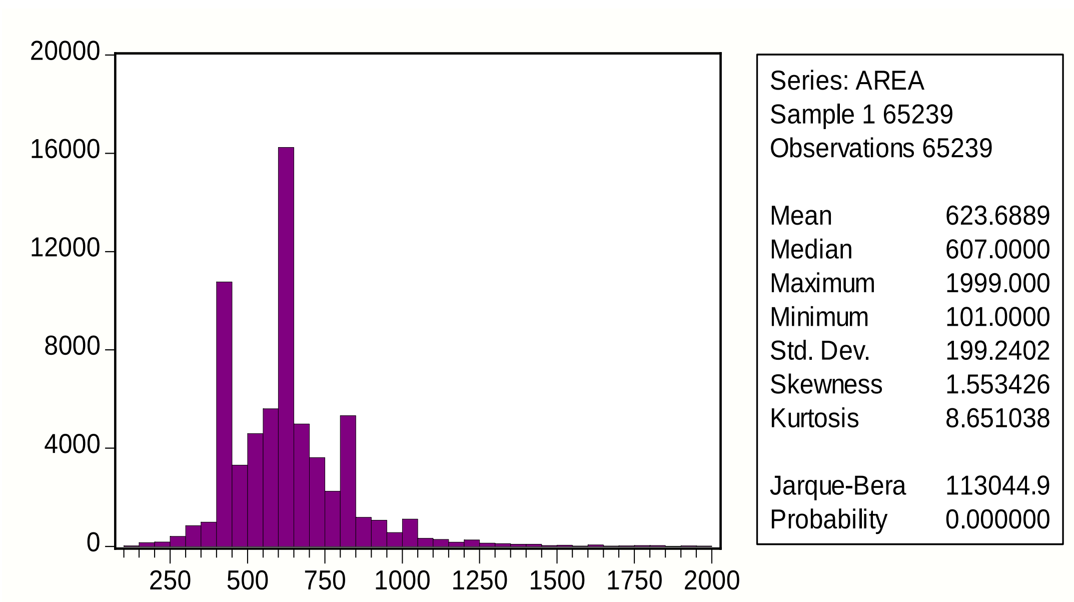

Figure 3: Area Variable (m^2)

The area variable is relatively symmetrical in its distribution however the right tail is longer than the left even though the number of observations in the range 1250 square meters and over is relatively small. Thus overall most of the houses in the data set are between 250 and 1250 centered around 600 square meters. Due to the outliers in the right hand tail of the distribution the range of the Area variable is quite large, 101 to 1999 square meters. To decrease the effect of the right hand tail and thus reduce the influence of large observations the log of the Area is used as a hedonic variable in the empirical comparison rather than the Area variable. In addition, the dependent variable will also be in the log form therefore the log form of the Area variable yields a convenient elasticity interpretation to the Area coefficient.

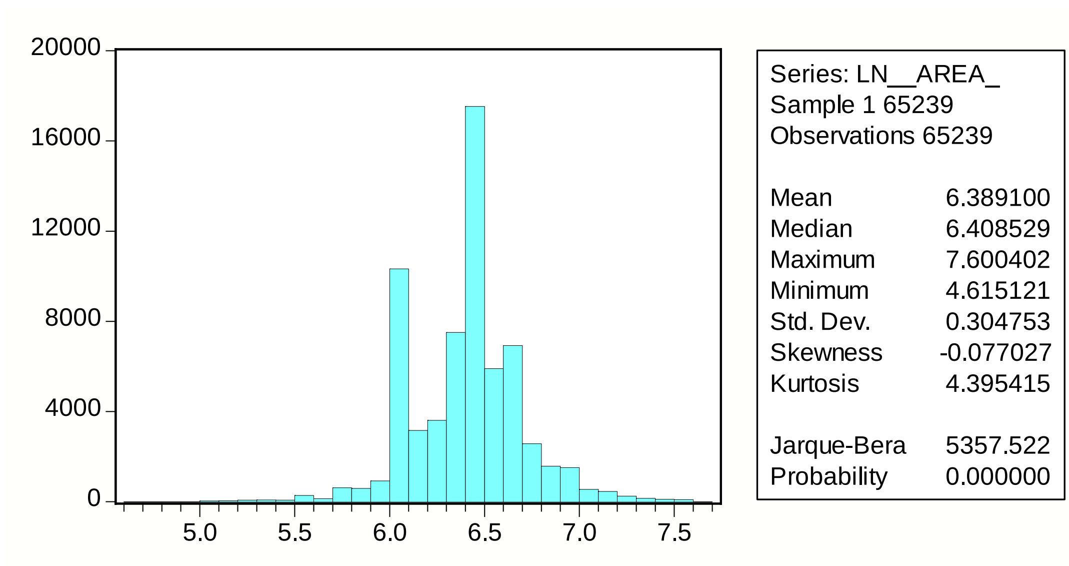

Figure 4: Ln(Area) Variable

Taking the log of the variable reduces its range by scaling large values more than small ones and therefore in this case may increase computational feasibility by reducing the values of some of the elements in the hedonic matrix.

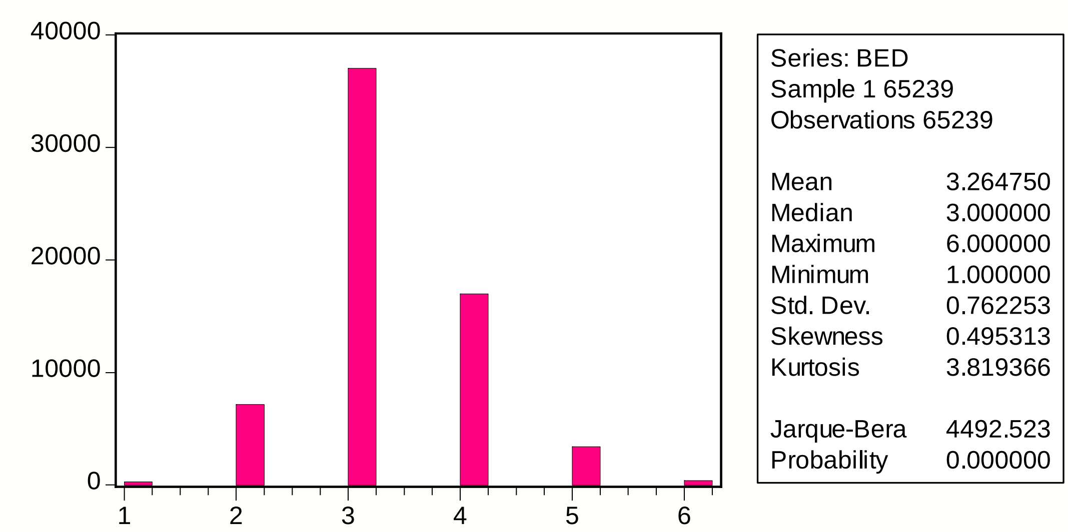

Figure 5: Number of Bedrooms Variable

The distribution of the Bedrooms variable is close to symmetrical with a slight right skew. The median number of bedrooms for the data set is three, which is a logical estimate for a typical house.

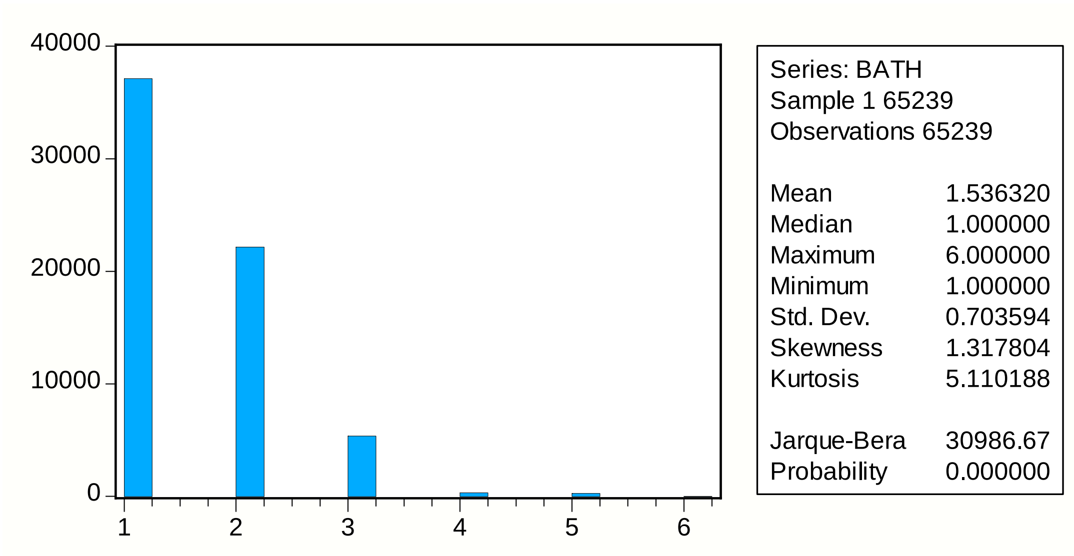

Figure 6: Number of Bathrooms Variable

The distribution of the Bathroom variable is skewed to the right indicating that although most houses have more than one bedroom the number of bathrooms tend to be mostly between one and three. Even for houses where six bedrooms are observed the number of bathrooms tends to be smaller. This result is not out of the ordinary as a typical house is not expected to contain a very large number of bathrooms.

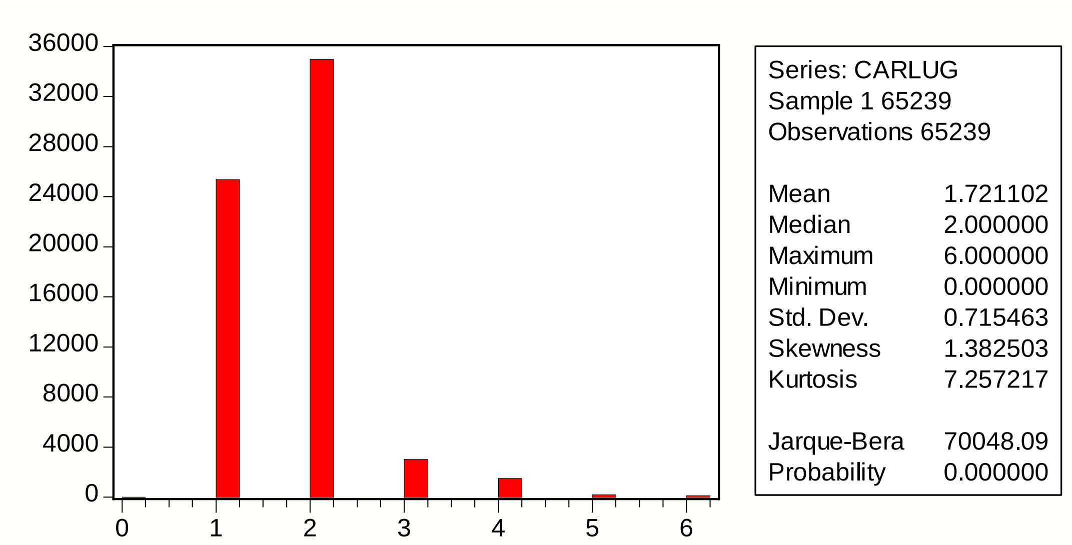

Figure 7: Number of Car Spaces and Lockup Garages Variable

In the discussion of the creation of the original data set, Cominos (2006) notes that the variable for car spaces is observed for some houses while others may contain a variable for a lock up garage. As these variables are very closely related the author combines the two variables into the CarLug variable which denotes the sum of the number of car spaces and the number of lock up garages. Most of the distribution for this variable is contained between values of one and two which is considered to be quite standard for a typical house in the Brisbane Metropolitan area.

The dependent variable in many hedonic models in the literature is given as log of the house price rather than just the house price to render the distribution more symmetrical. Normality is a common assumption for the error terms in the regression thus the dependent variable is expected to also be approximately Normal. For example, the Kalman Filter[1] applied in the estimation of the coefficients of the SSSEM model requires the normality assumption to be valid. The semi-log form of the model also serves to correct for heteroskedasticity between the house value and the residuals as the prediction error increases in size with more expensive dwellings.

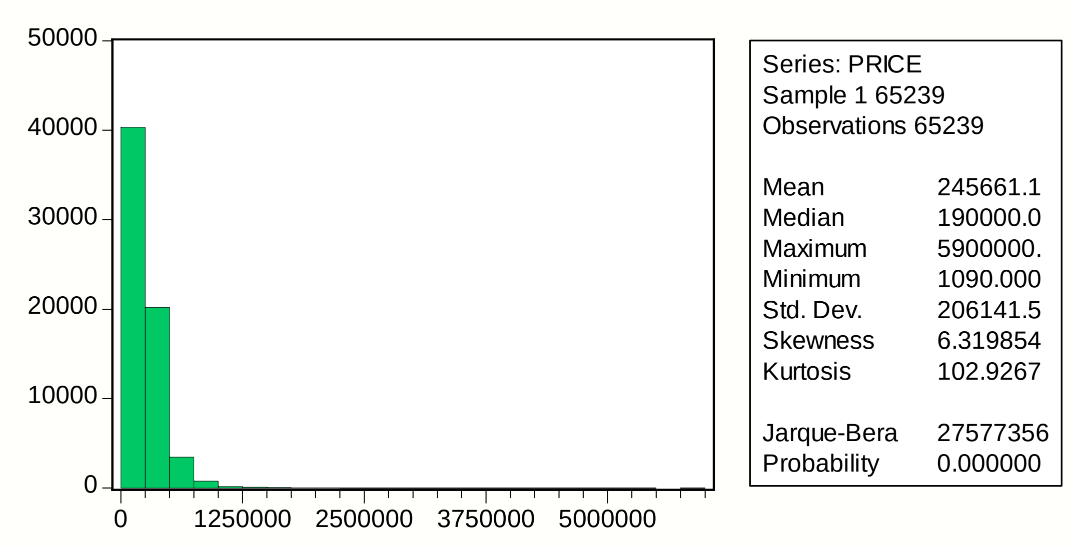

Figure 8: House Prices

Although the data set has been filtered further to exclude atypically large houses with a greater number of bedrooms, bathrooms or car spaces than six, it is difficult to exclude smaller atypical luxury homes. A luxury home may be determined partly by location however they are difficult to detect thus even the cleaned data distribution is heavily right skewed with a long, thin, right tail. Most of the typical houses in the data set range between $1090 (minimum value) and $100,000 however there are houses priced up to $5,900,000. The filtered data set contains ninety six houses which have a sales price below $10,000. Although this sales price seems too low to be realistic it may have resulted due to circumstances of the sale such as the house may have been in a bad state of disrepair or the seller wanted to sell immediately and was not concerned about the low price. As the underlying factors of these low price observations are unknown, it may not be appropriate just to exclude them from the data set as it may lead to biased results. Therefore they have not been filtered out from the data set. A more in depth investigation of the underlying reasons for such low prices is an avenue for further research however it is not investigated in this paper.

[1] Although the Kalman Filter may be altered to accommodate non-Gaussian data, the Kalman Filter specifications given in this paper assume normality.

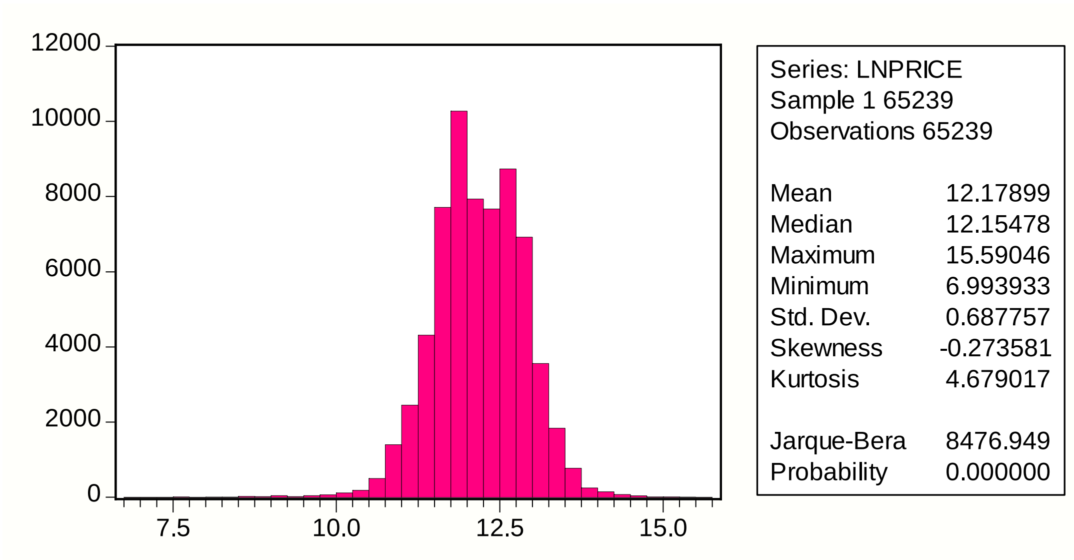

Figure 9: Log of House Prices

Taking the log of the house price renders the distribution of the variable approximately symmetric. The skewness of the distribution is reduced dramatically from 6.3 for the price to -0.27 for the log of the price.

The data may be tested for spatial autocorrelation via common spatial tests such as the Moran *I*, LR and LM tests. However as these tests have been performed on the data set by Cominos *et al*. (2007) only the conclusions are given in this paper. All the three tests performed on the data yield the result of the existence of spatial autocorrelation in the error of the hedonic model. Spatial autocorrelation is expected to occur in real estate data sets and is discussed in detail in the literature. On the other hand, testing the data for temporal autocorrelation yields more daunting problems. A Moran *I* type of spatial autocorrelation test is performed on data aggregated over some given time period thus, similarly to test for temporal autocorrelation data may be aggregated over space. However, the method of aggregation is unclear. A logical procedure to aggregation would be a submarket approach, where observations are aggregated over submarkets. Unfortunately the definition of submarkets requires in depth analysis of the spatial correlations in the data which is outside the scope of this paper. Therefore testing of the data for temporal autocorrelation is an avenue for further research.

3.3 Estimation Results

Both of the models have been coded in Matlab, the Matlab code for both models is available from the authors. As predicted via the analytical results the STLM specification leads to computational problems due to the large size of the data set therefore a moving window approach is taken to the estimation. The SSSEM specification yields monthly estimates for the hedonic coefficients therefore to compare the coefficient estimates of the SSSEM and the STLM, the monthly SSSEM estimates are averaged over each year. The averages of the estimated coefficients for the two models are relatively similar and the movements of the coefficients over time also confirm the results obtained in the previous section.

3.3.1 STLM Results

The application of the STLM to the Brisbane Metropolitan data set 1985 to 2005 resulted in computational difficulties. The attempt to run the program was made on three different computers however all attempts were unsuccessful. Although the model makes use of sparse matrices to aid computation it is nevertheless quite restrictive in the sample size it is able to handle. The problem lies in the process of the construction of the individual neighbour matrices. Once constructed, due to their sparsity their large dimensions should not cause computational problems however to construct these matrices the distance between each pair of observations must be calculated and recorded in an N by N matrix. As the potential neighbours only consist of previous observations this matrix will be lower triangular however, this still causes computational problems for sample sizes over 9000[2]. As the data set used in this application consists of over 60,000 observations a direct comparison of the models over the entire sample has proven impossible without access to a server.

To tackle this problem the model was estimated using an overlapping windows approach. As the number of observations rises at an increasing rate over the sample period (see Figure 2**)**, the later years, 2001 to 2005, are the most problematic for the STLM specification. This period alone contains nearly 30,000 observations. As a result for the purpose of this empirical comparison only 16 years as opposed to 21 years of data are used, accounting for the period from 1985 to 2000. Although it would have been interesting to see the results of the two models for the more recent years, 16 years of data are still enough to conduct the empirical comparison effectively. Even with the exclusion of the more recent five years of data, to apply a window of an equal duration over the period, each window could not be extended to over two years.

The STLM coefficients were estimated for each two year window over the period 1985 to 2000. As the windows overlap for all the years in this period with the exception of the first and last year, the coefficient estimates were taken as averages of the two overlapping windows for each year. The dependent variable in the hedonic regression equation of the STLM is the log of the house price rather than just the house price therefore the direct interpretation of the coefficients is not the shadow price of each characteristic. The coefficients for the Bedroom, Bathroom and CarLug variables may be interpreted as the percentage change in the house price given a one unit change in the respective variable. Cominos *et at* (2007) highlight an identification problem in the original model between the intercept and Area coefficient, as in this paper no attempt has been made to change the original specification, no interpretation of the Area coefficients is provided in the empirical comparison.

The hedonic coefficient estimates for each window with the exception of the Area variable are transformed into the shadow prices of each characteristic by multiplying the coefficients obtained via the STLM specification by the average price of a house in each estimation window. The average house prices have increased nearly three fold over the period 1985 to 2000 therefore the expectation is that the shadow prices of the hedonics should also increase over the period even if there is a downward trend in the coefficients in the semi-log form of the model. Table 2 illustrates the trend in house prices over the sample period.

[2] The data set used by Pace et al (2000) had 5243 houses.

Table 2: Average House Prices

Time Period	Average House Price
1985-1986	$66,675
1986-1987	$70,905
1987-1988	$84,616
1988-1989	$103,538
1989-1990	$126,199
1990-1991	$134,825
1991-1992	$138,924
1992-1993	$147,531
1993-1994	$156,114
1994-1995	$160,260
1995-1996	$161,739
1996-1997	$166,454
1997-1998	$174,506
1998-1999	$184,184
1999-2000	$192,814

Table 3 shows the shadow prices of the three house characteristics obtained from the STLM.

Table 3: STLM Shadow Prices

Time Period	Bedroom	Bathroom	CarLug
1985-1986	$2,613.66	$6,140.78	$2,166.94
1986-1987	$2,517.11	$5,516.37	$808.31
1987-1988	$4,112.32	$6,422.33	$2,293.08
1988-1989	$5,466.83	$9,349.52	$4,503.92
1989-1990	$7,723.35	$10,562.82	$2,700.65
1990-1991	$8,871.47	$12,296.02	$3,047.04
1991-1992	$8,002.05	$13,461.78	$4,751.22
1992-1993	$6,919.21	$13,956.45	$4,175.13
1993-1994	$6,790.98	$15,751.94	$2,997.40
1994-1995	$8,926.48	$19,231.19	$1,234.00
1995-1996	$11,176.14	$16,125.34	$3,509.73
1996-1997	$10,669.70	$15,946.29	$4,161.35
1997-1998	$10,226.05	$18,061.37	$5,043.22
1998-1999	$11,990.36	$20,131.27	$5,912.30
1999-2000	$13,921.14	$20,881.70	$5,379.50

The results illustrate plausible estimates, all of which are significant, for the Bedroom, Bathroom and CarLug shadow prices. However, the value of an extra bathroom relative to an extra bedroom seems high. This result may stem from the effect of missing hedonics that are significant contributors to the overall price of a house. Another possible explanation for this effect may be that an extra bathroom implies a larger house than an extra bedroom or a more luxurious house. The problem of missing variables is discussed by Cominos (2006) in the application of the SSSEM to the original data set. However, this problem stems from the availability of information on hedonic characteristics and not from the model specification. The Bathroom, Bedroom, CarLug and Area variables were used as the information on these variables was available for most of the houses in the Brisbane Metropolitan area. Thus, as stated, the Bathroom coefficient is likely to be correlated with other hedonics that have not been included in the model and is therefore results in a much higher shadow price than expected by picking up the shadow prices of the missing hedonics as well as its own. There is an upward trend over the sample in all three shadow prices illustrated in Table 3 as expected. The shadow price for an extra car space fluctuates over the sample period however the shadow price for an additional bedroom and an additional bathroom illustrate a steady upward trend over the sample period.

3.3.2 SSSEM Results

The estimation of the SSSEM did not result in any computational problems. However as the STLM could only be applied to the period 1985 to 2000, the SSSEM was also estimated for this period for the purpose of the comparison. One period for the SSSEM is taken to be one month, therefore this model specification yields one hundred and ninety two estimates for each of the hedonic coefficients. The spatial weight matrices are constructed using a Delaunay Triangulation Algorithm. The construction of the weight matrix by the Delaunay Triangulation Algorithm allows a different number of spatial neighbours for each property depending on the position of the property within the spatial domain and the variation in the density of points over the spatial domain. As discussed the application of the Kalman Smoother is considered inappropriate in the context of predictions of real estate prices. Therefore, the SSSEM coefficients are given either via the conditional estimates or the unconditional estimates yielded by the Kalman Filter. The use of the unconditional estimates implies the agents in the market mold their preferences based on current period and previous periods' information. On the other hand, the utilization of the conditional estimates implies that consumers base their preferences only on previous periods' information.

Similarly to the STLM specification the dependent variable is given as the log of the house price therefore to obtain the shadow prices of the hedonics the coefficients obtained via the SSSEM are multiplied by the average house price each period. Table 4 illustrates the average shadow prices for each year in the sample period for the hedonics (excluding the Area variable as stated in the previous section) using the conditional Kalman Filter coefficient estimates. Table 5 illustrates the results obtained if the unconditional results are utilized. The monthly estimates seem to move in cycles therefore the model may require a seasonal component. Attempts to account for the seasonality by considering one period as a quarter or specifying the SSSEM with seasonal dummies did not lead to promising results. Therefore, to account for the seasonality in the estimates a more formal modeling technique than simply inserting seasonal dummies into the modeling structure may be required. This investigation is outside of the scope of this paper however the results of the inclusion of the seasonal dummies and quarterly estimates are given in Appendix 3 of Svetchnikova (2007).

Table 4: SSSEM Conditional Shadow Prices

Conditional Estimates of Coefficients			
Year	**Bedroom**	**Bathroom**	**CarLug**
1986	$2,691	$9,461	$2,262
1987	$3,888	$10,560	$3,332
1988	$6,844	$9,484	$3,256
1989	$5,587	$18,598	$6,061
1990	$12,324	$16,264	$2,264
1991	$9,724	$18,061	$4,940
1992	$8,337	$20,877	$5,742
1993	$6,049	$22,540	$7,353
1994	$9,390	$25,732	$3,574
1995	$12,203	$29,993	$300
1996	$14,392	$24,031	$8,172
1997	$15,599	$24,084	$3,486
1998	$14,694	$30,628	$6,796
1999	$18,052	$29,712	$8,358
2000	$19,539	$34,110	$7,656

Due to the specification of the SSSEM[3] the conditional and unconditional estimates of the coefficients obtained via the Kalman Filter are not dramatically different. As a result the estimated shadow prices of the hedonics differ only slightly between the conditional and unconditional specifications.

Table 5: SSSEM Unconditional Shadow Prices

Unconditional Estimates of Coefficients			
Year	**Bedroom**	**Bathroom**	**CarLug**
1986	$2,451	$9,834	$1,903.19
1987	$4,128	$10,666	$2,718.67
1988	$6,998	$10,406	$4,137.89
1989	$5,742	$16,904	$5,677.43
1990	$12,308	$17,183	$2,176.88
1991	$9,009	$18,854	$6,976.16
1992	$8,610	$20,472	$4,571.01
1993	$6,137	$23,098	$6,693.39
1994	$9,358	$25,392	$3,664.22
1995	$12,381	$30,119	$597.67
1996	$15,306	$23,568	$7,860.60
1997	$14,404	$25,450	$3,850.20
1998	$16,124	$28,177	$6,068.90
1999	$17,319	$31,320	$8,131.03
2000	$19,204	$34,495	$7,261.85

Although the trend in all shadow prices is upward for the SSSEM as well as the STLM, the SSSEM specification illustrates a greater change over the sample period. The estimates near the beginning of the sample period are very close for the two modeling specifications however towards the end of the sample period the estimates produced by the SSSEM specification are significantly greater than those of the STLM especially for the Bathroom and

[3] This results from the transition equation in the SSF of the SSSEM being defined as a random walk.

Bedroom variables. The estimates of the shadow prices obtained via the SSSEM are all significant and plausible with the exception of the shadow price for a car space in 1995. The value for an extra car space in 1995 seems too low for both the conditional and unconditional estimates. A closer examination of the data in that year reveals several outliers. The presence of these observation results in some of the monthly estimates for the CarLug coefficient of the SSSEM to be negative and therefore the average coefficient for 1995 is unrealistically low. The presence of these observations warrants a review of the data obtained for the year 1995 to determine if these observations are a result of measurement error or some very specific circumstances.

Similarly to the STLM specification the relative sizes of the shadow prices of the Bedroom and Bathroom variables do not seem realistic. The Bathroom shadow price is too high relative to the Bedroom shadow price for the reasons already discussed.

The analytical derivations indicate that the estimates obtained via the STLM specification should be close to the average of the coefficients obtained via the Kalman Filter in the SSSEM. The data set used for this comparison states the time of sale by month thus the STLM estimates would incorporate the information of all the previous months for each observation respectively. The unconditional estimates given by the Kalman Filter incorporate the information obtained in the previous months as well as the current month in question. On the other hand the conditional estimates obtained via the Kalman Filter only incorporate the information obtained in the previous months and thus the average of the conditional estimates should be approximately the same as the STLM estimate for a given period. A disadvantage of using the Kalman Filter estimates to define the coefficients in the SSSEM is that the coefficient estimates near the beginning of the sample are unreliable and heavily dependent on the initialization values of the Kalman Filter, thus the year 1985 is not included in the comparison of the models. As the STLM estimates are obtained via overlapping two year windows (due to computational difficulties already stated), the yearly estimates are obtained as an average of the values obtained via the two overlapping windows for each year. The yearly estimate of the Bedroom coefficient for the two models is given in Table 6**.** Figure 10 illustrates the movement of the SSSEM conditional and unconditional monthly Bedroom coefficient estimates and the yearly Bedroom coefficient estimates obtained via the STLM.

Table 6: Bedroom Coefficient Comparison

	Bedroom Coefficient		
Year	**SSSEM(conditional)**	**SSSEM(unconditional)**	**STLM**
1986	0.039	0.036	0.037
1987	0.054	0.057	0.042
1988	0.074	0.076	0.051
1989	0.048	0.049	0.057
1990	0.092	0.092	0.064
1991	0.072	0.066	0.062
1992	0.059	0.061	0.052
1993	0.040	0.040	0.045
1994	0.059	0.059	0.050
1995	0.076	0.077	0.062
1996	0.089	0.094	0.067
1997	0.092	0.085	0.061
1998	0.082	0.090	0.062

1999	0.096	0.092	0.069
2000	0.099	0.097	0.072

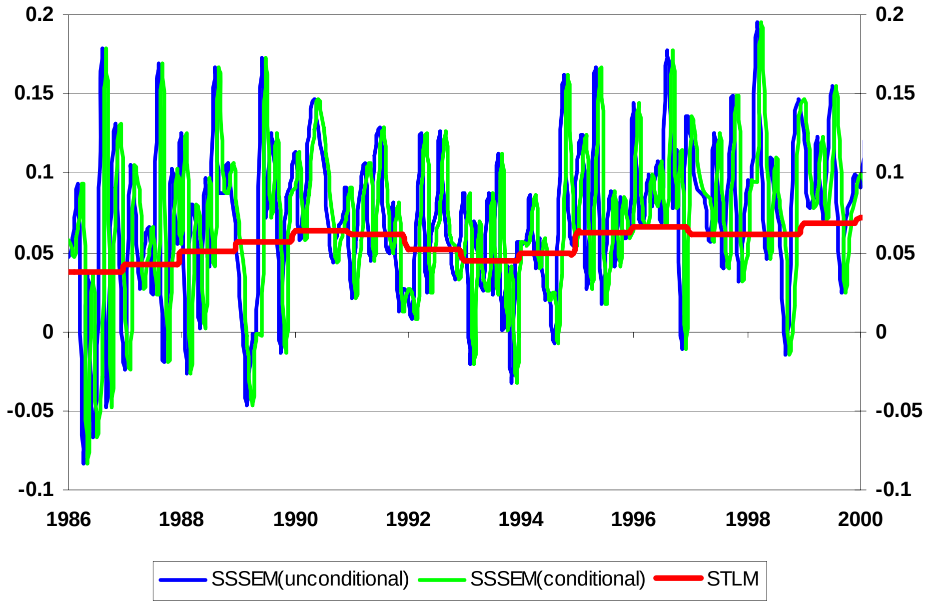

Figure 10: Bedroom Coefficient over Time

The conditional and unconditional estimates obtained via the Kalman Filter are very similar. The estimates obtained via the STLM and the average of the estimates obtained via the SSSEM are relatively similar, which confirms the results obtained in Section 2. Figure 10 illustrates that the time path of the STLM estimate is around the average of the SSSEM estimates. The standard errors of the estimates are not presented here as it is unclear how to determine the appropriate measure of the standard errors given the overlapping windows approach of estimation for the STLM. Although the standard errors for the monthly SSSEM coefficients are obtained via the Kalman Filter, as with the overlapping windows approach for the STLM, an appropriate measure for the standard error for the average of the coefficients each year is also unclear.

The empirical results for the Bathroom coefficient differ more significantly between the two models than the Bedroom coefficient. The estimates of the Bathroom coefficient are slightly higher for the conditional and unconditional estimates of the SSSEM specification with respect to the estimates obtained via the STLM. Table 7 illustrates the average of the monthly estimates obtained via the SSSEM for each year and the average of the overlapping windows of the STLM for each year. Figure 11 shows the time series of the coefficient estimates for the STLM and the SSSEM. Although the time series of the two modeling techniques are not as close as those of the Bedroom coefficient, the STLM estimates are still relatively close to the average of the SSSEM estimates.

Table 7: Bathroom Coefficient Comparison

Bathroom Coefficient			
Year	SSSEM(conditional)	SSSEM(unconditional)	STLM
1985	0.105	0.104	0.092
1986	0.139	0.144	0.085
1987	0.146	0.147	0.077
1988	0.103	0.113	0.083
1989	0.159	0.144	0.087
1990	0.121	0.128	0.087
1991	0.133	0.139	0.094
1992	0.147	0.144	0.096
1993	0.148	0.151	0.098
1994	0.161	0.159	0.110
1995	0.187	0.187	0.110
1996	0.148	0.145	0.098
1997	0.142	0.150	0.100
1998	0.171	0.157	0.106
1999	0.158	0.167	0.109
2000	0.173	0.175	0.108

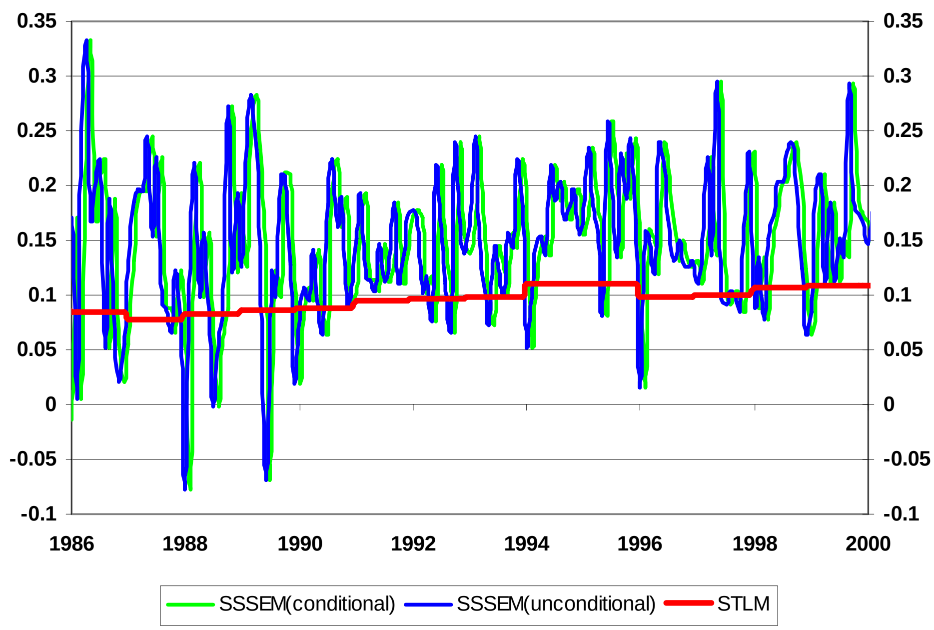

Figure 11: The Bathroom Coefficient over Time

The estimates of the CarLug coefficient are also quite similar for the two modeling specifications. Similarly to the Bathroom coefficient, the average of the unconditional as well as the conditional estimates of the SSSEM for each year are higher than the estimates obtained via the STLM. These results are illustrated by Table 8. The time path of the coefficient estimates is shown via Figure 12**.** The path of the STLM coefficients over time is approximately at the average of the SSSEM coefficients over time. These results are encouraging in light of the analytical findings of Section 2 and they seem confirmed via these empirical results.

Table 8: CarLug Coefficient Comparison

CarLug Coefficient			
Year	**SSSEM(conditional)**	**SSSEM(unconditional)**	**STLM**
1986	0.033	0.028	0.022
1987	0.046	0.038	0.019
1988	0.035	0.045	0.035
1989	0.052	0.049	0.032
1990	0.017	0.016	0.022
1991	0.036	0.051	0.028
1992	0.040	0.032	0.031
1993	0.048	0.044	0.024
1994	0.022	0.023	0.013
1995	0.002	0.004	0.015
1996	0.050	0.048	0.023
1997	0.021	0.023	0.027
1998	0.038	0.034	0.031
1999	0.044	0.043	0.030
2000	0.039	0.037	0.028

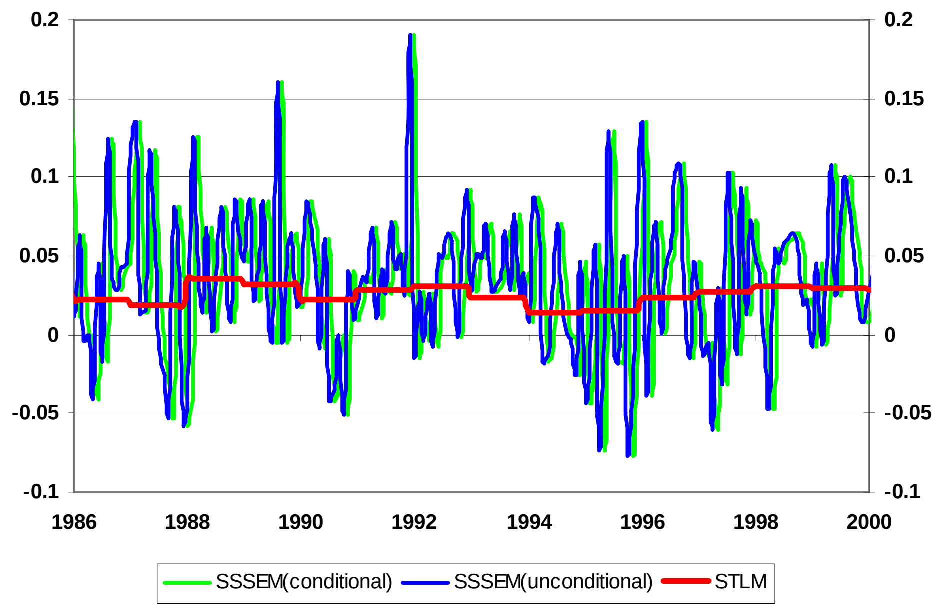

Figure 12: CarLug Coefficient over Time

Given the SSSEM has time varying coefficients, changes in preferences over time would be picked up by this specification as a trend (even if small). By having estimated the STLM, a fixed parameter model, with rolling windows, the coefficients have been allowed to evolve over time albeit in a more haphazard fashion than that imposed by the formal transition equation of the SSSEM.

3.4 Forecasting Accuracy

Attempting to compare the accuracy of prediction for the two models leads to difficulties as the best method of comparison is unclear. Due to the different treatment of time by the two models they may be compared via the Root Mean Squared Error (RMSE) using the monthly coefficients of the SSSEM or averaging the coefficients to obtain an average estimated shadow price for each characteristic over the sample period. Here the two models are compared on their accuracy of predicting the log of the price of a typical house for each two year window for which the STLM was estimated. The prediction may also be compared for observations outside the sample however this is outside the scope of this paper. The predicted log of the price obtained via each of the models is compared to the price of a typical house in that time window. A typical house is given by the median values for each of the characteristics each period. During the sample period 1985 to 2000 the median house stays the same with each window, the plot is of 607 meters squared and the house has 3 bedrooms, 1 bathroom and 2 car spaces. Although the typical characteristics of a house do not change through time the log of the price for a typical house increases over time as illustrated by Table 10. Table 10 illustrates the log of the price of a typical house as implied by the data and as predicted by the two models.

Table 11: Predicted Log Prices

Time Period	Ln(Price) Data	SSSEM conditional	SSSEM unconditional	STLM
1986-1987	11.0429	10.9927	10.9940	11.2078
1987-1988	11.2118	11.1218	11.1398	11.4485
1988-1989	11.3964	11.3748	11.3918	11.6110
1989-1990	11.5712	11.5320	11.5443	11.4859
1990-1991	11.6699	11.6268	11.6371	11.6827
1991-1992	11.7519	11.7157	11.7199	12.0108
1992-1993	11.8056	11.7701	11.7706	12.0212
1993-1994	11.8494	11.8101	11.8103	12.0489
1994-1995	11.8706	11.8072	11.8085	12.1566
1995-1996	11.8699	11.8056	11.8066	12.0648
1996-1997	11.9016	11.8243	11.8264	12.1791
1997-1998	11.9512	11.8554	11.8589	12.2415
1998-1999	11.9829	11.8992	11.8801	12.3131
1999-2000	12.0257	11.9391	11.9428	12.3253

The first window from 1985-1986 is excluded from the comparison as the Kalman Filter requires a burn-in period and therefore the coefficients obtained for the beginning of the sample are not considered. The predictions for the models are kept in the log form rather than converted into prices as simply taking the exponential of the log of the predicted price leads to biased results.

To obtain the unbiased estimates for the predicted prices of the two models an estimate of the variance of each of the coefficients is required. As the STLM is estimated via an overlapping windows approach it is unclear how an appropriate measure for the variance of the coefficients should be determined. The investigation of variance measures for an overlapping windows approach is outside the scope of this paper and therefore the prediction comparison is conducted in relation to the log of the price rather than the price.

The computed RMSE for each case are presented in Table 12. If the coefficients are estimated as the conditional estimates of the Kalman Filter is 0.06349. The RMSE for the SSSEM when the coefficients are estimated via the unconditional estimates of the Kalman Filter is 0.06134. The unconditional specification of the coefficients is expected to give more accurate results as consumers in the real estate market are likely to take the information of the current month into account. The RMSE obtained via the STLM specification is 0.2345. Therefore the STLM provides a less accurate prediction for the log of the price than both of the SSSEM specifications.

Table 12. Root Mean Square Comparison

	SSSEM (conditional)	**SSSEM (unconditional)**	**STLM**
RMSE	0.06349	0.06134	0.2345

4. Conclusions

Increased accuracy and improvements in the reporting and maintenance of data sets containing spatial and temporal domains as well as powerful computers have opened the way for Spatial-Temporal (ST) models and estimation techniques in many disciplines. ST methods have clear advantages over purely spatial or time-series methods as they do not require data to be pooled over either domain and thus do not lead a loss of information through aggregation. ST modeling has recently been introduced to real estate market analysis although it has been popular in other disciplines such us environmental sciences.

The ST real estate models have developed as extensions of existing spatial models in the literature. One of the pioneering models was the Spatial Temporal Linear Model (STLM) proposed by Pace et al**.** (2000) in the *International Journal of Forecasting*. The filtering of spatial and temporal components is handled through a covariance structure with both spatial and temporal weight matrices. The parameters of the model are constant over time and estimated through a generalized least squares estimator.

This compares both analytically and empirically the STLM model to a Spatial Errors Model (SEM) cast in state-space form, denote by SSSEM. In the SSSEM the covariance has a time varying spatial structure and the parameters are time-varying. The SSSEM model can be estimated through classical likelihood methods or Bayesian methods.

The analytical results show how the time and spatial domain are handled by the STLM and SSSEM, and demonstrate that the processes of information assimilation of each of the models leads to the result that the averages of the time varying coefficients' estimates obtained from the application of the Kalman filter to the SSSEM are comparable to the fixed coefficient estimates obtained via the STLM, although the Kalman smoothed estimates are not. We also show that the SSSEM is better suited to handle large data sets than the STLM. An illustration of the results is provided with a real estate sample for the Brisbane metropolitan area. The number of pair-wise comparisons required to construct the spatial neighbours weight matrices in the STLM model makes the estimation of it computationally unfeasible for samples that are larger than 9,000. This is not a problem for the SSSEM model as such comparisons are only required for houses sold within the same time period. The forecasting performance of the two models is evaluated through their RMSE. The SSSEM model provides significantly more accurate forecast than the STLM.

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