Imperfectly-Credible Disinflation of Small Inflations

Lynne Evans * — Newcastle University

Anamaria Nicolae † — Durham University

June 2010

Abstract

In this paper we study and quantify the effects of a disinflationary monetary policy on output and welfare in a New Keynesian DSGE model with both time varying velocity and imperfect credibility. The nonlinear solution method reveals that: i) the early output losses due to surprise announcement of a disinflation may be more pronounced and more prolonged than previously suggested in the literature, ii) the disinflationary output boom by Ball (1994) and Ireland (1997) may still appear, and iii) the optimal speed of disinflation is decreased.

JEL Classification: E31, E51, E52

Keywords: price stability, small inflation, imperfect credibility, time-varying velocity, gradual disinflation, optimal speed of disinflation.

*Lynne Evans is at Newcastle University Business School and is an Honorary Fellow at Durham University. E-mail: Lynne.Evans@ncl.ac.uk.

†Anamaria Nicolae is at Durham University, Department of Economics, 23/26 Old Elvet, Durham, DH1 3HY, UK, tel. +441913346358. E-mail: Anamaria.Nicolae@durham.ac.uk. Corresponding Author.

1. Introduction

In this paper, we study the output response to a disinflationary monetary policy and the optimal speed of disinflation in a New Keynesian DSGE model with time-varying velocity and imperfect credibility. We assume that policy makers are committed to price stability in the strict sense of achieving and maintaining a constant price level. The analysis takes place in a New Keynesian environment, where the supply-side of the economy is characterized by monopolistically competitive firms and there is rigidity in the setting of prices.

The research conducted on monetary contractions (Ball (1994), Ireland (1997), King and Wolman (1999), and Khan, King and Wolman (2003)) establishes two key results. The first result is that real output initially falls below its new long run equilibrium level. The second result is that a gradual disinflation from small inflation may result in a temporary output boom after the initial decline, as output may rise above its new steady-state level. This is the so-called 'disinflationary boom' identified by Ball (1994) and Ireland (1997), and much discussed in the literature, as it is not only counterintuitive but, also rarely observed in the data.

Since the output response to monetary contractions are of first-order policy importance, it is not surprising that interest has been shown in exploring the robustness of these results to the relaxation of key assumptions. Nicolae and Nolan (2006) relax the assumption of perfect credibility in Ireland's (1997) model[1] as the steady state of the economy may well be changing and policymakers may not enjoy complete credibility. They demonstrate that the early output losses are more marked, and that the disinflationary output boom may disappear in an environment characterized by imperfect credibility, depending on the speed

[1] Ball (1995), Ireland (1995) and Erceg and Levin (2003) all argue that higher output losses are the price of imperfect credibility during a period of disinflation. Ball (1994) had suggested that the appearance of the disinflationary boom may be due to perfect foresight.

of learning relative to the speed of disinflation. However, a characteristic of the new Keynesian literature referred to above is the hypothesis of constant unitary velocity which occurs because money demand is not formally modelled but is postulated. Unitary velocity implies that the policymaker chooses a time path of the money supply which just supports nominal GDP while making strong assumptions about money demand behaviour. Yet it is well known that velocity is not a constant. A limitation of their model is that it assumes constant velocity of circulation, an assumption not acceptable when different rates of inflation are involved.

Evans and Nicolae (2010) address this issue and relax the assumption of constant velocity in Ireland's model. They endogenise time-varying velocity by making the demand for money explicit. In this set up, they find that the early out losses are now bigger and that even under perfect foresight, the disinflationary booms' (dis)appearance depends on the degree of time varying velocity. However, their model is limited to the case of perfect foresight and does not allow to examine the output response to disinflation in the context when policymakers initially do not enjoy complete credibility and the steady state of the economy is changing.

Another important result from the above mentioned literature which we discuss in our paper is the optimal speed of disinflation. One of the main findings of Ireland (1997) is that 'small' (3%) inflations are best disinflated gradually while 'big' (200%) inflations are best disinflated immediately. The result that 'big' inflations are best disinflated immediately holds when each of the assumptions of perfect foresight and constant velocity have been separately relaxed (see Nolan and Nicolae (2006) and Evans and Nicolae (2010)) in his model (a result of the pricing strategy adopted). However, while supporting the finding that small disinflations are best approached gradually, these papers also show that the relaxation of each of these assumptions resulted in bigger early output losses and also raised the question of the (dis)appearance of the disinflationary output boom, suggesting an even more gradual approach to disinflation of small inflation.[2] Therefore the focus of this paper is on further exploring output responses to gradual disinflations from small inflation rates (as well as the optimal speed of disinflation).

The contribution of this paper is that it builds a model which allows for the relaxation of both assumptions of perfect credibility and constant velocity simultaneously. The model builds on Ireland (1997) and Evans and Nicolae (2010). The approach we adopt to implement imperfect credibility draws on that employed by Nicolae and Nolan (2006). In this new model, the monetary policymakers make a surprise announcement of the disinflation and doggedly pursue the goal of price stability while agents take time to believe the announced policy in the face of this imperfect, but improving, credibility. Thus, in this paper we can examine the output response to a disinflationary monetary policy where the steady state of the economy is changing and when policymakers initially do not enjoy complete credibility, and in which, moreover, velocity is time varying.

Within this new, more complex set up we explore output responses to gradual disinflation from a small inflation, focusing on the two key results in the literature regarding the early output loss and the disinflationary output boom during a disinflation. Moreover, the non-linear solution method employed is also used to calculate the optimal speed of disinflation in this new context, and quantify the effect of this joint relaxation of assumptions.

The following section of this paper presents the model and the parameter values used in the calibration. Section 3 presents benchmark results from the existing literature showing the output response to (gradual) disinflations (from small

[2] Ireland (1997) shows that disinflating small inflations immediately brings about bigger output losses than gradual disinflations. Ireland's finding is robust to individually relaxing the assumptions of perfect credibility (Nicolae and Nolan (2006)) and constant velocity (Evans and Nicolae (2010)) in his model.

inflation) when velocity is assumed constant and there is imperfect credibility, and also when velocity is time varying under perfect foresight. Section 4 analyses the output response to (gradual) disinflation in the model with time-varying velocity and imperfect credibility. Section 5 discusses the optimal speed of disinflation, and section 6 concludes the paper.

2. The Model

The framework employed for this analysis extends the perfect foresight model developed in Evans and Nicolae (2010). The component parts of this model are now familiar in the literature.

2.1. The Representative Agent

Each period, the representative agent makes plans for consumption and leisure/labour to maximize the expected present discounted utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\alpha} - 1}{1-\alpha} - \gamma L_t \right\} \qquad \alpha, \gamma > 0, \tag{1}$$

which is separable in consumption, C_t, and employment, L_t. $\beta \in (0,1)$ is a discount factor, α is the intertemporal elasticity of substitution and γ is the disutility of work. Consumption, is defined over a continuum of goods

$$C_t = \left[\int_0^1 c_t(i)^{\frac{b-1}{b}} di \right]^{\frac{b}{b-1}} \qquad b > 0,$$

where $c_t(i)$ is, in equilibrium, the number of units of each good i from firm i that the representative agent consumes and b is the price elasticity of demand. Labour supply, L_t, is

$$L_t = \int_0^1 l_t\,(i)\,di,$$

where $l_t(i)$ denotes the labour supplied by the household to each firm i, at the nominal wage W_t, during each period.

Households face an aggregate price level, P_t, given by:

$$P_t = \left[\int_0^1 p_t(i)^{1-b} di \right]^{\frac{1}{1-b}},$$

where $p_t(i)$ is the nominal price at which firm i must sell output on demand during time t. Households supply labour to *all* firms, which, together with the budget constraint below (equation (2), ensures that the marginal utility of wealth equalizes across agents.

Each period, the representative household faces a budget constraint of the following form:

$$\int_0^1 \left[Q_t\left(i\right) s_{t-1}\left(i\right) + \Phi_t\left(i\right) + W_t l_t\left(i\right) \right] di \geq \int_0^1 \left[p_t\left(i\right) c_t\left(i\right) + Q_t\left(i\right) s_t\left(i\right) \right] di, \qquad (2)$$

where $Q_t\left(i\right)$ denotes the nominal price of a share in firm i, $s_t(i)$ denotes the quantity of shares, $\Phi_t\left(i\right) = D_t(i)s_t(i)$, where $D_t(i)$ is the dividend associated with a unit share, and $\int_0^1 p_t\left(i\right) c_t\left(i\right) di = P_t C_t$ denotes total nominal expenditure on non-durable consumption. We assume that for $t = 0$, $s_{-1}(i) = 1$, for all $i \in [0, 1]$. Also, we assume that each household owns an equal share of all the firms. The constraint (2) says that, in each period, income (financial plus labour) must be less than the value of expenditure (on non-durable consumption plus financial investment). The household chooses $c_t(i), l_t(i), s_t(i)$ so as to maximize (1) subject to the constraint (2) and the relevant initial and transversality conditions. Additionally, its optimal allocation across differentiated goods $c_t(i)$ must satisfy:

$$c_t(i) = C_t \left(\frac{p_t(i)}{P_t} \right)^{-b}. \qquad (3)$$

The aggregate equilibrium nominal magnitudes are determined by a quantitytheory type relation:

$$M_t V_t = \int_0^1 p_t\left(i\right) c_t\left(i\right) di = P_t C_t, \tag{4}$$

where the velocity of circulation, V_t is time-varying. It is given by:

$$V_t = \Omega C_t^{\delta} e^{\rho \pi_t}, \qquad \delta \in [0,1), \rho \in [0,1), \tag{5}$$

where $(1-\delta)$ is the scale elasticity of money demand and ρ is the semi-elasticity with respect to the opportunity cost variable, $\pi_t = \frac{P_t}{P_{t-1}} - 1$, inflation. Different values of parameters δ and ρ capture different degrees of time varying velocity and Ireland's case of a constant velocity is nested as a special case (for $\delta = 0$ and $\rho = 0$). For any non-zero positive values of δ or ρ, velocity is time-varying and endogenous to the model.

The agent solves the maximization problem, yielding the following first order conditions:

$$C_t^{-\alpha} = \lambda_t P_t; \tag{6}$$

$$\gamma = \lambda_t W_t; \tag{7}$$

(from (6) and (7))

$$W_t = \gamma P_t C_t^{\alpha}. \tag{8}$$

And for all i

$$Q_t(i) = D_t(i) + \beta(\lambda_{t+1}/\lambda_t) Q_{t+1}(i), \tag{9}$$

where λ_t is an unknown multiplier associated with the budget constraint (2)[3].

[3]For simplicity, Ω is here set equal to unity.

2.2. The Corporate Sector

The supply-side of the economy consists of monopolistically competitive firms and there is price rigidity. A continuum of firms, indexed by i over the unit interval, each produces a different, perishable consumption good, indexed by $i \in [0,1]$, where firm i produces good i. The representative household trades shares in each firm i, which sell at the nominal price $Q_t(i)$ at the beginning of time t, and pay a nominal dividend $D_t(i)$ at the end of time t.

We assume a simple linear production technology $y_t(i) = x_t(i)$, where $y_t(i)$ and $x_t(i)$ are the output of firm i and the labour used to produce it, respectively. Y_t is aggregate output. Equilibrium returns to shareholders at time t for firm i are given by:

$$D_t(i) = [p_t(i) - W_t(i)] \left(\frac{p_t(i)}{M_t} \right)^{-b} C_t^{1-b(1-\delta)} - I_t(i) W_t(i) k, \tag{10}$$

where

$$I_t(i) = \begin{cases} 1, & \text{if the firm pays the cost of price adjustment } k \text{ at moment } t; \\ 0, & \text{if the firm does not pay the cost } k \text{ at moment } t. \end{cases}$$

Costly price adjustment is central to this model, in which time-dependent and state-dependent strategies are both present. Firms are divided into two categories, such that at time t firms belonging to the first category can freely change their prices, $p_{1,t}(i)$, while firms belonging to the second must sell output at the same price as they set a period before, $p_{2,t}(i) = p_{2,t-1}(i)$, unless they pay the fixed cost $k > 0$, measured in terms of labour. At time $t+1$, the roles are reversed and the first category of firms keeps prices unchanged, $p_{1,t+1}(i) = p_{1,t}(i)$, unless they are willing to pay the fixed cost k, while the second category of firms can freely set new prices.

Firms are constantly re-evaluating their pricing strategy, weighing the benefits of holding prices fixed against the alternative of changing prices and incurring the fixed penalty. At moment t the firms that can freely change price are able to choose between two strategies, depending on whether the inflation rate is moderate or high. At moderate rates of inflation, they are more likely to keep their prices constant for two periods and hence avoid the cost k (single price strategy). On the other hand, in the case of a high inflation, or in the face of sharp changes in the monetary stance, firms are more likely to choose a new price and pay the cost k (two price strategy). The price-setting decision at time t maximizes the return to shareholders.

The *equilibrium* in the model is given by the market clearance conditions for the three markets present in this model (goods market, labour market and asset market). Clearance in two markets assures clearance in the third. From the market clearance conditions for the goods and labour markets we have:

$$C_t = Y_t = X_t.$$

The clearance condition for the asset market is $s_{t-1}(i) = 1, \forall i \in [0,1]$, in each period.

2.3. The pricing strategies

There are two pricing strategies the firm can follow. Under the *single price strategy,* firm i chooses the price $p_t(i)$ to maximize the expression:

$$\Pi_t(i) = D_t(i) + \beta \left(\frac{\lambda_{t+1}}{\lambda_t} \right) D_{t+1}(i), \tag{11}$$

which follows from (9), and implies that prices are set to maximize market value. Substituting (4) and (8) into (10), and then this into equation (11), yields the price firm i will use for two consecutive time periods:

$$p_t(i) = \frac{b}{b-1} \gamma e^{\rho \pi_t} \frac{M_t^b Y_t^{1-b(1-\delta)} + \beta M_{t+1}^b Y_{t+1}^{1-b(1-\delta)}}{M_t^{b-1} Y_t^{2-b(1-\delta)-\alpha-\delta} + \beta M_{t+1}^{b-1} Y_{t+1}^{2-b(1-\delta)-\alpha-\delta}}.$$

This equation, familiar from the New Keynesian economics literature, shows that the optimal price is a function of current and future anticipated demand and cost conditions; and that, in steady-state, price is a fixed mark-up over marginal costs. As is familiar in models of monopolistic competition, the markup is constant and determined by the elasticity of demand (that is, it is tied down via the preference side of the model): the lower the elasticity, the higher the mark-up.

Under the *two price strategy,* firm i chooses the price $p_t(i)$ to maximize the expression:

$$\Pi_t(i) = D_t(i) \tag{12}$$

and now the optimizing price is:

$$p_t(i) = \frac{b}{b-1}\gamma e^{\rho\pi_t}\frac{M_t}{Y_t^{1-\alpha-\delta}}.$$

Again, prices are a mark-up, but now only current period demand and cost conditions are relevant since only current dividend matters.

2.4. Monetary Policy

The disinflationary policy employed in this paper follows the approach adopted by Ball (1994), Ireland (1997) and Evans and Nicolae (2010). The monetary policy is designed to bring money growth to zero over some time horizon. Specifically, at period 0, the authorities make a surprise announcement about the path for the money supply, $\left\{M_t^A\right\}_{t=0}^T$, such that by time period T inflation will be zero. This announced path for the money supply implies a decrease in the growth rate of the money supply.

Let

$$\theta_t = \frac{M_t}{M_{t-1}}$$

denote the gross rate at which the money supply increases at time t. A horizon of time $T = 1$ implies immediate disinflation, while for $T > 1$ the policymakers engineer a more gradual path towards price stability. We adopt a disinflationary process of the following sort:

$$\theta_t = \begin{cases} \theta_{t-1} - \varphi^{T-1}\left(\pi^{initial} - \pi^*\right), & t < T-1 \\ 1, & t \geq T \end{cases}, \qquad \varphi \in (0,1)\,, \tag{13}$$

where $\pi^{initial}$ is the initial rate of inflation from which the disinflation process starts, π^* is the final (target) inflation to be set here at $\pi^* = 0$.

To facilitate comparison with the existing literature we employ a linear disinflationary policy following Ireland (1997), Nicolae and Nolan (2006) and Evans and Nicolae (2010), which we obtain for $\varphi = (1/T)^{\frac{1}{T-1}}$.

2.5. Imperfect Credibility

In this paper imperfect credibility is modelled in the style of Nicolae and Nolan (2006) in which credibility is imperfect, but nevertheless improving over time. The probability mass characterising agents' subjective expectations is shifting through time onto the central bank's announced money supply path,$\left\{M_s^A\right\}_{s=0}^{T+J}$, $J \geq 0$. It is assumed that agents perceive of two possible outcomes regarding the path for the money supply: the monetary authority's announced path for the money supply and a more inflationary path for the money supply. For the more inflationary path agents perceive the authorities as reverting to the previous steady state inflation rate.

As it is assumed that the authorities stick to the announced path of disinflation, the expected money supply is now:

$$E_{t+j-1}M_{t+j} = \sigma_{t+j}\theta_{-1}M_{t+j-1}^A + (1-\sigma_{t+j})M_{t+j}^A,$$

where $\{\sigma_s\}_{s=0}^{T+J}$ is given by two plausible characterizations of the transition between imperfect credibility and perfect foresight. These are given by:

$$\sigma_t = \begin{cases} (-1)^{\delta}\,\tau(N^2 - (t-\eta N)^2)^{\frac{1}{2}} + \eta\sigma_0, & t < N-1 \\ 0, & t \geq N \end{cases}, \tag{14}$$

where N captures the time it takes for agents to believe completely the central bank's announcements (we assumed in this paper $N \leq T$) , σ_0 is a measure of the initial level of credibility, $\sigma_0 \ \in [0,1]^4$ and $\tau = N/\sigma_0$. For $\eta = 0$ we have what is labeled as 'concave' learning[5] and $\eta = 1$ 'convex' learning[6]. Thus, σ plots a concave or convex function of time in the (x, y) plane, where the $x-$axis measures time and y degree of credibility.

For the purposes of comparison with the existing literature we assume that agents start the learning process from total lack of credibility ($\sigma_0 = 1$) and that agents take the central bank announcment at face value (implying that agents finally believe completely the announcement when, and only when, price stability is actually achieved) after three years ($N = 6$) [7]. Introducing uncertainty into our framework results in some computational complexity which requires a more complex nonlinear solution method to solve the model.

2.6. Model Calibration

This section presents the calibration of the model. To facilitate comparison with the existing literature, we employ parameter values drawn from the wider literature. For ease of reference, Table 1 sets out the parameter values used in the

[4] For $\sigma_0 = 0$ at the moment of the change in policy were is perefect credibility (this is the perfect foresight case). For $\sigma_0 = 1$ the learning process starts from total lack of credibility (there is imperfect credibility) .

[5] This captures the intuitive idea that agents may be reluctant to update their priors initially. However, as time goes by and the central bank sticks to its announced money supply targets, they increasingly come to believe the announced target path. This case is refered to as *concave (expectations) updating.*

[6] This reflects a population, although happy to accept that the monetary authority dislikes the current relatively high rate of inflation, nevertheless worries that as the slope of the short-run Phillips curve flattens the monetary authority may be tempted to renege. The importance of the exploitability of the Phillips curve has been emphasized by Ball, Mankiw and Romer (1988) and is a crucial factor in high inflation equilibria in games of the Barro and Gordon (1983) sort. This case is refered to as *convex (expectations) updating.*

[7] If σ_t takes a longer time to reach zero, output obviously also takes a longer time to reach its new steady state level.

calibration. The parameters δ and ρ to take a number of different values in order to explore the effect of time varying velocity on output.

Parameter	**Value**	**Description**
α	0.1	intertemporal elasticity of substitution (value as in Ball, Mankiw and Romer, 1988)
b	6	price elasticity of demand (value as in Rotemberg and Woodford, 1992)
k	0.1075	cost of price adjustment (value as in Ireland, 1997)
β	0.97	discount factor; each interval of time corresponds to 6 months (value as in Ball and Mankiw, 1994)
γ	1	degree of disutility from work (value as in Nicolae and Nolan, 2006)
Parameters	*capturing*	*the degree of time varying velocity*
δ	$[0, 1)$	$(1-\delta)$ is the scale elasticity of money demand
ρ	$[0, 1)$	opportunity cost semi-elasticity of money demand

Table 1. *Parameter values used in the model calibration.*

In the following section, we present benchmark results from the existing literature. These describe the behaviour of output during immediate and gradual disinflations starting from low initial inflation rates, where velocity is assumed constant and time varying and where, of course, there is perfect credibility. The subsequent section analyses the output response to gradual disinflation in the model when velocity is allowed to vary and there is imperfect credibility.

3. Benchmark Results

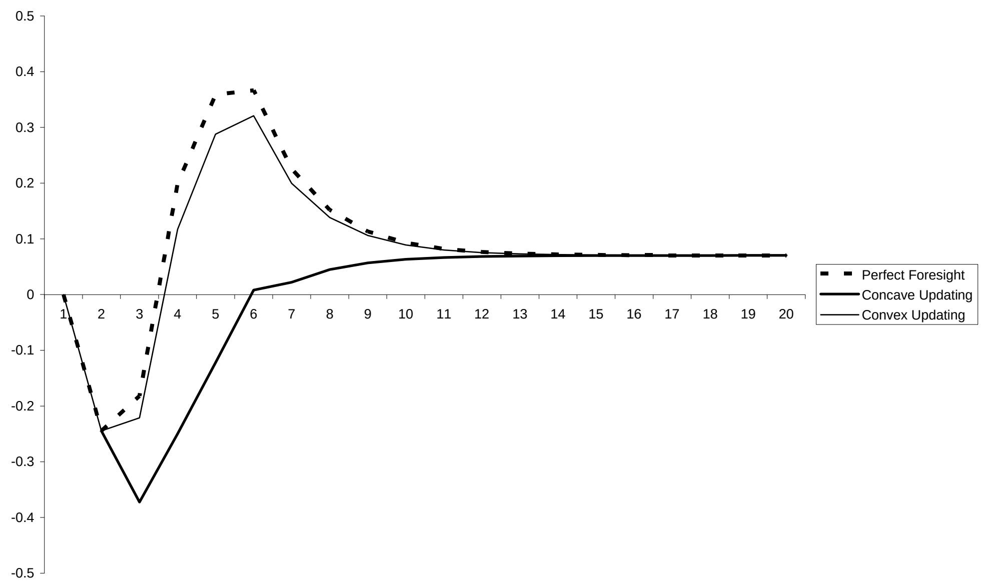

Figure 3.1: **Benchmark Result (Ireland 1997, Nicolae and Nolan 2006):** *Output response to a gradual ($T = 6$) disinflation of a 'small' (3%) initial annual inflation rate under perfect foresight and imperfect credibility (concave and convex expectations updating) with constant velocity.*

This section presents results of output responses to disinflationary monetary policy from the existing literature. First we present results when the velocity is assumed constant and there is imperfect credibility, and second when velocity is time varying under perfect foresight. For comparison purposes, we choose to present and analyze the case of a 3 year long disinflation ($T = 6$) from a 3% initial annual inflation rate.

Figure 3.1 presents the output response to a gradual disinflation (of the 'small' (3%) initial annual inflation rate) under perfect foresight and imperfect credibility (concave and convex expectations updating) when the velocity of circulation is constant. The short dashed line shows that a gradual disinflation from a low initial inflation rate brings about an early fall in output for two consecutive periods (0.24% in the first period and 0.18% in the second one) which is then followed by a compensatory output boom before the new steady-state is reached (Ireland 1997).[8] The thick solid line shows the output under imperfect credibility when there is concave expectations updating. The output keeps falling in the second period as well (to almost 0.4% below the initial steady state) and most importantly, there is no output boom. The thin solid line shows the output under imperfect credibility when there is convex expectations updating. The output fall in the second period is much less (0.22% below the initial steady state). After this, the output rises above the intial steady state, resulting in a boom, though smaller that that under perfect foresight.

Figure 3.2 shows output responses to the same 3 year long gradual disinflation of the 'small' (3%) initial annual inflation rate achieved under perfect foresight when velocity is time varying. The short dashed line shows the output response to disinflation when velocity of circulation is constant under perfect foresight, as in Figure 3.1. The solid lines show the output response when velocity is time-varying to various degrees, captured here through the scale elasticity of money demand parameter δ, and the opportunity cost parameter ρ. Relative to Ireland's early output loss and compensating boom (the short dashed line), the impact of having endogenised time varying velocity is to induce much larger early output losses (of

[8]Such disinflationary booms are typically understood as follows. Under perfect credibility, agents respond in advance of the change in policy by lowering their prices, knowing that inflation is going to be lower in the future. Because agents set prices for two periods, and because inflation will be lower in the future, they set lower prices today, inducing a boom (Ball 1994).

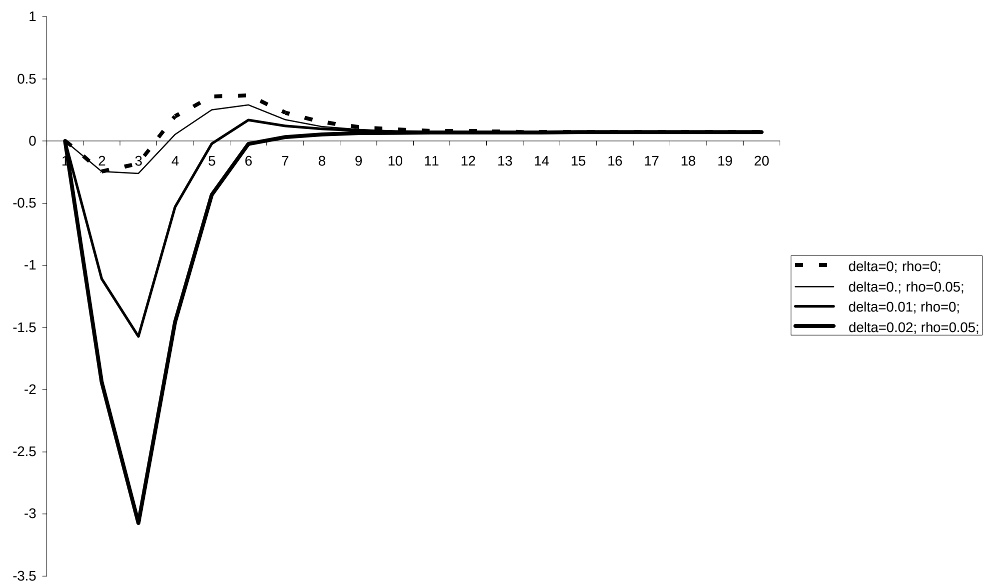

Figure 3.2: **Benchmark Result (Ireland 1997, Evans and Nicolae 2010):** *Output response to a gradual ($T = 6$) disinflation of a 'small' (3%) initial annual inflation rate under perfect foresight with time-varying velocity.*

almost 1.1% below the initial steady state in the second period and 1.6% in the second period for $\delta = 0.01$, $\rho = 0$) and much moderated, to almost nonexistent, booms.

In the next section we explore these benchmark results in the context of a model which simultaneously allows for both time-varying velocity and imperfect credibility. Specifically, we will explore: i) the size of the early output losses and, ii) the size of the disinflationary boom and its (non)appearance. Furthermore, we address the question of whether any output boom can compensate the early output loss when a 30 year time horizon is considered. The issue of the optimal speed of disinflation in this context is addressed in Section 5.

4. Output Response to Disinflation with Time-Varying Velocity and Imperfect Credibility

In this section we examine the output response to the gradual disinflationary monetary policy when velocity is time-varying and the policymakers pursue the goal of price stability in the face of imperfect, but improving credibility, captured here through both concave and convex expectations updating.[9] For ease of comparison with the benchmark results, we present and analyze the case of a 3 year long disinflation ($T = 6$) from a 3% initial annual inflation rate. Since there are two sets of benchmark results, we embed the one - which relates to relaxing the perfect foresight assumption in Ireland's model (Fig. 3.1.) - within Figures 4.1.and 4.2, and cross reference to the benchmark results which relate to relaxing the constant velocity assumption in Ireland's model (Fig.3.2).

Concave Expectations Updating

Figure 4.1 presents output responses to a gradual disinflation from a small (3%) initial annual inflation rate from the model with both time-varying velocity and imperfect credibility with concave expectations updating.[10] This figure shows that when time varying velocity is captured through the opportunity cost parameter only ($\delta = 0$, $\rho = 0.05$), in the first period after the change in policy is announced, the output loss is the same as under perfect foresight (the short dashed line) and

[9] We also analyse the output response to a gradual disinflationary for the case in which the more inflationary path for the money supply agents perceive is that that the government will 'run out of steam' (the growth rate of money will be that between $t-1$ and t) with concave and convex expectations updating. These results are available on request. However, for the purpose of this paper, the results presented are sufficient to demonstrate the effect of time-varying velocity and imperfect crediblity on output in our model.

[10] Different values for the parameters δ and ρ capture different degrees of time varying velocity. For comparison purposes we follow Evand and Nicolae (2010), where these parameters have been set to reflect empirical estimates of the relevant opportunity cost and income elasticities of the demand for money. $\rho = 0.05$ (as in Ball, 2001) while δ takes the minimum value for the range of values $[0.01, 0.03]$ as reported in a survey of recent empirical money demand studies (Sriram, 2001).

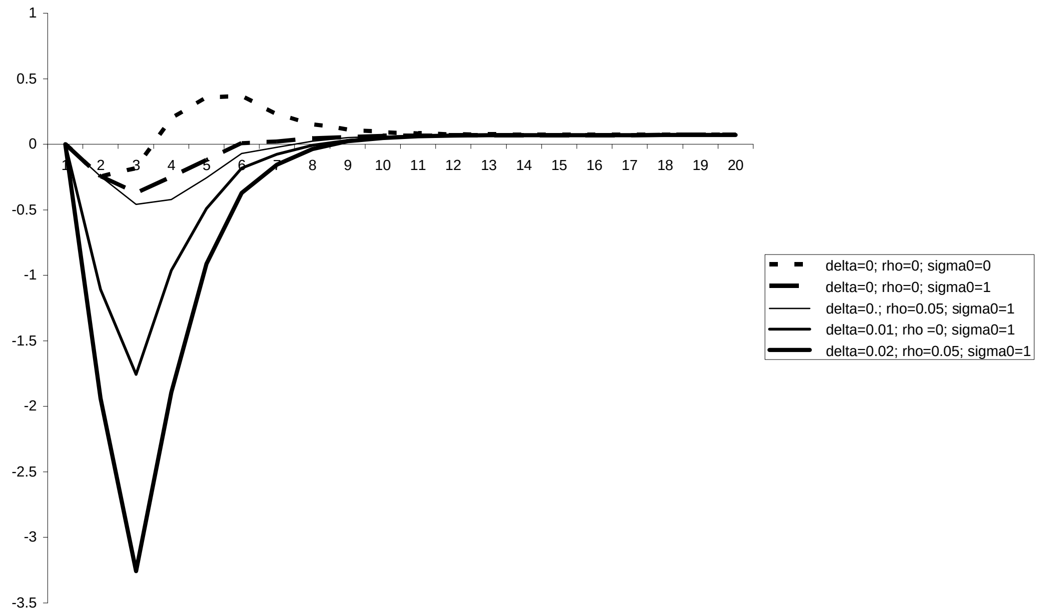

Figure 4.1: **Imperfect Credibility Concave Expectations Updating and Time-Varying Velocity Result:** *Output response to gradual disinflation from a 'small' (3%) initial annual inflation rate.*

imperfect credibility (the long dashed line) - both with constant velocity. The drop in output is of the same magnitude when there is time-varying velocity and perfect foresight (the thin solid line in Figure 3.2). This result is due to price stickiness and money demand sensitivity (through the parameter ρ).[11] Here, at this low level of inflation, the opportunity cost semi-elasticity of money demand (ρ) does not affect the price. However, in the second period, the output drop (of 0.5% below the initial steady state) is larger than that under perfect foresight and imperfect credibility both with constant velocity (0.25% below the initial steady state and 0.18%, respectively). Comparing Figure 4.1 with Figure 3.2 it can be seen that

[11] The pricing strategy adopted when inflation is low is the *single price strategy,* under which a firm chooses the price such as to maximize the profit for two consecutive time periods.

this drop in output is also bigger than that obtained when there is time-varying velocity and perfect foresight (1.4% below the initial steady state). Since firms are allowed to change prices in the second period, they do so by taking into account the decrease in the demand they face due to velocity variability, their expectations of future money supply and, importantly, forecasts of their covariances. From the second period onwards, the output recovers to its long run equilibrium level over 2 years, but it never rises above the new steady state level, so there is no record of output boom.

Figure 4.1 also shows that the early output loss is more marked when time-varying velocity is captured through both the scale elasticity of money demand and the opportunity cost parameters ($\delta > 0$, $\rho \geq 0$). For example, for $\delta = 0.01$, $\rho = 0$, the output falls more compared to all the other cases, starting from the first period after the change in policy is announced (as it is evident from comparison of Figure 3.2 and Figure 4.1). Even though prices are sticky, the scale elasticity parameter is now at work. In the second period the fall in output deepens further before raising to its long run equilibrium level, which will reach after a prolonged period. Again, there is no record of output boom.

These results shows that the simultaneous relaxation of the two assumptions of perfect foresight and constant velocity reveals that with concave expectations updating, there is a further deepening of the early output loss and no hint of an output boom for either of the (low) degrees of time varying-velocity considered here. In this case, we cannot reinforce the result of Evans and Nicolae (2010) that the appearance of the disinflationary output boom depends on the degree of time-varying velocity. Given the significance of this finding, we next ask whether it generalises to the case of convex expectations updating.

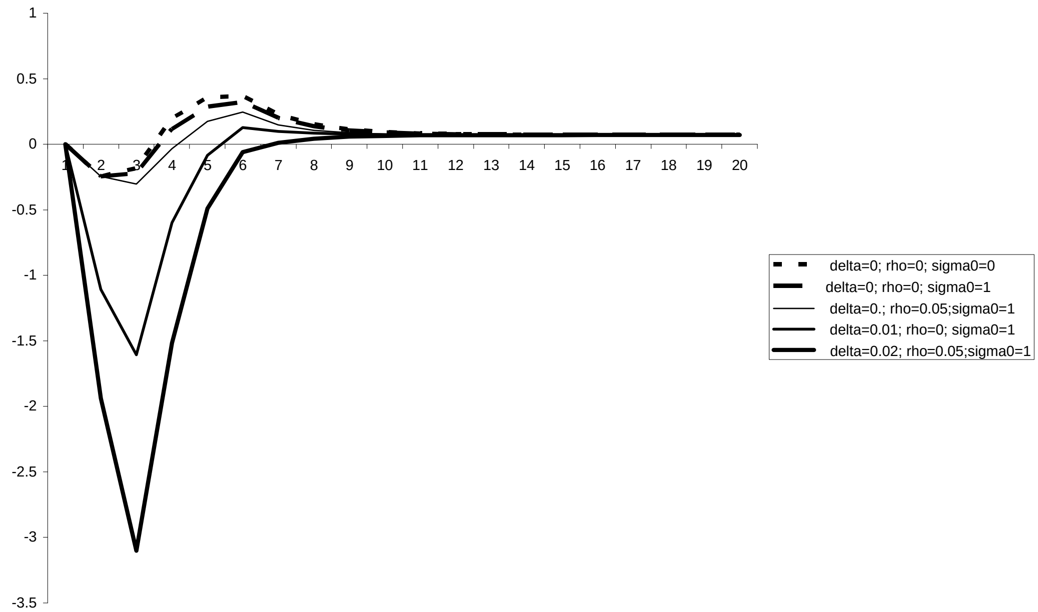

Figure 4.2: **Imperfect Credibility Convex Expectations Updating and Time-Varying Velocity Result:** *Output response gradual disinflation from a 'small' (3%) initial annual inflation rate.*

Convex Expectations Updating

Figure 4.2 presents output responses to a gradual disinflation from the same small (3%) initial annual inflation rate where we have both time-varying velocity and imperfect credibility with convex expectations updating. Again, we embed the benchmark results from Figure 3.1 within Figure 4.2. This relates to relaxing the perfect foresight assumption in Ireland's model. We can then cross reference to the benchmark results which relate to relaxing the constant velocity assumption in Ireland's model (Figure 3.2). Relative to the previous case, we find that when convex expectations updating replaces concave expectations updating, the early output losses are smaller. For example, for $\delta = 0.01$, $\rho = 0$, the output loss in the second period is 1.6% below the initial steady state, compared to the 1.76% under concave expectation updating. For $\delta = 0.02$, $\rho = 0.5$, this is 3.1% below the initial steady state compared to 3.25% under concave expectation updating (as now both parameters are at work).

Most notably, we can see that for $\delta = 0$, $\rho = 0.5$, the disinflationary output boom still appears, even though for low degrees of time-varying velocity. For $\delta = 0.01, \rho = 0$, the output barely rises above the new steady state after 3 years before settling at its new long run equilibrium level within the next 2 periods. For $\delta = 0.02, \rho = 0.5$, clearly there is no disinflationary output boom. The result that the appearance of the disinflationary output boom depends on the degree of time-varying velocity is consistent with that found in Evans and Nicolae (2010). We can now argue that even when velocity is time varying and there is imperfect credibility, the disinflationary output boom may still appear. To be more precise, the disinflationary output boom appears for low degrees of time varying velocity and when there is convex expectations updating - when agents accept that the monetary authority dislikes the current rate of inflation, but nevertheless worries that the monetary authority may be tempted to renege. When compared with the concave expectations updating result presented in Figure 4.1 it can be seen that the output cost imposed by imperfect credibility in this case is smaller.

Notwithstanding these differences, for $\delta > 0, \rho \geq 0$ the simultaneous relaxation of both the assumptions of constant velocity and perfect foresight results in much bigger early output losses than those suggested in the literature. This raise the question whether a gradual disinflation of small inflations is beneficial.

To explore this issue further, we construct a crude measure of the overall impact on output by projecting forward over a 30 year time horizon and calculating the net output gain.[12] Table 2 sets out the value of the area between the 'output path' and the x axis for a range of δ values for the two cases of perfect and imperfect credibility respectively.[13] The area below the axis gives the output loss, and that above the axis gives the output gain. The absolute size of the overall impact is noted in the final column and defined to be the net output gain. We can see that for sufficiently high values of δ the overall impact on output is negative. (If we were to calculate present values, overall net losses would arise at even lower levels of δ). Also, we can see that imperfect credibility serves to lower the value of δ at which the overall impact on output is negative. What is clear is that early output losses from disinflation may not be compensated in even a 30 year horizon.

$\boldsymbol{\delta}$	**Loss**		**Gain**		**Net Output Gain**	
	$\sigma_0 = 0$	$\sigma_0 = 1$	$\sigma_0 = 0$	$\sigma_0 = 1$	$\sigma_0 = 0$	$\sigma_0 = 1$
0	(0.42)	(0.98)	4.97	3.63	4.55	2.64
0.001	(0.65)	(1.33)	4.82	3.60	4.17	2.26
0.005	(1.72)	(2.76)	4.38	3.54	2.66	0.77
0.01	(3.22)	(4.58)	4.00	3.49	0.77	(1.08)
0.02	(6.60)	(8.23)	3.61	3.44	(2.99)	(4.79)
0.03	(10.22)	(11.89)	3.49	3.41	(6.73)	(8.48)
0.05	(17.56)	(19.17)	3.40	3.36	(14.15)	(15.81)

Table 2. *Overall impact on real output of a gradual disinflation from a 3% initial annual inflation rate under perfect foresight* ($\sigma_0 = 0$) *and imperfect credibility* ($\sigma_0 = 1$) *with concave expectations updating for different values of the velocity parameter* δ (with $\rho = 0$).

In the light of these results, Ireland's (1997) conclusion that small inflations

[12] This analysis is conducted for concave expectations updating as this type of expectation updating is considered to be a more plausible one.

[13] δ has been allowed to vary but $\rho = 0$ is maintained. For values of $\rho \in (0, 1)$ the calculated output loss is yet higher.

are best ended gradually may need to be qualified: it seems that even disinflating a low inflation gradually may be undesirable since the net 'overall impact' on the real economy may be negative. This shift in the potential policy conclusion is attributable to the introduction of time varying velocity and imperfect credibility.

The result of having endogenized time varying velocity is that a disinflation reduces both inflation and consumption in the short run and it is also associated with a decline in velocity (from (5)). However, this decline in velocity means equivalently, that for given levels of prices and consumption, nominal money demand would have to rise. This increase in money demand exacerbates the excess demand created by the disinflationary reduction in the money supply. Since prices are slow to adjust, the output loss is magnified, making time-varying velocity costly on output. As a result of having allowed for imperfect credibility, agents only gradually come to realize that the price-level is to grow at a zero rate—a realization that is all the more tardy because of the gradualness of the disinflationary process itself. This tardiness results in more of the necessary adjustment being borne by output losses than prices. Under imperfect credibility, the initial contraction in output is more severe for any initial inflation rate than under perfect credibility. When simultaneously relaxing the two assumptions, these two processes serve to reinforce each other.

The analysis in this section raises important questions for policymakers faced with low inflation. When Ireland considered the simple the choice between disinflating gradually and immediately, he advocated a gradual disinflation since, in his model, the immediate disinflation from 3% generated unambiguously bigger output losses (reaching a trough of 1.7% below the initial steady state as compared with 0.27% - all under the assumptions of perfect credibility and constant velocity). However, our model shows that of itself, this scale of bigger losses can ensue from the gradual disinflation considered throughout these analyses.

Our focus therefore shifts to the policy question of alternative rates of gradual disinflations. Put more succinctly, we turn to the question of the optimal speed of disinflation. In doing this we broaden the range of initial low inflation rates beyond the 3% and use our extended non-linear solution method to calculate the level of utility associated with different speeds of disinflation for a range of initial inflation rates from 1% to 20%, which characterises low inflation in this particular literature.

5. Optimal Speed of Disinflation

The optimal speed of disinflation is important information for policymakers who must decide on the time horizon over which they bring about price stability. We therefore attempt to address this issue by calculating the optimal speed of disinflation in the model in which we have both time-varying velocity and imperfect credibility. The optimal speed of disinflation is given by the length of time for which utility is maximized.[14]

In Figure 5.1, the dashed line shows the optimal speed of disinflation calculated for the case in which constant velocity and perfect credibility are assumed, as in Ireland (1997). In this case, given the price stickiness, a gradual disinflation results in output rising above the new steady-state for some time after the early output loss and utility derives from the output gains following an initial contraction in activity. However, the higher is the initial inflation rate, the more pronounced is the contraction in output at the beginning of the disinflation period and the utility gain from any subsequent boom offsets this increasing the optimal speed

[14] To calculate the optimal speed of disinflation, we maximize utility which derives from labour as well as consumption. (In this framework, consumption follows the same path as output.) The calculation was conducted for lengths of the disinflation period ranging from immediate $T = 1$ to more gradual $T = 45$, where T is the length of the period of disinflation measured in half yearly intervals.

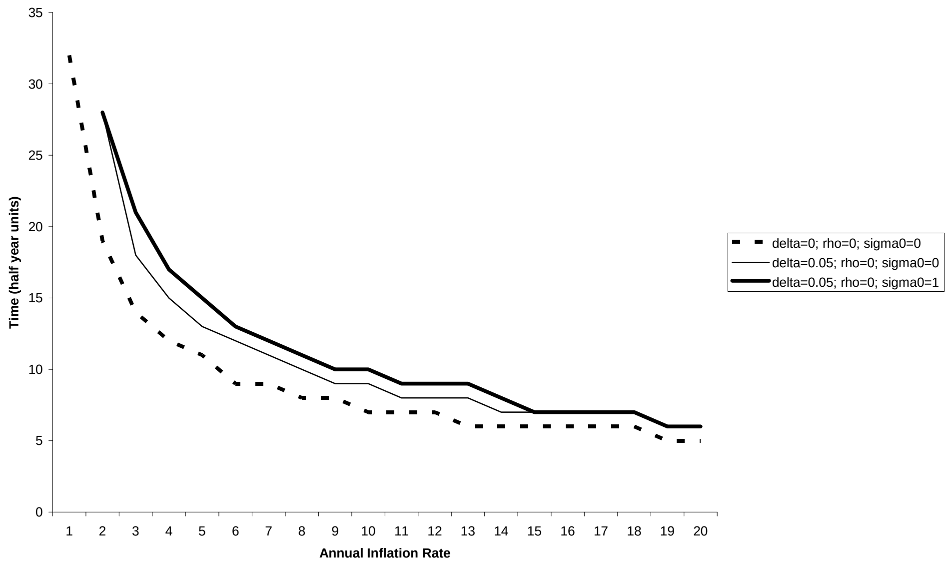

Figure 5.1: **Optimal Speed of Disinflation**

of disinflation.

The thin solid line plots the optimal speed of disinflation for the case in which we have time varying velocity and perfect credibility, as calculated by Evans and Nicolae (2010). They show that the effect of allowing for time-varying velocity, through the scale elasticity of money demand parameter (δ) and the opportunity cost parameter (ρ), affects the optimal speed of disinflation by decreasing it.[15] The time-varying velocity effect on output makes the early output loss more severe and, depending on the degree of time-varying velocity, may even eliminate any disinflationary output boom. Furthermore, utility is much lower than in the case where velocity is constant and there is perfect credibility. Since a more gradual disinflation boosts utility, the time needed for the disinflation needs to be longer,

[15] They also show that the optimal speed of disinflation is affected less through the scale elasticity of money demand parameter than through the opportunity cost parameter.

and thus, the optimal speed of disinflation slower, compared to the case of constant velocity.

Both of these results are obtained under the assumption of perfect credibility. The thick solid line, however, shows the optimal speed of disinflation calculated in the model developed in this paper. This shows that the optimal speed has to be even further decreased. For the case in which we have time-varying velocity and imperfect credibility, the early output loss is more severe for any initial inflation rate than is the case under perfect foresight only and the utility gain from the disinflationary boom may not even arise. Certainly, for intial inflation rates less than 15%, the optimal speed of disinflation is decreased. For example, for an initial inflation rate of around 10%, approximately an additional half year is required for the disinflationary time period when allowing for imperfect credibility ($\delta = 0.05$, $\rho = 0$[16], $\sigma_0 = 1$) as compared to when there is perfect credibility ($\delta = 0.05$, $\rho = 0$, $\sigma_0 = 0$). However, for initial inflation rates greater than 15% the optimal speed of disinflation is the same as under the case of time varying velocity and perfect foresight.[17]

The central message is that with time-varying velocity and imperfect credibility, the optimal speed of inflation is decreased. This is because utility maximizing policymakers cannot necessarily rely on a disinflationary boom to compensate the (greater) early losses in output. To avoid the early extra costs imposed by both time-varying velocity and imperfect credibility, a yet more

[16] The calculation of the optiamal speed of disinflation has been carried out here for these values for δ and ρ ($\delta = 0.05$ and $\rho = 0$) for comparison purposes with the existing litarature. Evans and Nicolae (2010) use the same values for their calcualtion fo the optimal speed of disinflation under perfect foreisght. They also carry out the calcualtion of the optimal speed of disinlfation for $\delta = 0$, $\rho = 0.02$, however, due to computational complexity this has not been carried out in this paper.

[17] Nicolae and Nolan (2006), found that allowing for imperfect crediblity has an effect of a similar magintude on their calcualtion of the optimal speed of disinflation and that a more gradual period of disinflation is optimal up until an initial inflation rate of around 12%. Their model however, does not allow for time-varying velocity.

gradual disinflation is needed, as this makes the contractions in activity in the early period of the disinflation less sharp. Once again, it appears that once inflation has risen, time-varying velocity and imperfect crediblity makes sizeable output losses in the transition to price stability highly likely, even when the speed of disinflation is 'optimal'.

Thatcher Disinflation Experiment

To understand further the role imperfect credibility may play in this model and strengthen the message of this paper we look at the "Thatcher disinflation" experience of the UK during the early 1980s.[18] Clearly, one needs to be cautious when attempting to relate real world data to a highly simplified modelling framework. As in Evans and Nicolae (2010), we assume that the initial steady-state inflation was 15%, and that Thatcher intended to reduce inflation to 1%[19] and we use a narrow definition of money (notes and coin) from 1979 until 1986 as the path for the money stock during the disinflation.

The long dashed line in Figure 5.2 is the UK output gap as measured by the OECD. The short dashed line is the output gap from the benchmark model in which velocity is assumed constant and there is perfect credibility (Ireland's model). The thin solid line shows the output gap of the model when velocity is time varying and there is perfect credibility. This is the result in Evans and Nicolae (2010) who found that for $\delta = 0.024$ and $\rho = 0.05$. the model output gap was closest to the actual OECD output gap (assuming, of course, perfect credibility). Having allowed for time varying velocity, the model performance was much improved yet it could not quite generate the full contraction indicated in

[18]This 'inflation first' policy strategy was an attempt to put 'monetarism' into practice and focused on monetary targeting. In the period to 1986, inflation fell from above 15% to comfortably below 5% and the early 1980s saw a deep economic recession.

[19]The Thatcher disinflation pre-dates explicit inflation targeting. At that time, the focus was on reducing inflation to a low level (1% reflects average UK inflation of the late 1950s).

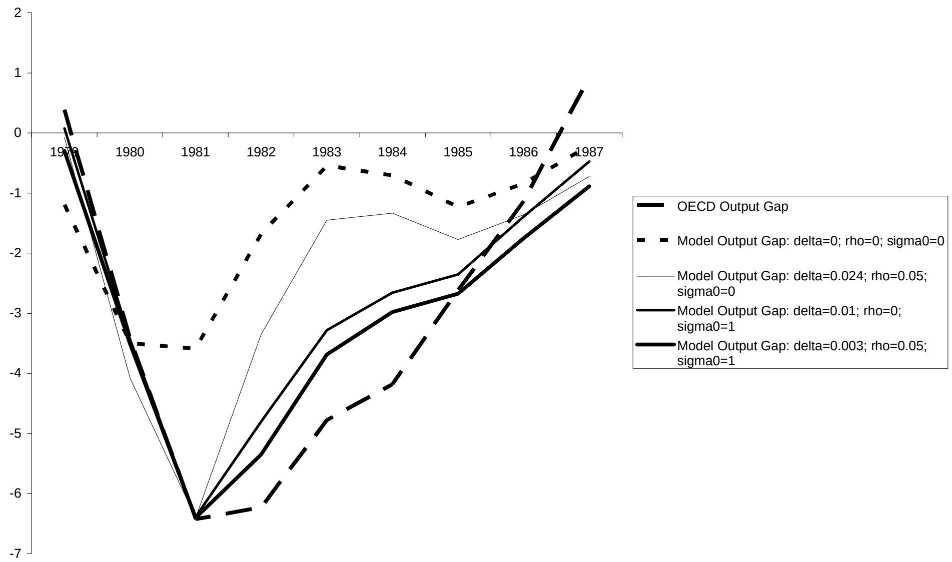

Figure 5.2: The Output Gap of Thatcher Disinflation

the OECD data. The thick solid line shows the output gap of the model when velocity is time varying and when there is imperfect credibility.

We tracked the OECD output gap for various degrees of time varying velocity (setting $\rho = 0$ and $\rho = 0.05$). We have found that under imperfect credibility ($\sigma_0 = 1$), for this specific Thatcher disinflation experiment, the OECD output gap was best tracked by our model for a smaller degree of time varying velocity and only through the scale elasticity of money demand parameter ($\delta = 0.01$ and $\rho = 0$). When both parameters of time varying velocity are nonzero, the OECD output gap was best tracked by our model for a smaller degree of time varying velocity degree of time varying velocity characteried by a the scale elasticity of money demand parameter of 0.003 and the opportunity cost parameter set at 0.05.for ($\delta = 0.003$ and $\rho = 0.05$)[20].

[20]For this specific Thatcher disinflation experiment, under imperfect crediblity, the OECD

The model's performance is evidently further improved: the trough of the drop in output is fully captured and some depth of the ongoing recession is better tracked. Although not fully, the output loss is better explained by the model which allows for time varying velocity and imperfect credibility. From this study it is evident that imperfect credibility, seems to play a role in explaining the output loss during the Thatcher disinflation.

6. Conclusion

In this paper we make a contribution to the literature on stopping inflation through monetary contraction. We have developed a New Keynesian DSGE model and used it to explore the output response to gradual disinflation of a small inflation. Well known and influential work on monetary contractions consistently found that a gradual disinflation may result in a 'disinflationary output boom' [Ball (1994), Ireland (1997), King and Wolman (1999), Khan, King and Wolman (1999)]. Ball (1994) attributed the disinflationary boom to the assumption of perfect credibility prompting Nicolae and Nolan (2006) to relax the perfect credibility assumption. They found that the (dis)appearance of the disinflationary boom depends on the speed of learning relative to the speed of disinflation. More recently, Evans and Nicolae (2010) relaxed another strong assumption made in this literature – that of constant velocity. They found that, even with perfect foresight, the appearance of the disinflationary boom depends on the degree of time-varying velocity.

In the model developed in this paper, we have simultaneously relaxed both assumptions of perfect foresight and constant velocity. We find that when velocity is time varying and imperfect credibility is allowed for, Nicolae and Nolan's

output gap was best tracked by our model for a degree of time varying velocity characteried by a the scale elasticity of money demand parameter of 0.003 and the oportunity cost parameter set at 0.05.

result - that the (dis)appearance of the disinflationary boom depends on the speed of learning relative to the speed of disinflation - still holds, but only for low degrees of time-varying velocity. We find that under concave expectations updating, regardless of the degree of time-varying velocity, the disinflationary boom disappears. In the light of this result, we cannot reinforce the Evans and Nicolae (2010) result that output boom (dis)appearance depends on velocity. Their finding however, continues to hold under convex expectations updating where the existence of the disinflationary boom depends on time-varying velocity.

However, the appearance/disappearance of the boom is not the only issue here - the overall output effect of simultaneously relaxing the two assumptions (of perfect foresight and constant velocity) is important. The nonlinear solution method employed reveals that, with both assumptions relaxed, the early output losses which follow a disinflationary monetary policy announcement may be more pronounced and more prolonged than previously suggested in the literature – and this is a result that holds for both concave and convex expectations updating. The message is consistent – when agents take time to believe a disinflationary policy action and velocity is allowed to vary, the cost of taking a small inflation out of the economy over a three year time horizon, is a sizeable early output loss and a wait of some many years for compensating output. Indeed, we find that these early output losses may not (ever) be compensated by later output gains, suggesting that it is important to consider alternative time horizons for the disinflation. We therefore calculated the optimal speed of disinflation.

These calculations show that in the model developed here, an even more gradual period of disinflation is recommended relative to that found by Evans and Nicolae (2010) who had already shown that a more gradual period of disinflation is appropriate by comparison with that found in Ireland (1997). It appears that some of the familiar findings and policy implications of the influential work on gradual disinflation from small inflations are not robust to some modifications of the modelling framework. Given the practical importance of the underlying policy issue, further research on model specification would seem warranted.

References

[1] Ball, Laurence. (1994) "Credible Disinflation with Staggered Price-Settings." *American Economic Review*, 84:1, 282-89.

[2] Ball, Laurence (1995), "Disinflation with Imperfect Credibility", *Journal of Monetary Economics*, Vol. 35, pp.5-23.

[3] Ball, Laurence. (2001) "Another Look at Long-Run Money Demand." *Journal of Monetary Economics*, 47:1, 31-44.

[4] Ball, Laurence, N. Gregory Mankiw, and David Romer. (1988) "The New Keynesian Economics and the Output-Inflation Trade-Off." *Brookings Papers on Economic Activity*, 1:1, 1-65.

[5] Ball, Laurence, and N. Gregory Mankiw. (1994) "Asymmetric Price Adjustment and Economic Fluctuations." *Economic Journal*, 104:423, 247-61.

[6] Barro, Robert J. and David B. Gordon. (1983) "Rules, Discretion and Reputation in a Model of Monetary Policy." *Journal of Monetary Economics*, 12:11, 101-21.

[7] Erceg, Christopher J. and Andrew T. Levin (2003), 'Imperfect Credibility and Inflation Persistence', *Journal of Monetary Economics*, 50 pp. 915-944.

[8] Evans, Lynne and Anamaria Nicolae. (2010) "The Output Effect of a Transition to Price Stability When Velocity is Time-varying." *Journal of Money, Credit and Banking*, 42:5, pp859-878.

[9] Ireland, Peter N., (1995), "Optimal Disinflationary Paths", *Journal of Economic Dynamics and Control*, vol. 19, pp.1429-1448.

[10] Ireland, Peter N. (1997) "Stopping Inflations, Big and Small." *Journal of Money, Credit and Banking*, 29:4, 759-75.

[11] Khan, Aubik, Robert G. King, and Alexander L. Wolman. (2003) "Optimal Monetary Policy." *Review of Economic Studies*, 70:4, 825-60.

[12] King, Robert G, and Alexander L. Wolman. (1999) "What Should the Monetary Authority Do When Prices Are Sticky?" In *Monetary Policy Rules*, edited by John Taylor, pp. 349-404. Chicago: University of Chicago Press for NBER.

[13] Nicolae, Anamaria, and Charles Nolan. (2006) "The Impact of Imperfect Credibility in a Transition to Price Stability." *Journal of Money, Credit and Banking*, 38:1, 47-66.

[14] Rotemberg, Julio J, and Michael Woodford. (1992) "Oligopolistic Pricing and the Effects of Aggregate Demand on Economic Activity." *Journal of Political Economy*, 100:6, 1153-207.

[15] Sriram, Subramanian S. (2001) "A Survey of Recent Empirical Money Demand Studies." *IMF Staff Papers*, 47:3, 334-65.