CPI Inflation Targeting and the UIP Puzzle: An Appraisal of Instrument and Target Rules

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Abstract:

Employing an optimizing framework, this paper shows that a target rule dominates a simple instrument rule when the focus of monetary policy is on CPI inflation. The target rule approach produces a systematic relationship between the current CPI inflation rate and the lagged policy instrument that renders the former immune to the stochastic risk premium. No matter how policy parameters are set, the optimal simple instrument rule cannot replicate the superior stabilization results achieved by the target rule approach. The optimal simple instrument rule also fails to account for the UIP puzzle. In contrast, the target rule approach can motivate the widely reported phenomenon whereby high interest rate currencies tend to appreciate. In fact the degree of openness and the central bank's relative aversion to CPI inflation variability determine the sensitivity of observed changes in the nominal exchange rate to the lagged interest rate differential.

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There are two primary ways in which rule-based monetary policy can be specified in macroeconomic models. A central bank can follow a pre-specified rule which ties the setting of the policy instrument, typically a short-term interest rate, to the underlying target(s) of monetary policy. This mechanical rule compels the central bank to respond to the observed deviation of one or multiple target variables from its respective target value. The speed with which the central bank counters observed deviations from target depends on the central bank's willingness to tolerate target misses. To the extent that the central bank adjusts the policy instrument gradually over time it must also decide on the size of the smoothing parameter attached to the lagged value of the policy instrument in this mechanical rule. Any policy parameters that appear in such a stand-alone rule can be set with some discretion. In the literature a rule of this type is referred to as a simple *instrument rule*. A clear and obvious advantage of a simple instrument rule is that its specification does not depend on any particular model or knowledge of the central bank's objective function. A further advantage in the eyes of some is that the implementation of a simple instrument rule is not predicated on optimizing behavior by the central bank.

The alternative approach to implementing monetary policy could not be more different. The target rule approach is firmly grounded in an optimizing framework. The successful implementation of a *target rule* rests on full knowledge of the central bank's objective function, i.e. the target variables proper, the associated target levels, and the weight attached to each target variable in the objective function. In addition, the target rule approach requires the specification of a model of the economy as one or more of its components represents the constraint the central bank faces in the execution of optimal monetary policy. The central bank minimizes the objective function subject to the constraint with respect to the target variables. The target rule is obtained by combining the optimizing conditions of the target variables. As such, the target rule is a clear, succinct, and arguably rigorous specification of optimal policy as it prescribes how the target variables are related to each other. An *implicit reaction* function can be backed out by substituting the components of the model into the target rule and solving for the policy instrument. By following this reaction function mechanically, the central bank implements policy optimally. The coefficients on the variables and shocks that appear in the implicit reaction function depend on the weights attached to the target variables in the central bank's loss function and the structural parameters of the model economy.

This paper adds to the ongoing discussion about the merits of instrument versus target rules in the implementation of monetary policy. Svensson (1999, 2002, 2003, 2005), Svensson and Woodford (2003, 2005) voice strong support for the target rule approach while McCallum (1999a,b) and McCallum and Nelson (2004, 2005) argue in favor of instrument rules. Comparing *optimal* instrument rules to target rules in both the aggregate demand-aggregate supply framework and the New Keynesian model, Froyen and Guender (2010) find that as long as the same information set underlies policymaking the two policy strategies produce the same optimal stabilization response.

The target rule approach does, however, have one distinct advantage over the *optimal* instrument rule which responds directly to all shocks of the model.[1] If policy is conducted from a timeless perspective, the target rule approach can motivate the history-dependence and inertial character of monetary policy in the standard New Keynesian model. While an optimal instrument rule can replicate the optimal stabilization response of policy from a timeless perspective, it cannot explain the inclusion of the lagged output gap in the instrument rule. A common thread running through all of these discussions is that they take place in a closed economy framework.

This paper shifts the focus of the debate from a closed to a small open economy framework to address two questions pertinent to the conduct of monetary policy. The first question tackles the issue of the specification of *optimal* monetary policy rules in a small open economy where the central bank is concerned about the stability of the CPI inflation rate. Are there any obvious advantages or disadvantages associated with conducting optimal monetary policy by way of a simple instrument rule as opposed to a target rule? The second question addresses the uncovered interest rate parity (UIP) puzzle.[2] Specifically, the paper asks whether the choice between a simple instrument and a target rule in a small open economy materially affects the linkage between the interest rate differential and changes in the nominal exchange rate. The focus of this part of the analysis rests squarely on whether there are obvious and acute differences between *optimal* simple instrument and target rules that have important implications for the behavior of nominal interest and exchange rates in open economies.

There has been a long-standing interest in the second issue, the role of monetary policy as a key driver of the linkage between changes in the nominal exchange rate and the interest rate differential. McCallum (1994) recognizes that interest rate smoothing by the central bank can successfully explain the widely reported empirical failure of the UIP hypothesis. In a simple rational expectations framework, the central bank, intent on countering pressure on the domestic currency to depreciate by raising the setting of the policy instrument, causes the emergence of an inverse relationship between observed changes in the nominal exchange rate and the lagged interest rate differential.[3]

[1] Because of the requirement that the policymaker be in a position to observe and respond to all shocks of the model, optimal instrument rules are dismissed as being impracticable. Strictly speaking, the target rule approach suffers from the same problem as it relies on an implied reaction function that responds to all shocks of the model.

[2] Absent any risk premium, the UIP hypothesis suggests that a positive interest rate differential (domestic interest rate – foreign interest rate) should be compensated in full by a depreciation of the domestic currency over the investment horizon. In practice, the presumed one-for-one relationship between the interest rate differential and the expected change in the nominal exchange rate is not apparent in the data. In fact, many studies report a negative association between movements in the interest rate differential and observed changes in the nominal exchange rate. The observed violation of the UIP condition has been labeled the "UIP Puzzle". The "Carry Trade" phenomenon whereby high interest rate currencies are seen appreciating is symptomatic of the failure of the UIP condition.

[3] To be precise, McCallum emphasizes the important distinction between UIP and tests of the unbiasedness hypothesis. What the literature generally regards as a failure of UIP is actually a failure of the unbiasedness hypothesis according to which the forward premium is not an accurate predictor of the change in the spot rate.

Employing an optimizing two-country framework standard in modern finance, Backus et al (2009) assume log-normal distribution of domestic inflation and introduce stochastic volatility into a simple instrument rule to show that uncovered interest rate parity need not hold. In their analysis, the inverse link between the interest rate differential and changes in the nominal exchange rate comes about if the domestic central bank ignores exchange rate movements in setting the policy instrument but the foreign central bank raises its policy instrument in response to its currency appreciating! In other words, seeing its currency gaining strength, the foreign country tightens monetary policy. This seems paradoxical.

In both McCallum (1994) and Backus et al (2009) the central bank is viewed as following a simple instrument rule. The policy parameters are taken as given and assumed to satisfy basic criteria. Neither contribution concerns itself with the determination of optimal policy whereby the central bank minimizes an objective function to derive the values of the policy parameters. The current paper fills this gap. It embeds a simple instrument rule into a basic open-economy macro model and shows how an optimizing central bank determines the optimal policy parameters. This lays the groundwork for conducting a comparison of the performance of optimal policy based on a simple instrument rule with optimal policy based on a target rule. In a nutshell, this paper seeks to provide a fresh perspective on some of the issues involved in choosing between an instrument rule and a target rule in an expanded optimizing framework where the central bank is concerned about the variability of the policy instrument and the variability of CPI inflation. The central bank operates in an environment where the uncovered interest rate parity relationship is assumed to hold and inflation can be controlled by varying the policy instrument.

The paper shows that the target rule has three distinct advantages over the optimal simple instrument rule. First, the target rule is a well-specified form of optimal policy while the simple optimal instrument rule suffers from a mathematical complexity that renders it inoperable. Second, in contrast to the ad hoc simple instrument rule, the target rule approach provides a clear rationale for why the lag of policy instrument enters the model. Third, the target rule approach can explain the UIP puzzle while the optimal simple instrument rule cannot.

The paper is structured as follows. Section II introduces the model. Section III compares and contrasts optimal policy based on an instrument versus a target rule. This section also looks at the implications for the test of the UIP hypothesis of basing the conduct of optimal policy on an instrument as opposed to a target rule. Section IV concludes.

II. The Model

This section lays out the model that serves as the frame of reference for examining the merits of simple instrument and target rules in a small open economy. The model consists of five equations. Variables marked by an asterisk denote the foreign counterpart of the domestic variable. Foreign variables are treated as exogenous random variables. The first equation is the definition of the policy instrument (x_t).The policy instrument is defined as the difference between the domestic interest rate (i_t) and the foreign interest rate (i_t^*) . Adopting this convention simplifies the analysis and allows us to compare our results directly with those reported by McCallum (1994). The second equation is the UIP condition in nominal terms. It allows for the existence of a risk premium (ρ_t). The two remaining equations describe the behavior of inflation. Equation (3) is a condensed equation of the rate of domestic inflation. It is obtained by combining three different elements: an open economy IS equation, UIP in real terms, and a Phillips Curve.[4] The rate of domestic inflation is inversely related to the policy instrument but reacts positively to the difference between expected domestic inflation and expected inflation abroad in period *t+1.* The composite stochastic disturbance captures the effect of demand-side and cost-push shocks on domestic inflation. Equation (4) is the definition of CPI inflation.

$$x_t = i_t - i_t^* \qquad (1)$$

$$x_t = Es_{t+1} - s_t + \rho_t \qquad (2)$$

$$\pi_t = -\alpha x_t + \alpha(E_t\pi_{t+1} - E_t\pi_{t+1}^*) + u_t \qquad (3)$$

$$u_t = \kappa(v_t - a_1(i_t^* - E_t\pi_{t+1}^*) + (a_2 - a_1\gamma)\rho_t) + w_t$$

$$\alpha = \kappa((1-\gamma)a_1 + a_2)$$

$$\pi_t^{CPI} = (1-\gamma)\pi_t + \gamma(\Delta s_t + \pi_t^*) \qquad (4)$$

$$0 \leq \gamma \leq 1$$

III. Instrument vs. Target Rules

1. A Simple Instrument Rule to Target the CPI Inflation Rate

McCallum (1994) investigates the nexus between the instrument rule that the central bank follows to implement monetary policy and the behavior of the nominal exchange rate. He proposes the following instrument rule:

[4]Equation (3) is based on a simple Phillips Curve, i.e. one without forward-looking inflationary expectations. Likewise, the expected output gap next period has been dropped from the IS relation. This simplification is not crucial to the results reported. Forward-looking expectations of the rate of domestic inflation do appear in equation (3), however, because in the IS relation the output gap depends inversely on the expected real rate of interest. The additive disturbance is a composite term consisting of random disturbances and exogenous variables that appear in the IS relation, the UIP relation, and the Phillips curve. For further details on the specification of the Phillips curve, IS relation and the UIP condition, the reader is referred to part D of the appendix.

$$x_t = \lambda_1 \Delta s_t + \lambda_2 x_{t-1} \qquad (5)$$

His simple instrument rule has two key features. The central bank adjusts the setting of the instrument if the change in the nominal exchange rate deviates from its fixed target value (which for simplicity is assumed to be zero). The central bank also adjusts the policy setting gradually over time. Thus, the current setting of the policy instrument depends on the setting in the previous period. Both λ_1 and λ_2 are policy parameters that the central bank controls. McCallum combines the instrument rule (eq.(5)) with the UIP condition (eq. (2))to show that interest rate smoothing by a central bank, i.e. $\lambda_2 \neq 0$, leads to a negative coefficient on the lagged interest rate differential in the reduced form equation for the change in the nominal exchange rate. To be precise, the coefficient on the lagged interest rate differential in said equation equals (minus) the ratio of the policy parameters, i.e. $-\frac{\lambda_2}{\lambda_1}$. One would expect both policy parameters to be positive as central banks typically smooth interest rates and "lean against the wind", i.e. attempt to stem a depreciating domestic currency by raising the short-term interest rate.

McCallum's theoretical example provides a plausible explanation for why empirical tests of the UIP hypothesis typically reject its validity. The standard test of the joint hypothesis of UIP and rational expectations consists of a regression of the change in the nominal exchange rate on the lagged interest rate differential. Employing this test regression, scores of empirical papers report either statistically insignificant or statistically significant *negative* coefficients on the interest rate differential for a large number of countries over different sample periods.[5]

But is the exchange rate really the focus of monetary policy in the small open economies of the developed world? McCallum's specification of the policy target in the instrument rule is arguably at odds with conventional practice. In most industrialized countries, central banks have a target for the CPI inflation rate. They are less worried about changes in the nominal exchange rate. Indeed, the nominal exchange rate needs to be flexible as it acts as a shock absorber under CPI inflation targeting. If the central bank engages in interest smoothing and is intent on keeping the CPI inflation rate in check, then the instrument rule takes the following form:

$$x_t = \lambda_1 (\pi_t^{CPI} - \pi^{CPI^T}) + \lambda_2 x_{t-1} \qquad (6)$$

Here π^{CPI^T} represents the fixed target for the CPI inflation rate. In the remainder of the paper we assume this fixed target value to be zero. Positive values for the two policy parameters seem plausible.

[5] See, for instance, Fama (1984), Froot and Thaler (1990), Lewis (1995), Engel (1996), Chinn and Meredith (2004), Burnside et al (2008).

This instrument rule can be combined with equations (2) –(4) of the model to produce the reduced form solutions for the policy instrument and the endogenous variables of the model:[6]

$$x_t = \frac{\lambda_1 \gamma}{\lambda_1 \gamma + \lambda_2} \rho_t \tag{7}$$

$$\pi_t = \left(-\frac{\alpha \lambda_1 \gamma}{\lambda_1 \gamma + \lambda_2} + \kappa(a_2 - a_1 \gamma) \right) \rho_t + \kappa(v_t - a_1 i_t^*) + w_t \tag{8}$$

$$\pi_t^{CPI} = \frac{\gamma}{\lambda_1 \gamma + \lambda_2} \rho_t - \frac{\lambda_2}{\lambda_1} x_{t-1} \tag{9}$$

$$\Delta s_t = \left(\frac{1 + \lambda_1 (1-\gamma)\alpha}{\lambda_1 \gamma + \lambda_2} - \frac{(1-\gamma)}{\gamma} \kappa(a_2 - a_1 \gamma)\right) \rho_t - \frac{\lambda_2}{\lambda_1 \gamma} x_{t-1} - \frac{(1-\gamma)}{\gamma} (\kappa(v_t - a_1 i_t^*) + w_t) - \pi_t^* \tag{10}$$

Several observations about the above solutions are noteworthy. First, according to equation (7), the policy instrument responds only to the risk premium and does not depend on its own lag even though the central bank engages in interest smoothing. At first sight this may strike the reader as a surprising result. However, it can be explained by scrutinizing the behavior of the CPI inflation rate under the instrument rule. Substitute equation (9) into the instrument rule (eq. (6)) and notice that the lags of the policy instrument cancel so that the current instrument setting responds only to the risk premium.

Second, notice that the rate of CPI inflation responds only to the risk premium apart from the lagged interest rate differential. With the rate of CPI inflation being immune to the other shocks of the model, it is left to the two components of the CPI inflation rate – the rate of domestic inflation and particularly the nominal exchange rate to act as shock absorbers. Inspection of equations (8) and (10) reveals that the rate of domestic inflation and the exchange rate react to the cost-push shock (w_t), the IS shock (v_t), and the foreign interest rate (i_t^*) so that the CPI inflation rate is not affected. Again, this can be easily verified by substituting equations (8) and (10) into the definition of the CPI

[6] For simplicity, we assume that all shocks are white noise with a constant variance. The first step of the solution procedure involves setting up putative solutions for those variables whose forward-looking expectations appear in the model. They are: the change in the nominal exchange rate and the domestic rate of inflation. The lagged policy instrument appears in both putative solutions. Following the procedure suggested by McCallum (1994), we solve equation (2) for the policy instrument after substituting for the expected change in the exchange rate. This equation is then substituted into the instrument rule (eq. (6)) where the CPI inflation rate has been replaced with equation (4). The resulting expression is finally combined with equation (3) to yield the solution for Δs_t. Given the solution for Δs_t, we can proceed to solve for the remaining endogenous variables and the policy instrument. The appendix provides further details on the solution procedure.

inflation rate, equation (4). It is worth noting that the (change in the) exchange rate responds to all shocks of the model and is therefore expected to be rather volatile under CPI inflation targeting.

Third, consider the coefficient on the lagged interest rate differential in equation (10), the reduced form equation for the change in the nominal exchange rate. If monetary policy focuses on a CPI inflation target, then this coefficient depends not only on the two policy parameters in the instrument rule, λ_1 *and* λ_2 , as in McCallum's set-up, but also on the weight (γ) of the foreign consumption good in the CPI. This weight is often interpreted as measuring the degree of openness of the economy. The degree of openness matters now as the focus of monetary policy in the instrument rule is not on the change in the nominal exchange rate but on the CPI inflation rate. And the degree of openness determines the extent to which the CPI inflation rate changes in response to a change in the nominal exchange rate.

Fourth, consider the solutions for the policy instrument and the CPI inflation rate, the two variables that appear in the instrument rule. The coefficient on the risk premium in both solutions depends only on the two policy parameters λ_1 and λ_2 and the degree of openness γ but not on α. The parameter $\alpha = \kappa((1-\gamma)a_1 + a_2)$ is a summary measure of the potency of the interest and exchange rate channels on aggregate demand and its flow-on effect on domestic inflation. Thus, the behavior of the CPI inflation rate and the policy instrument is completely insensitive to key features of the monetary policy transmission mechanism through which changes in policy affect the ultimate targets of monetary policy.

2. Monetary Policy Based on a Optimal Simple Instrument Rule

In the previous section, we specified a simple instrument rule without giving a detailed account of how the central bank determines the values of the policy parameters λ_1 and λ_2. In this section we examine how an optimizing central bank chooses the values of both policy parameters. An analysis of optimizing behavior on the part of the central bank necessitates the specification of the central bank's objective function. It is customary to assume that central banks wish to minimize fluctuations in CPI inflation and the output gap. In addition, they wish to avoid huge swings in the setting of the policy instrument for fear of unsettling financial markets. We simplify matters by specifying an objective function for a central bank which is concerned only about the variability of the CPI inflation rate and the policy instrument, respectively. The policy problem for the central bank then becomes:[7]

$$Min_{\lambda_1,\lambda_2} E(L_t) = V(x_t) + \mu V(\pi_t^{CPI}) \tag{11}$$

[7] The target for the policy instrument and the rate of CPI inflation, respectively, is usually constant.

The central bank chooses the two policy parameters λ_1 and λ_2 to minimize the variances of the policy instrument and the CPI inflation rate. The parameter μ measures the central bank's aversion to CPI inflation variability relative to instrument variability.

The optimal values for the two policy parameters are:

$$\lambda_1^* = -\frac{\sqrt{\mu}}{\sqrt{-1-\mu\gamma^2}} \quad \lambda_2^* = \frac{\gamma\sqrt{\mu}}{\sqrt{-1-\mu\gamma^2}}$$

$$\text{and} \tag{12}$$

$$\lambda_1^* = \frac{\sqrt{\mu}}{\sqrt{-1-\mu\gamma^2}} \quad \lambda_2^* = -\frac{\gamma\sqrt{\mu}}{\sqrt{-1-\mu\gamma^2}}$$

These solutions are problematic. The existence of two pairs of solutions for the optimal policy parameters is far less disconcerting than the fact that all four roots are complex numbers. With $\mu \geq 0$ and $0 \leq \gamma \leq 1$, the term in the square root in the denominator of all four solutions is negative. From a practical point of view, optimal instrument rules such as equation (6) are clearly inoperative. From a theoretical perspective, the optimal instrument rule is inconsistent with a well-defined rational expectations equilibrium as the characteristic equation of the relevant coefficient matrix does not produce two roots outside the unit circle.[8]

Irrespective of the solution pair chosen, the linear combination of $\lambda_1^*\gamma + \lambda_2^*$ adds up to zero. This linear combination appears in the denominator of the coefficient on the risk premium in the solutions of both inflation rates, the change in the nominal exchange rate, and the policy instrument. The variances of the policy instrument and the CPI inflation rate – both of which appear in the objective function - would literally explode! As would the variances of the other two variables, the domestic rate of inflation and the exchange rate. A third characteristic of the solutions is that the ratio of the two optimal policy parameters equals $\frac{\lambda_2^*}{\lambda_1^*} = -\gamma$. Substituting this result into the coefficient on the lagged interest rate differential in equation (10) reduces the coefficient to 1. But this suggests that the optimal instrument rule, which assumes interest rate smoothing, cannot explain the systematic failure of standard tests of UIP which rely on regressing the first difference of the nominal exchange rate on the lagged interest rate.

Taken altogether, the following conclusion emerges. If the model economy and the policymaker's objective function, described in Section 1, capture the essence of their

[8] This conclusion follows from applying the procedure outlined by Woodford (2003) in the addendum to chapter 4. Part E of the appendix of the current paper provides further details.

real world counterparts, it is not advisable to base the conduct of monetary policy on an instrument rule that seeks to respond optimally to deviations of the CPI inflation rate from target and chooses the optimal degree of persistence. Implementing monetary policy in this fashion leads to unstable behavior of the policy instrument, the CPI inflation rate, the domestic inflation rate, and the nominal exchange rate.

3. Optimal Policy Based on a Target Rule

The cornerstone of the target rule approach to formulate monetary policy is the policymaker's objective function. The objective function of the central bank was introduced in the previous section and consists of the variance of the policy instrument and the variance of the CPI inflation rate:

$$V(x_t) + \mu V(\pi_t^{CPI}) \tag{13}$$

Associated with the quadratic objective function is a linear target rule. This rule embodies a systematic relationship between the variables that appear in the objective function. The specific form that the target rule takes depends in part on the way monetary policy is implemented. For the case at hand, the linear target rule is a simplified version of the one that underlies policy from a timeless perspective. Employing this simple target rule has two important advantages. First, it permits the derivation of closed form solutions of the endogenous variables of the model and, second, it identifies the key parameters in the UIP puzzle.

A noteworthy feature of the rule proposed below is that the same three variables appear in it as in the simple instrument rule. As such the target rule looks deceptively similar to the instrument rule of the previous section. However, there is a fundamental difference between the two policy rules concerning the interpretation of the policy parameters.

$$\theta_1 x_t + \theta_2 x_{t-1} + \pi_t^{CPI} = 0 \tag{14}$$

θ_1 and θ_2 are relative weights that the central bank treats as policy parameters. They represent the importance that the central bank attaches in the target rule to the policy instrument in the current and previous period, respectively, compared to the current rate of CPI inflation.[9]

The policymaker's objective is to minimize the expected loss function by choosing the optimal values of θ_1 and θ_2:

$$\min_{\theta_1,\theta_2} E[L_t] = V(x_t) + \mu V(\pi_t^{CPI}) \tag{15}$$

[9] The appendix shows how the target rule is determined from an intertemporal perspective. The appendix shows further that the proposed target rule achieves almost the same stabilization results as the target rule underlying policy from a timeless perspective.

To solve the model for the endogenous variables, substitute first the definition of the CPI inflation rate, equation (4), into the target rule. Next, eliminate the current-period rate of domestic inflation by substituting equation (3) into the target rule. The policy instrument can be eliminated by the amended UIP condition where the expected change in the exchange rate next period has been disposed of with the help of the putative solution for the exchange rate. Finally, solve for the change in the nominal exchange rate. Once we have the solution for the change in the nominal exchange rate, we can solve for the remaining endogenous variables of the model. The solutions appear in equations (16) – (19).

$$x_t = \frac{\gamma}{\gamma+\theta_2}\rho_t \tag{16}$$

$$\pi_t = -\left(\frac{\kappa\left(((1-\gamma)a_1+a_2)\gamma-(a_2-a_1\gamma)(\gamma+\theta_2)\right)}{\gamma+\theta_2}\right)\rho_t + \kappa(v_t - a_1 i_t^*) + w_t \tag{17}$$

$$\pi_t^{CPI} = -\,\theta_2 x_{t-1} - \frac{\theta_1}{\gamma+\theta_2}\rho_t \tag{18}$$

$$\Delta s_t = -\frac{\theta_2}{\gamma}x_{t-1} - [\left(\frac{\theta_1+(1-\gamma)(-\alpha)}{\gamma}\right)\frac{\gamma}{\gamma+\theta_2} + \frac{(1-\gamma)}{\gamma}\kappa(a_2 - a_1\gamma)]\rho_t - \frac{(1-\gamma)}{\gamma}(\kappa(v_t - a_1 i_t^*) + w_t) - \pi_t^* \tag{19}$$

Substituting the variances of the CPI inflation rate and the policy instrument into the objective function and minimizing with respect to the two policy parameters yields their respective optimal value:

$$\theta_1^* = 0 \qquad \theta_2^* = \frac{1}{\mu\gamma} \tag{20}$$

These optimal settings give rise to a few interesting observations. First, the target rule 14) and equation (18) reduce to

$$\pi_t^{CPI} = -\frac{1}{\mu\gamma}x_{t-1} \tag{21}$$

Under a target rule, there is no contemporaneous relationship between the rate of CPI inflation and the policy instrument; the rate of CPI inflation is pre-determined. The current rate of CPI inflation depends only on the setting of the policy instrument in the previous period and does not respond to the current risk premium. With the CPI inflation rate being immune to the shock of the UIP relation, it follows that the change in the nominal exchange rate and the rate of domestic inflation share the burden of adjusting to the risk premium optimally.[10] After substituting the optimal policy parameters into equations (17), (18), and (19), one finds that the response of the policy instrument, the rate of domestic inflation, and the change in the nominal exchange rate to the risk premium is well-defined. As such the target rule avoids introducing the serious complexity into policymaking that occurs under the instrument rule.

Second, just like under the simple instrument rule approach, the optimal responses of the rate of CPI inflation and the policy instrument to the risk premium depend on the degree of openness and the preference parameter in the objective function but not on the parameter α. Third, there is indeed an inverse relationship between changes in the nominal exchange rate and the lagged policy instrument under *optimal* policy that is based on a target rule. The coefficient on x_{t-1}in equation (19) equals $-\frac{1}{\mu\gamma^2}$. The sensitivity of changes in the nominal exchange rate to the lagged policy instrument depends on the central bank's relative aversion to CPI inflation variability and the degree of openness. The greater this aversion, the weaker the negative association between the lagged interest rate differential and the change in the nominal exchange rate. In countries where strict CPI inflation targeting is the norm, standard regression-based tests of the UIP hypothesis should find no evidence for its validity.

4. A Comparison of the Two Approaches

The findings yielded by the target rule approach afford the opportunity to explore the connection between the optimal policy parameters of the instrument rule approach (λ_1, λ_2) and the two parameters that help define the target rule approach, μ and γ. The information presented in Table 1 is key to understanding the fundamental difference between the instrument rule and the target rule approach in the simple open economy framework of this paper. Attention focuses on the optimal response of $x_t, \pi_t, \Delta s_t$ and π_t^{CPI} to the lagged interest rate differential and the risk premium. For the remaining disturbances the instrument and target rule produce identical optimal responses.

Table 1A shows the response of the current policy instrument and domestic inflation, the two variables that do not depend on past information, to the risk premium under both policy rules. A simple comparison of the two coefficients on the risk premium in the solution for x_t reveals that choosing the ratio $\frac{\lambda_1}{\lambda_2}=\mu\gamma$ generates identical responses under both implementation schemes. Substituting the same ratio into the coefficient on ρ_t in the solution for π_t under the instrument rule approach also yields the same optimal response as under the target rule. The instrument rule approach can *hypothetically* deliver a well-defined and optimal response of the policy instrument and the domestic rate of inflation to the risk premium if the ratio of the optimal policy

[10] To see this, multiply the solution for the domestic inflation rate (eq.17) and the change in the nominal exchange rate (eq.19) by $(1-\gamma)$ and γ, respectively, and add the two components.

parameters equals the weight on squared CPI inflation rate deviations in the objective function multiplied by the degree of openness. The obvious problem with the instrument rule is, however, that its implementation rests on choosing the optimal settings for λ_1 and λ_2 separately. Determining the optimal weights in this fashion results in a different ratio of the two optimal policy parameters as shown in sub-section 2: $\frac{\lambda_1^*}{\lambda_2^*} = -\frac{1}{\gamma}$.

This is not the only drawback associated with the instrument rule. A further complication that besets the instrument rule approach is that it cannot deliver the optimal response of the CPI inflation rate and the change in the exchange rate to the risk premium generated by the target rule. The problem with the instrument rule arises because the central bank is not free to choose the optimal policy parameters independently of each other. For argument's sake, suppose that the central bank implements policy by the instrument rule and chooses $\frac{\lambda_1}{\lambda_2} = \mu\gamma$.

Consider the entries in Table 1B which describes the behavior of the CPI inflation rate and the change in the exchange rate. Picking this particular ratio for the policy parameters in the instrument rule evokes the same response to x_{t-1} in both π_t^{CPI} and Δs_t as under the target rule. Thus the instrument rule can match the optimal response to the lagged policy instrument produced by the target rule.

Consider next the response of the CPI inflation rate to the risk premium under the target rule and the instrument rule, respectively. Both coefficients appear in the top-half of Table 1B. The CPI inflation rate is immune to the risk premium under the target rule approach but not under the instrument rule approach. For the latter, the numerator and denominator of the coefficient on the risk premium have been divided by λ_2. Observe that the numerator of the coefficient can approach zero only if λ_2 tends towards infinity. But letting λ_2 take on this extreme value forces μ towards zero for a fixed value of λ_1 if $\frac{\lambda_1}{\lambda_2} = \mu\gamma$ is to be maintained. Alternatively, letting λ_2 take on extreme values would require λ_1 to do so, too, which makes the instrument rule inoperable. A similar argument applies to the response of the exchange rate change to the risk premium. The bottom row of Table 1B shows that the instrument rule coefficient differs from the target rule coefficient by $\frac{1}{\lambda_2}$. To make the two response coefficients equal requires $\lambda_2 \to \infty$. But this would require $\lambda_1 \to \infty$ which in turn leaves the ratio of the two policy parameters undefined. However one looks at the issue, the fact remains that the *hypothetical* instrument rule cannot deliver the optimal response of the CPI inflation and the change in the exchange rate to the risk premium while the target rule approach can.

At a more general level, there is an information asymmetry that results in different outcomes under the two approaches. The superior performance of the target rule derives from its ability to respond optimally to *all* shocks of the model. Underlying the target rule approach is an implied reaction function that delivers this optimal response.

The implied reaction function can be obtained by substituting into the target rule, first, the definition of the CPI inflation rate and, second, the equation describing the behavior of domestic inflation. Solving this equation for the policy instrument x_t shows that the policymaker at time t observes the composite shock (u_t).

$$x_t = \frac{1}{(1-\gamma)\alpha}\left(\theta_2 x_{t-1} + (1-\gamma)(\alpha(E_t\pi_{t+1} - E_t\pi^*_{t+1}) + u_t) + \gamma(\Delta s_t + \pi^*_t)\right) \tag{22}$$

While the composite shock, which includes ρ_t, is in the policymaker's information set at time t under a target rule it is not under an instrument rule. When implementing the latter, the policymaker responds only to a deviation of the target variable from its fixed target but not directly to the shock that causes the deviation.[11]

IV. Conclusion

Recent discussions of the merits of instrument and target rules in the conduct of monetary policy focus on the closed economy New Keynesian framework. The current paper shifts the debate to an open economy framework. Employing a basic optimizing framework, this paper shows that a target rule dominates a simple instrument rule when the CPI inflation rate is the focus of monetary policy in a small open economy.

The target rule approach produces a systematic relationship between the current rate of CPI inflation and the lagged policy instrument. The weight the central bank places on the CPI inflation rate in the target rule is the product of two parameters: the relative aversion to inflation variability in its objective function and the degree of openness of the economy. The dominance of the target rule approach manifests itself in its ability to shield CPI inflation from a risk premium shock. This shock affects only the two endogenous components of the CPI inflation rate, domestic inflation and the change in the nominal exchange rate.

Specifying monetary policy in terms of a simple instrument rule suffers from the drawback that it is impossible for the central bank to choose the two optimal policy parameters independently of each other. No matter how policy parameters are set, the optimal simple instrument rule cannot replicate the superior stabilization results achieved by the target rule approach.

The optimal simple instrument rule also fails to account for the UIP puzzle in the simple optimizing framework of this paper. In contrast, the target rule approach can explain the widely reported empirical phenomenon whereby high interest rate currencies tend to appreciate. In fact the degree of openness of the economy and the central bank's relative aversion to CPI inflation variability – the same parameters that

[11] We make this point to highlight the information asymmetry between the two policy rules. As shown in the paper, ultimately the policy instrument does not respond to the composite shock as its direct effect is cancelled by the indirect effect which works through the change in the nominal exchange rate. This property is due to the simplicity of our model framework. See Froyen and Guender (2007, 2010) for a more elaborate discussion of instrument versus target rules in the conduct of optimal monetary policy.

appear in the target rule - determine the sensitivity of observed changes in the nominal exchange rate to the lagged interest rate differential.

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Table 1A: The Response of the Policy Instrument and Domestic Inflation to the Risk Premium.

	Instrument Rule	Target Rule
Coefficient on\ Solution for	$\boldsymbol{x_t}$	
$\boldsymbol{\rho_t}$	$\frac{\lambda_1\gamma}{\lambda_1\gamma+\lambda_2}$	$\frac{\mu\gamma^2}{\mu\gamma^2+1}$
	$\boldsymbol{\pi_t}$	
$\boldsymbol{\rho_t}$	$\frac{-\alpha\lambda_1\gamma}{\lambda_1\gamma+\lambda_2}+\kappa(a_2-a_1\gamma)$ **If** $\frac{\lambda_1}{\lambda_2}=\mu\gamma$ **then above equals** $-\frac{\kappa(a_1\gamma(\gamma\mu+1)-a_2)}{\gamma^2\mu+1}$	$-\frac{\kappa(a_1\gamma(\gamma\mu+1)-a_2)}{\gamma^2\mu+1}$

Table 1B: The Response of CPI inflation and the Change in the Exchange Rate to the Lagged Policy Instrument and the Risk Premium.

	Instrument Rule	Target Rule
Coefficient on\ Solution for	$\boldsymbol{\pi_t^{CPI}}$	
$\boldsymbol{x_{t-1}}$	$-\frac{\lambda_2}{\lambda_1}$	$-\frac{1}{\mu\gamma}$
$\boldsymbol{\rho_t}$	$\frac{\gamma}{\lambda_1\gamma+\lambda_2}$ **If** $\frac{\lambda_1}{\lambda_2}=\mu\gamma$ **then above equals** $\frac{\frac{\gamma}{\lambda_2}}{\mu\gamma^2+1}$	$\mathbf{0}$
	Instrument Rule	Target Rule
Coefficient on\ Solution for	$\boldsymbol{\Delta s_t}$	
$\boldsymbol{x_{t-1}}$	$-\frac{\lambda_2}{\lambda_1\gamma}$	$-\frac{1}{\mu\gamma^2}$
$\boldsymbol{\rho_t}$	$\frac{1+\lambda_1(1-\gamma)\alpha}{\lambda_1\gamma+\lambda_2}$ **If** $\frac{\lambda_1}{\lambda_2}=\mu\gamma$ **then above equals** $\frac{\frac{1}{\lambda_2}+\alpha(1-\gamma)\mu\gamma}{\mu\gamma^2+1}$	$\frac{\alpha(1-\gamma)\mu\gamma}{\mu\gamma^2+1}$

Appendix: A. The Intertemporal Optimization Problem

The starting point of discussing optimizing behavior from an intertemporal perspective is the policymaker's objective function. The policymaker seeks to minimize an intertemporal loss function that consists of squared deviations of the CPI inflation rate and the policy instrument. The constraint the policymaker faces is based on the condensed equation that describes the behavior of the domestic rate of inflation. After manipulating equation (1.3), we can express it in terms of the CPI inflation rate and the policy instrument.[12] The policy problem that the central bank faces can then be stated as:

$$\min_{\pi_t^{CPI}\ x_t} \quad E_t \sum_{j=0}^{\infty} \beta^j (x_{t+j}^2 + \mu \pi_{t+j}^{CPI^2}) \tag{23}$$

subject to

$$\pi_t^{CPI} = \alpha E_t \pi_{t+1}^{CPI} - \alpha x_t - \alpha E_t \pi_{t+1}^* + \gamma(\Delta s_t + \pi_t^*) + \alpha\gamma\rho_t + (1-\gamma)u_t \tag{24}$$

The policymaker implements monetary policy from a timeless perspective, a form of optimal policy under commitment. The key characteristic of this type of optimal policy is that the policymaker can successfully manipulate the forward-looking expectations formed by agents. Another important characteristic is that the policymaker ignores the initial period when choosing the rate of CPI inflation and the policy instrument. In setting up the constraint that appears in the Lagrangean, we replace future changes in the nominal exchange rate by the policy instrument minus the risk premium. After all, it is assumed that UIP holds in the model.

The Lagrangean takes the following form:

$$\mathcal{L} = E_t\{x_t^2 + \mu\pi_t^{CPI^2} + \delta_t(\gamma(\pi_t^* + \Delta s_t) + \alpha\pi_{t+1}^{CPI} - \alpha(x_t + \pi_{t+1}^*) + \alpha\gamma\rho_t + (1-\gamma)u_t - \pi_t^{CPI})$$

$$+\beta[x_{t+1}^2 + \mu\pi_{t+1}^{CPI^2} + \delta_{t+1}(\gamma(\pi_{t+1}^* + x_t - \rho_t) + \alpha\pi_{t+2}^{CPI} - \alpha(x_{t+1} + \pi_{t+2}^*) + \alpha\gamma\rho_{t+1} + (1-\gamma)u_{t+1} - \pi_{t+1}^{CPI})]$$

$$+\beta^2[x_{t+2}^2 + \mu\pi_{t+2}^{CPI^2} + \delta_{t+2}(\gamma(\pi_{t+2}^* + x_{t+1} - \rho_{t+1}) + \alpha\pi_{t+3}^{CPI} - \alpha(x_{t+2} + \pi_{t+3}^*) + \alpha\gamma\rho_{t+2} + (1-\gamma)u_{t+2} - \pi_{t+2}^{CPI})]$$

$$+ \cdots\}$$

Δs_t is predetermined as it depends on the stetting of the policy instrument in the previous period (plus ρ_{t-1} and an expectational error).

Suppose in time period *t* the policymaker sets policy for period *t+j, j=0,1,2,3, …….* . The optimizing condition for the rate of CPI inflation in period *t* is:

$$\delta_{t-1}\alpha + (2\mu\pi_t^{CPI} - \delta_t) = 0 \tag{25}$$

The optimizing condition for the policy instrument in the same period is:

[12] The derivation of the constraint appears in part D of the appendix.

$$(2x_t - \delta_t \alpha) + \beta \delta_{t+1} \gamma = 0 \tag{26}$$

δ_{t+j} denotes the Lagrange Multiplier in period *t+j*. Here it becomes apparent that the derivation of the target rule in the current context is somewhat more complex than in the canonical New Keynesian model. The added complexity is due to the presence of δ_{t+1} in equation (26). [13] To derive the optimal target rule, we begin by solving equation (25) for δ_t and equation (26) for δ_{t+1}.

$$\delta_t = 2\mu \sum_{j=0}^{\infty} \alpha^j \, \pi_{t-j}^{CPI} \tag{27}$$

$$\delta_{t+1} = -2 \sum_{j=0}^{\infty} \frac{\alpha^j}{(\beta\gamma)^{j+1}} x_{t-j} \tag{28}$$

After lagging equation (28) by one period, we set the two equations equal to each other. Doing so yields the target rule expressed solely in terms of the target variable and the policy instrument.

$$\mu \sum_{j=0}^{\infty} \alpha^j \, \pi_{t-j}^{CPI} = - \sum_{j=0}^{\infty} \frac{\alpha^j}{(\beta\gamma)^{j+1}} x_{t-j-1} \tag{29}$$

The target rule relates the weighted sum of current and lagged rates of CPI inflation to the weighted sum of lagged rates of the policy instrument. There is no place for $x_t$in the target rule. A parsimonious representation of the above target rule is obtained by restricting the number of terms of the sequence to one on both sides of equation (29) and setting $\beta = 1$. In this case, the target rule reduces to a simple expression that links the current rate of CPI inflation to the lagged policy instrument:

$$\pi_t^{CPI} = - \frac{1}{\mu\gamma} x_{t-1} \tag{30}$$

Greater emphasis on stability in the rate of CPI inflation and greater openness weaken the link between current CPI inflation and the lagged policy instrument. Equation (30) is the same target rule that appears in Section III.3 which discusses the implementation of monetary policy via the target rule approach from a more heuristic angle. There we

[13] Strictly speaking, the expectation of δ_{t+1} should appear in eq. (26). One can then solve this equation forward and express $\delta_t$as a sequence of current and expected future settings of the policy instrument. Doing so yields, however, a target rule where a sequence of current and past inflation rates is matched with a sequence of current and expected future settings of the policy instrument.

employ an expected loss function that consists of a weighted combination of the unconditional variances of the CPI inflation rate and the policy instrument. Equation (13) results if, first, one multiplies the intertemporal loss function by (1-β) and, second, takes the limit as $\beta \rightarrow 1$.

It can easily be verified that basing policy on the proposed simplified target rule generates results that approximate the globally optimal rule.

A numerical optimization procedure can be used to determine the variances of the endogenous variables and the policy instrument under the globally optimal rule, i.e. under policy from a timeless perspective. For $a_1 = 0.5$, $a_2 = 0.25$, $\gamma = 0.4$, $\kappa = 0.1$, $\mu = 1$, and unit variances for all exogenous disturbances, the variances of the relevant variables are shown in Table A1.[14]

Table A1: Results Based on Policy from a Timeless Perspective

V(x)	V(π^{CPI})	V(Δs)	E(L)=V(x)+$\mu V(\pi^{CPI})$	V(δ_1)	V(δ_2)
0.0191	0.1187	4.0216	0.1378	0.0763	0.4770

There are two Lagrange Multipliers in this set-up because DYNARE solves the model by imposing two constraints: the UIP condition and equation (24).

Table A2 shows the variances of the policy instrument, the CPI inflation rate and the change in the nominal exchange rate under simple commitment where the proposed target rule (equation (30)) governs the relationship among the target variables. All variances are based on the analytical results reported in Section III of the paper. It is apparent that these variances are very close to those reported in Table A1.

Table A2: Results Based on Simplified Target Rule

V(x)	V(π^{CPI})	V(Δs)	E(L)=V(x)+$\mu V(\pi^{CPI})$
0.0190	0.1189	4.0213	0.1379

B. A Proposed Solution Method: The Case of the Instrument Rule

For the change in the nominal exchange rate we pose the following putative solution:

$$\Delta s_t = \phi_{10} x_{t-1} + \phi_{11} \pi_t^* + \phi_{12} u_t + \phi_{13} \rho_t \qquad (31)$$

For the domestic rate of inflation we pose a similar trial solution:

[14] The entries reported in Table A1 were calculated with the help of DYNARE.

$$\pi_t = \phi_{20}x_{t-1}+\phi_{21}\pi_t^* + \phi_{22}u_t + \phi_{23}\rho_t \tag{32}$$

As all shocks of the model are assumed to be white noise processes, the forward-looking expectations are given by:

$$E_t\Delta s_{t+1} = \phi_{10}x_t \tag{33}$$

$$E_t\pi_{t+1} = \phi_{20}x_t \tag{34}$$

With the help of equation (33), we can solve the UIP equation for x_t.

$$x_t = \frac{1}{1-\phi_{10}}\rho_t \tag{35}$$

Combine (35) with the instrument rule and definition of CPI inflation rate:

$$\frac{1}{1-\phi_{10}}\rho_t = \lambda_1((1\text{-}\gamma)\pi_t + \gamma(\Delta s_t + \pi_t^*)) + \lambda_2 x_{t-1} \tag{36}$$

Next, substitute the domestic inflation equation (3) into (36). Replace the policy instrument by equation (35). Doing so results in an equation that describes the behavior of the change in the nominal exchange rate:

$$\lambda_1\gamma\Delta s_t = \left(\frac{1}{1-\phi_{10}} + \frac{\lambda_1}{1-\phi_{10}}(1-\gamma)\alpha(1-\phi_{20})\right)\rho_t - \lambda_1\big((1-\gamma)u_t + \gamma\pi_t^*\big) - \lambda_2 x_{t-1}$$

(37)

Next substitute the trial solution for the change in the nominal exchange rate into equation (37). Matching coefficients on both sides of the equation yields the following solutions for the coefficients in the trial solution:

$$\phi_{10} = -\frac{\lambda_2}{\lambda_1\gamma} \qquad \phi_{11} = -1 \qquad \phi_{12} = -\frac{1-\gamma}{\gamma} \tag{38}$$

$$\phi_{13} = \frac{1}{\lambda_1\gamma+\lambda_2}(1 + \lambda_1(1-\gamma)\alpha(1-\phi_{20}))$$

The solution for Δs_t can be substituted back into equation (36) and solved for the rate of domestic inflation.

$$\pi_t = -\alpha(1-\phi_{20})\frac{\lambda_1}{\lambda_1\gamma+\lambda_2}\rho_t + u_t \tag{39}$$

Replacing the rate of domestic inflation on the left-hand side with the trial solution (32) and matching coefficients result in the following solutions:

$$\begin{aligned} \phi_{20} &= 0 & \phi_{22} &= 1 \\ \phi_{21} &= 0 & \phi_{23} &= -\frac{\alpha\lambda_1\gamma}{\lambda_1\gamma+\lambda_2} \end{aligned} \tag{40}$$

To reconcile these results with those reported in the main part of the paper, replace the shock in the domestic inflation equation with $u_t = \kappa(v_t\text{-}a_1 i_t^* + (a_2 - a_1\gamma)\rho_t) + w_t$.

C. Derivation of Budget Constraint

Multiply the domestic inflation equation by $(1-\gamma)$:

$$(1-\gamma)\pi_t = (1-\gamma)(-\alpha x_t + \alpha(E_t\pi_{t+1} - E_t\pi^*_{t+1}) + u_t) \tag{41}$$

Add and subtract $\gamma(\Delta s_t + \pi^*_t)$on the left-hand side of above equation and restate resulting expression as:

$$\pi_t^{CPI} = \gamma(\Delta s_t + \pi^*_t) + \alpha(1-\gamma)E_t\pi_{t+1} + \alpha\gamma(E_t\Delta s_{t+1} + E_t\pi^*_{t+1}) - \alpha(E_t\Delta s_{t+1} + E_t\pi^*_{t+1}) + (1-\gamma)(u_t - \alpha\rho_t) \tag{42}$$

Making use of the definition of the CPI inflation rate and the UIP condition on the right-hand side allows us to express equation (42) as

$$\pi_t^{CPI} = \gamma(\Delta s_t + \pi^*_t) + \alpha E_t\pi^{CPI}_{t+1} - \alpha(x_t - \rho_t + E_t\pi^*_{t+1}) + (1-\gamma)(u_t - \alpha\rho_t) \tag{43}$$

or

$$\pi_t^{CPI} = \alpha E_t\pi^{CPI}_{t+1} - \alpha x_t - \alpha E_t\pi^*_{t+1} + \gamma(\Delta s_t + \pi^*_t) + \alpha\gamma\rho_t + (1-\gamma)u_t \tag{43'}$$

This is the constraint of the policymaker in the intertemporal optimization problem.

D. Derivation of Domestic Inflation Equation

We begin with the standard specification of the Phillips Curve and an open-economy IS relation:

$$\pi_t = \beta E_t\pi_{t+1} + \kappa y_t + u_t \tag{44}$$

$$y_t = E_t y_{t+1} - a_1\left(i_t - E_t\pi^{CPI}_{t+1}\right) + a_2(q_t - E_t q_{t+1}) + v_t \tag{45}$$

q_t = real exchange rate.

Assume perfect exchange rate pass-through. This allows us to replace the CPI inflation rated with

$$\pi_t^{CPI} = \pi_t + \gamma\Delta q_t. \tag{46}$$

The IS relation can then be restated as:

$$y_t = E_t y_{t+1} - a_1\left(i_t - E_t\pi_{t+1}\right) - (a_2 - a_1\gamma)E_t\Delta q_{t+1}) + v_t \tag{47}$$

Next, we impose real UIP to eliminate $E_t\Delta q_{t+1}$ from the equation.

$$y_t = E_t y_{t+1} - a_1\left(i_t - E_t\pi_{t+1}\right) - (a_2 - a_1\gamma)\left(i_t - i^*_t - (E_t\pi_{t+1} - E_t\pi^*_{t+1}) - \rho_t\right) + v_t \tag{48}$$

Add and subtract $a_1(i^*_t - E_t\pi^*_{t+1})$:

$$y_t = E_t y_{t+1} - a_1(i_t - i_t^*) + a_1(E_t\pi_{t+1} - E_t\pi_{t+1}^*) - (a_2 - a_1\gamma)(i_t - i_t^*) + (a_2 - a_1\gamma)[E_t\pi_{t+1} - E_t\pi_{t+1}^* + \rho_t]$$
$$+ v_t - a_1(i_t^* - E_t\pi_{t+1}^*) \qquad (49)$$

Making use of the definition of the policy instrument allows us to rewrite the above as:

$$y_t = E_t y_{t+1} - [a_1(1-\gamma) + a_2]x_t + [a_1(1-\gamma) + a_2]\left(E_t\pi_{t+1} - E_t\pi_{t+1}^*\right) + v_t - a_1(i_t^* - E_t\pi_{t+1}^*) + (a_2 - a_1\gamma)\rho_t \qquad (50)$$

Substitute this equation into the Phillips Curve:

$$\pi_t = \beta E_t\pi_{t+1} + \kappa\begin{bmatrix} E_t y_{t+1} - [a_1(1-\gamma) + a_2]x_t + [a_1(1-\gamma) + a_2]\left(E_t\pi_{t+1} - E_t\pi_{t+1}^*\right) \\ + v_t + (a_2 - a_1\gamma)\rho_t - a_1(i_t^* - E_t\pi_{t+1}^*) \end{bmatrix} + w_t \qquad (51)$$

Simplify by dropping the $\beta E_t\pi_{t+1}$ and $\kappa E_t y_{t+1}$ terms.

$$\pi_t = -\alpha x_t + \alpha\left[E_t\pi_{t+1} - E_t\pi_{t+1}^*\right] + \kappa\left[v_t - a_1(i_t^* - E_t\pi_{t+1}^*) + (a_2 - a_1\gamma)\rho_t\right] + w_t \qquad (52)$$

The presence of the forward-looking expectation of inflation in this equation implies that monetary policy works partly through the expectations channel. Expected future inflation affects current inflation because the formation of this expectation is influenced by the interest rate differential. The lagged interest rate differential enters the model either through the instrument rule or the target rule.

The above equation can be simplified to read

$$\pi_t = -\alpha x_t + \alpha\left[E_t\pi_{t+1} - E_t\pi_{t+1}^*\right] + u_t \qquad (53)$$

where

$$u_t = \kappa\left[v_t - a_1(i_t^* - E_t\pi_{t+1}^* + (a_2 - a_1\gamma)\rho_t)\right] + w_t$$

$$\alpha = \kappa[a_1(1-\gamma) + a_2].$$

E. Determinacy of Equilibrium Under a Simple Instrument Rule

The model is written in terms of the forward-looking expectation of the rate of inflation and the nominal exchange rate, respectively, and the policy instrument.

$$E_t\pi_{t+1} = (\frac{1}{\alpha} + \lambda_1(1-\gamma))\pi_t + \lambda_1\big(\gamma(\pi_t^* + \Delta s_t)\big) + \lambda_2 x_{t-1} - \frac{1}{\alpha}u_t \qquad (54)$$

$$E_t \Delta s_{t+1} = \lambda_1\big((1-\gamma)\pi_t + \gamma(\pi_t^* + \Delta s_t)\big) + \lambda_2 x_{t-1} - \rho_t \tag{55}$$

$$x_t = \lambda_1\big((1-\gamma)\pi_t + \gamma(\pi_t^* + \Delta s_t)\big) + \lambda_2 x_{t-1} \tag{56}$$

Now define $z_t = \begin{bmatrix} \pi_t \\ \Delta s_t \\ x_{t-1} \end{bmatrix}$ and $E_t z_{t+1} = \begin{bmatrix} E_t \pi_{t+1} \\ E_t \Delta s_{t+1} \\ x_t \end{bmatrix}$. Then the above three equations can be rewritten as $E_t z_{t+1} = A z_t + B e_t$ where e_t is a vector of the disturbances and A and B are coefficient matrices. The characteristic equation of the matrix A has three roots.

$$\Gamma^3 + (\frac{-1-\alpha\lambda_1-\alpha\lambda_2)}{\alpha}\Gamma^2 + \frac{(\gamma\lambda_1+\lambda_2)}{\alpha}\Gamma = 0 \tag{57}$$

$$A_2 = (\frac{-1-\alpha\lambda_1-\alpha\lambda_2)}{\alpha}$$

$$A_1 = \frac{(\gamma\lambda_1+\lambda_2)}{\alpha}$$

The rational expectations equilibrium is well-defined if two roots lie outside the unit circle and one within.

The three roots of the characteristic equation are:

$$\mathrm{T}_1 = 0$$

$$\Gamma_{2,3} = \frac{-1-\alpha(\lambda_1+\lambda_2) \pm \sqrt{4\alpha(-\gamma\lambda_1-\lambda_2) + (1+\alpha(\lambda_1+\lambda_2))^2}}{2\alpha}$$

According to (12) in the text, the optimal policy parameters satisfy the linear restriction $\gamma\lambda_1^* + \lambda_2^* = 0$.

Imposing this condition on the second and third root produces another zero root:

$$\mathrm{T}_2 = 0$$

$$\Gamma_3 = \frac{-1-\alpha(\lambda_1+\lambda_2)}{\alpha}$$

Hence the condition that two roots lie outside the unit circle is violated. A simple optimal instrument rule fails to establish a well-defined rational expectations equilibrium.

Analogously, one can check whether various conditions that apply to A_1 and A_2 are met. (For further details, see page 673 in Woodford (2003)). Here we consider condition C.15 as specified by Woodford.

According to C15:

$$A_1 > 1$$

$$\frac{(\gamma\lambda_1 + \lambda_2)}{\alpha} > 1$$

This condition is violated, however, as $\gamma\lambda_1^* + \lambda_2^* = 0$.