It never rains but it pours: Modelling the persistence of spikes in electricity prices

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Abstract

During periods of market stress, electricity prices can rise dramatically. This paper treats these abnormal episodes or price spikes as count events and attempts to build a model of the spiking process. In contrast to the existing literature, which either ignores temporal dependence in the spiking process or attempts to model the dependence solely in terms of deterministic variables (like seasonal and day of the week effects), this paper argues that persistence in the spiking process is an important factor in building an effective model. A Poisson autoregressive framework is proposed in which price spikes occur as a result of the latent arrival and survival of system stresses. This formulation captures the salient features of the process adequately, and yields forecasts of price spikes that are superior to those obtained from naïve models in which persistence in the spiking process is not accounted for.

Keywords
Electricity Prices, Extreme Events, Poisson Regressions, Poisson Autoregressive Model.

JEL classification numbers
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1 Introduction

Electricity retailers generally buy electricity from an electricity grid at a market price, known as the spot price, but sell electricity to customers at a price that is heavily regulated. Spot prices cannot, therefore, be passed on to the final consumer directly, and retailers end up bearing the price risk. Consequently, the accurate forecasting of electricity prices is one of particular practical importance to risk management in the energy sector. From the perspective of academic research, modelling electricity prices also provides a compelling challenge because time-series data on electricity prices display a number of interesting idiosyncracies that stem mainly from the lack of practical ways to store electricity. It is not surprising, therefore, that there is a large and growing literature on modelling and forecasting electricity spot prices.

Most models of electricity prices start from the generally accepted view that the price process is stationary and mean reverting, that it displays predictable fluctuations over daily, weekly and yearly frequencies and also exhibits sudden and extreme changes in price, often referred to as price 'spikes' (Barlow, 2002; de Jong and Huisman, 2002; Geman and Roncorni, 2006; Mount *et al.*, 2006). Historically, models fall into three broad categories, namely, traditional autoregressive time series models, nonlinear time series models with particular emphasis on Markov-Switching models and continuous-time diffusion models. Each class of model, with few exceptions, expresses the spot price of electricity or its logarithm as the sum of a mean-reverting autoregressive component together with a seasonal components. Where the models differ is in terms of the treatment of spikes.

Traditional autoregressive time-series models treat spikes either in terms of: a latent threshold (Misiorek *et al.*, 2006); a homogeneous binomial process for jumps (Crespo Cuaresma *et al.*, 2004); a basic time-varying Poisson process for jumps with autoregressive error structure (Knittel and Roberts, 2005); a switching process and as a process with either Gaussian-mixture errors or GARCH errors (Swider and Weber, 2007); or a process with autoregressive error structure with errors drawn from a Gaussian or extreme-value distribution (Contreras *et al.*, 2003; Garcia *et al.*, 2005; Byström, 2005).

Nonlinear time-series models, in particular Markov-switching models, tackle the problem of spikes by proposing different regimes, at least one of which is consistent with a state of system stress in which a spike is more likely to occur (Becker *et. al*, 2007). De Jong and Huisman (2002), Huisman and Mahieu, (2003), Weron *et al.*, (2004), de Jong (2006), Kosater and Mosler, (2006) and Bierbrauer *et al.*, (2007) found that the performance of models incorporating Markov-switching is often superior to models that do not incorporate switching. This improved performance is achieved despite the fact that some of these studies ignore seasonality in the transition probabilities as observed by, for example, Kanamura and Ōhashi (2007) and Escribano *et al.* (2002). In an attempt to overcome this problem, Mount *et al.* (2006) and Kanamura and Ōhashi (2007) propose models in which regime-switching probabilities are dependent upon fundamental exogenous variables representative of the interplay between system load and capacity constraints.

In continuous-time diffusion processes, spikes are normally captured by the addition of a Poisson process with either a constant intensity parameter (Weron *et al.*, 2004; Cartea and Figueroa,

2005) or a time-inhomogeneous intensity parameter (Escribano *et al.*, 2002; Knittel and Roberts, 2005) in which seasonality is incorporated by representing the intensity of the process as a sum of temporal dummy variables. A well-acknowledged problem with these models is that simultaneous inclusion of mean-reversion and a jump process is incompatible with the large downward swings that necessarily follow shortly after price spikes. This empirical reality confounds parameter estimation algorithms, leading in turn to unreliable and unrealistic results (Barlow, 2002; de Jong and Huisman, 2002; Bunn and Karakatsani, 2003; Burger *et al.*, 2003). The most common method of circumventing this problem is to propose a more complex (possibly multiple factor) diffusion process (Barlow, 2002; Burger *et al.*, 2003; Geman and Roncorni, 2006).

Each of these approaches to modelling spot electricity prices has its own particular strengths and weaknesses, and a comprehensive comparison is beyond the scope of this paper. All the models considered so far, however, share the common property that they treat price as a continuous variable and attempt to model its trajectory. This apparently innocuous assumption needs to be reconsidered. Deregulated electricity markets are segregated in time by market rules making each unit of time each day a separate market, a concept acknowledged early by Lucia and Schwartz (2002). Thus, the notion of modelling price as a continuous variable in time appears to be at odds with the way in which the electricity market functions. Consequently, this paper adopts and approach in which each price spike is a discrete event, with the time series of events being regarded as a realisation of a point process. This approach is consistent with the physical model of electricity generation and dispatch and also interesting in the sense of enabling the econometric tools relating to point processes to be applied to the problem.

This paper makes two major contributions to the existing state of knowledge in the modelling of extreme movements in electricity prices. The first concerns the validity of the basic assumptions made in the literature about the spiking process. It is shown that one of the most common methods used in the literature to characterize the spiking process, that is a Poisson process with an intensity parameter that is either constant or driven solely by deterministic seasonal variables, is in fact a misspecification. In particular, it is found that the intensity of the true process is significantly related to a historical component, and that this persistence must be accounted for if the resulting model is to be credible. This finding contradicts a fundamental assumption made by Poisson process models, namely that events occur independently across time.

The second, more fundamental contribution of this paper relates to the use of a variant of the Poisson autoregressive (PAR) model introduced by Al-Osh and Alzaid (1987) and McKenzie (1988), which has proven useful when dealing with low-count integer-valued time series (Freeland and McCabe, 2004a). In the current context, the PAR model is used to capture persistence in the spiking process, and hence provide a better characterisation of the behaviour of electricity prices. The motivation for using the model lies in the idea that spikes occur as a result of a number of unobserved stresses acting on the system simultaneously (Geman and Roncorni, 2006; Mount *et al.*, 2006). The behaviour of these concurrent, but unobserved stresses is described by a PAR model with Bernoulli-distributed residuals. This novel adaptation allows estimation of the model parameters despite the underlying process remaining latent.

The institutional framework of the paper is that of the Australian national electricity market

(NEM), which has been operating since 1998. It comprises six state grids, five of which are physically linked by inter-connectors. It should be noted, however, that price spikes are a generic feature of electricity markets worldwide (Escribano, *et al.*, 2002), and so the work reported in this paper should be of general interest and applicability, despite the Australian focus of the data underlying the analysis.

The remainder of the paper is structured as follows. Section 2 provides an introduction to some of the important characteristics of electricity price data, particularly with respect to the definition and nature of price spikes. An analysis of the deterministic temporal drivers of the spiking process is performed via count regressions in Section 3, where it is demonstrated that a spiking process constructed in this way does not display the same characteristics as that observed in the Australian electricity market. Section 4 provides estimates of the suitably-adapted Poisson autoregressive model and compares the forecasting performance of this model against naïve forecasts of price spikes implied by an inhomogeneous Poisson process. Section 5 is a brief conclusion.

2 Data

In the NEM, electricity prices change every five minutes to match demand with the schedule of bids offered by suppliers (generators) of electricity. Transactions in the market are settled each half-hour at the spot price, which is calculated as the average of the prices at which electricity was supplied in the six preceding five-minute intervals. This paper investigates the behaviour of spot prices for the regions of New South Wales (NSW), Queensland (Qld), South Australia (SA), the Snowy Mountains (Snowy) and Victoria (Vic) for the period from the opening of the market on December 13, 1998 to May 1, 2007, a data set spanning 3,061 days or 146,928 half-hours for each region. While the regions are interconnected, physical constraints on the amount of power transmittable between regions means that each region essentially operates as a separate market. Table 1 gives the summary statistics of the time series of spot price and its logarithm for each region. Figures 1 and 2, respectively, illustrate these series.

Table 1 demonstrates that the effect of spikes is to introduce a high level of skewness in the distribution of electricity prices. This property of extreme skewness has also been observed in data from other markets (Escribano *et al.*, 2002). Other basic attributes possessed by data of this type are: mean reversion to a deterministic component with periodic behavior (Huisman and Mahieu, 2003); seasonal volatility (Lucia and Schwartz, 2002); and GARCH characteristics in both the price (Garcia *et al.*, 2005) and returns series (Byström, 2005). Electricity prices also have well-documented diurnal, weekly, monthly, seasonal and annual fluctuations. Attempts to model these permanent components have included combinations of daily dummy variables and monthly dummy variables, or trigonometric functions with appropriate period (Lucia and Schwartz, 2002; Knittel and Roberts, 2005) and wavelet decompositions (Weron *et al.*, 2004).

	NSW	Qld	SA	Snowy	Vic
mean	34.57	36.28	41.83	32.07	31.74
median	23.13	20.93	28.71	23.66	23.51
std. dev.	176.83	157.99	150.27	118.09	123.99
skewness	36.29	29.87	26.42	38.28	43.21
kurtosis	1545.67	1143.43	873.22	1773.24	2445.89
mean	3.21	3.19	3.40	3.21	3.18
median	3.14	3.04	3.36	3.16	3.16
std. dev.	0.52	0.59	0.58	0.50	0.55
skewness	2.32	2.53	1.64	1.82	1.22
kurtosis	18.49	15.61	12.57	15.15	11.73

Table 1: Full sample summary statistics for the wholesale electricity price series (upper panel) and its natural logarithm (lower panel). Observations that were not strictly positive had to be excluded from the logarithmic data, but these observations only constituted a tiny proportion of the overall set.

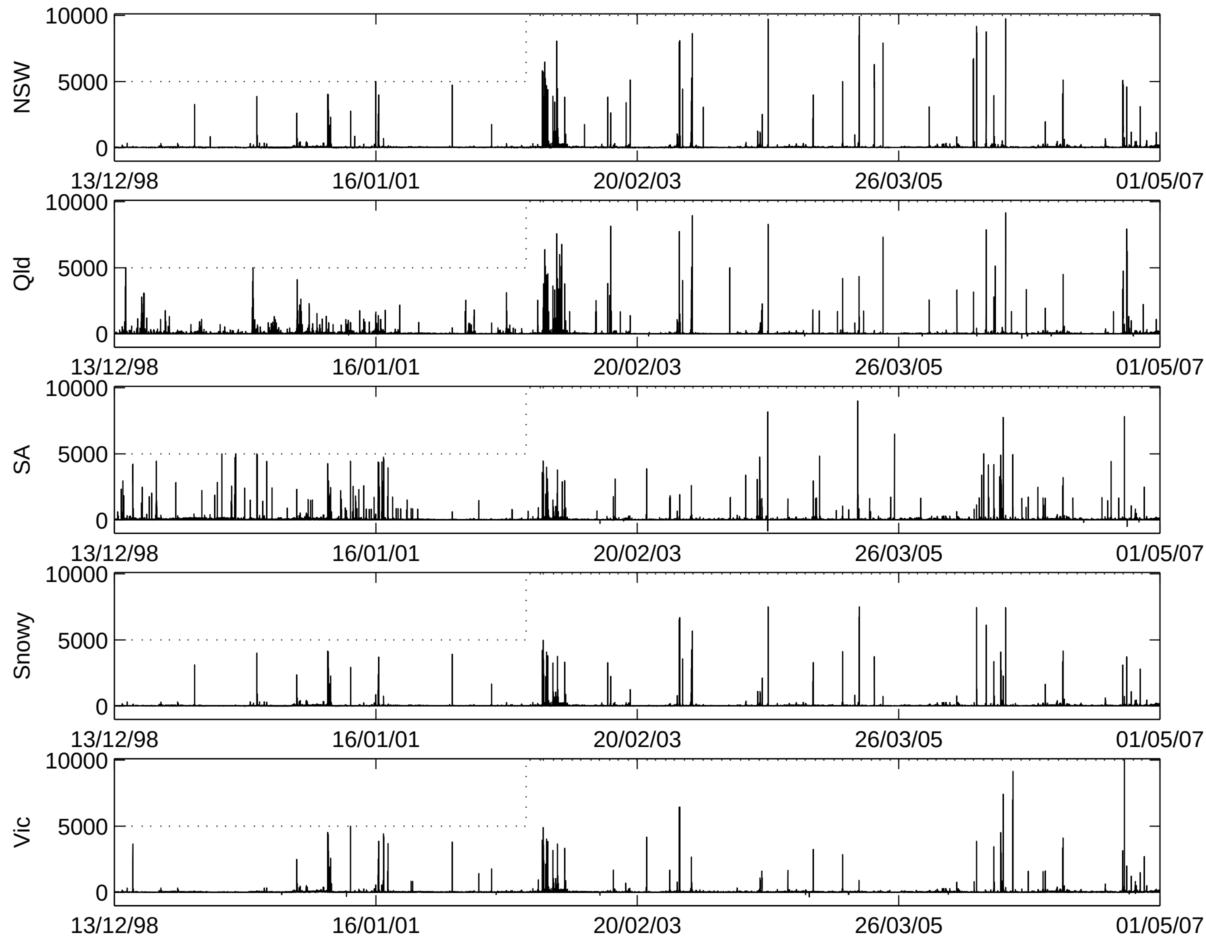

Figure 1: Full series of half-hourly spot prices by region. The dashed line indicates the maximum bid price for each five minute interval, increased from \$5,000/MWh to \$10,000/MWh during April 2002.

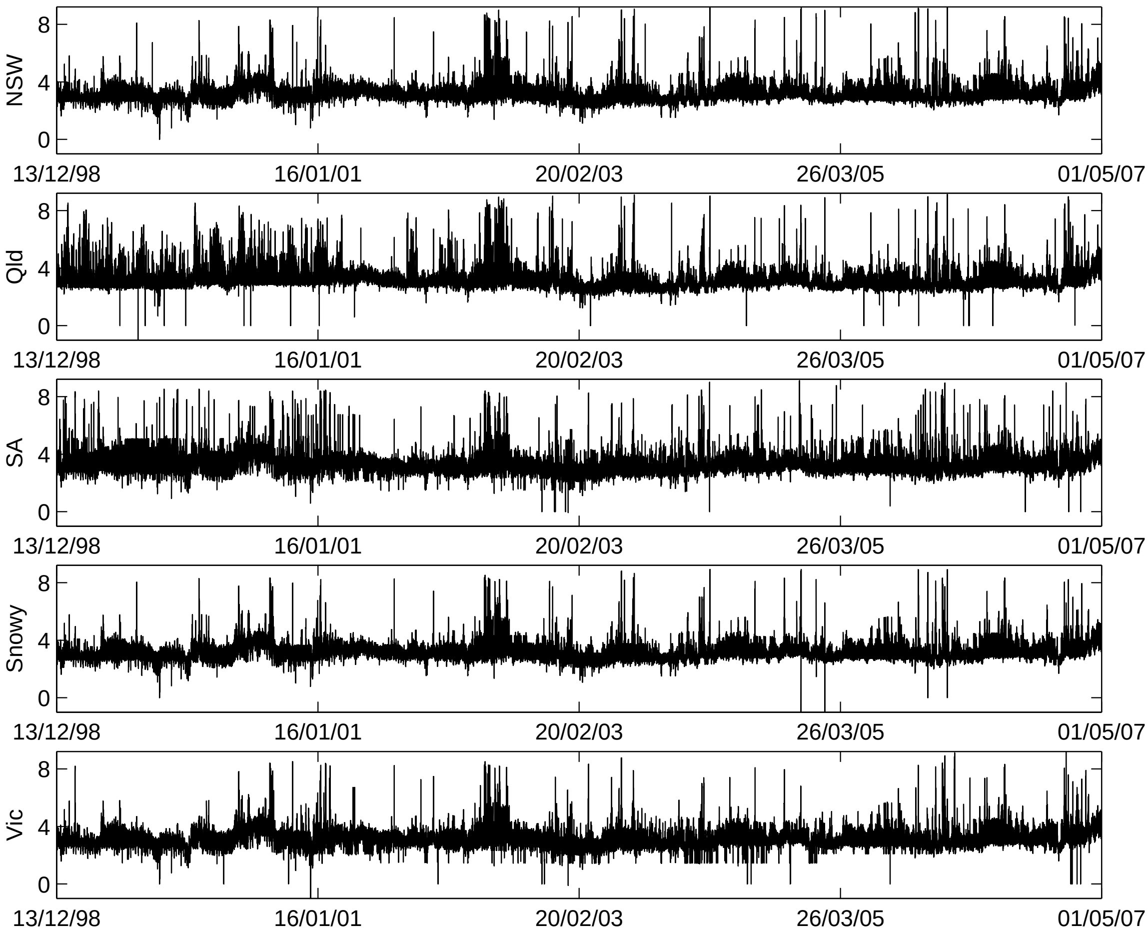

Figure 2: Series of the natural logarithm of half-hourly spot prices by region. Observations corresponding to zero and negative raw prices were discarded in the construction of this series, but these constituted only a tiny fraction of the full data set.

The abrupt price increases so characteristic of electricity markets are clearly evident in Figure 1. Price spikes are not a uniquely Australian phenomenon but are characteristic of deregulated electricity markets worldwide (Barlow, 2002; de Jong and Huisman, 2002; Lucia and Schwartz, 2002; Burger *et al.*, 2003; Byström, 2005; Cartea and Figueroa, 2005). For the purposes of this paper, a *price spike* or *exceedence* will be formally defined as a situation where spot electricity price exceeds a particular threshold value that is chosen to lie substantially outside the normal range of daily fluctuations. Figure 3 plots by region the medial half-hourly price (solid lines) and the 10th and 90th percentile half-hourly spot prices (dashed lines). If the interval of spot prices enclosed by these dashed lines is regarded as a 'natural' range for that region, then Figure 3 suggests that setting the threshold spot price at \$100/MWh will satisfy the conditions required of a threshold spot price for each region. This choice of threshold price is also in agreement with that used by Becker *et al.* (2007) in a recent study of the Australian electricity market.

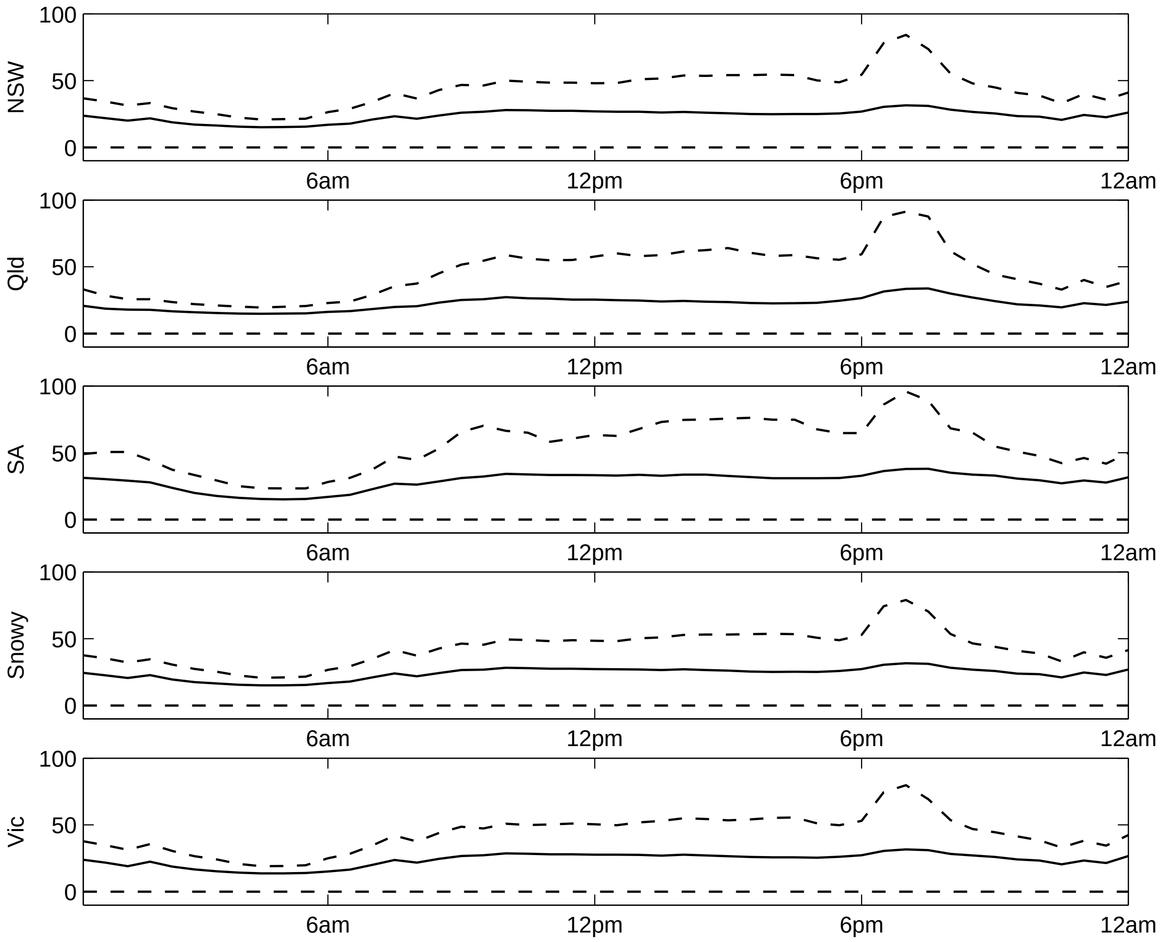

Figure 3: Median half-hourly spot price by region. The dashed lines indicate the 10th and 90th percentile half-hourly spot price.

Geman and Roncorni (2006) and Mount *et al.* (2006) explain the presence of spikes by analysing the generator bid curve, particularly the transition in bids from low-cost high-supply generators to high-cost low-supply generators. Systematic fluctuations in demand due to weather or business demands, systematic reductions in supply due to scheduled infrastructure maintenance and non-systematic reductions in supply due to generator or network failure are some of the factors that can shift the demand and supply curves. Given these potential explanations for abnormal price events, it is not surprising that the empirical literature has found that the occurrence of price spikes varies across time. For example, Escribano *et al.* (2002) and Knittel and Roberts (2005) show that for a number of electricity markets, the intensity parameter of the spiking process in electricity prices exhibits seasonal dependence. Kanamura and Ōhashi (2007) also observe a seasonal dependency in the transition probabilities between spike and non-spike regimes due to systematic fluctuations in demand.

Time	NSW	Qld	SA	Snowy	Vic
00:00 – 02:00	22	12	132	25	38
02:00 – 04:00	7	5	100	10	14
04:00 – 06:00	4	2	27	3	4
06:00 – 08:00	38	61	131	35	38
08:00 – 10:00	65	329	429	44	76
10:00 – 12:00	112	439	387	76	92
12:00 – 14:00	298	556	670	226	249
14:00 – 16:00	466	668	811	357	356
16:00 – 18:00	335	537	630	266	250
18:00 – 20:00	573	903	750	459	445
20:00 – 22:00	48	223	245	41	42
22:00 – 24:00	34	142	178	37	42
Total	2002	3877	4490	1579	1646

Table 2: Count of exceedences above $100/MWh by region, pooled by time of day.

Weekday	NSW	Qld	SA	Snowy	Vic
Sunday	163	272	327	138	95
Monday	382	722	694	278	326
Tuesday	332	834	782	261	300
Wednesday	310	700	710	238	254
Thursday	378	603	822	300	333
Friday	221	434	625	186	210
Saturday	216	312	530	178	128
Total	2002	3877	4490	1579	1646

Table 3: Count of exceedences above $100/MWh by region, pooled by weekday.

Month	NSW	Qld	SA	Snowy	Vic
January	332	697	641	298	281
February	311	418	670	249	269
March	110	651	237	89	94
April	277	363	336	163	144
May	159	198	256	142	151
June	174	284	307	142	166
July	169	311	443	146	162
August	81	218	405	66	103
September	36	45	119	33	42
October	54	222	226	38	35
November	145	151	519	110	102
December	154	319	331	103	97
Total	2002	3877	4490	1579	1646

Table 4: Count of exceedences above \$100/MWh by region, pooled by month.

Tables 2, 3 and 4, which display the respective number of exceedences above \$100/MWh on an intra-hour, intra-day and intra-month basis, also provide casual empirical evidence to support temporal dependency in the Australian market. These tables illustrate, for example, that in most regions an exceedence is at least twice as likely to occur on a 'working' day (Monday–Thursday) compated to a Sunday, and, in some regions, at least one hundred times more likely between 16:00–20:00 than between 02:00–06:00.

Section 3 provides a more formal analysis to support the observation that the intensity of exceedences in electricity prices exhibits temporal dependence. In order to isolate the deterministic factors that are thought to drive exceedences, the total number of abnormal price events arising per day may be regarded as a counting measure, quantifying the stress acting on the system, thus enabling the formal framework of count regression to be used to determine the statistical significance of possible factors driving the occurrence of price spikes.

3 Modelling temporal dependence

Count regressions may be used to capture the properties of series of observed counts provided these are independent draws from an underlying discrete distribution. For example, the primitive Poisson regression assumes that if $y_1, y_2, \ldots, y_n$ is a series of counts, then each observation is an independent draw from a Poisson distribution with constant intensity parameter λ. One generalisation of the basic Poisson regression incorporating the possibility that the series of observed counts may depend on a set of exogenous variables, say $\boldsymbol{X} = (X_1, \cdots, X_m)'$, is to propose that the intensity parameter of the process at instant i is given by the ansatz $\lambda_i = \exp(\boldsymbol{X}_i' \boldsymbol{\beta})$ in which $\boldsymbol{\beta} = (\beta_1, \cdots, \beta_m)'$ is a parameter vector to be estimated.

In the current research, a series $y_1, y_2, \ldots, y_n$ ($n = 3061$) of counts was constructed for each region with y_i denoting the number of daily exceedences of the spot price above \$100/MWh on the i-th day of the data. An important property of these constructed series was that the frequency with which the observation zero occurred was significantly in excess of that which would be expected from a Poisson or Negative Binomial distribution. To resolve this difficulty, a zero-inflated Poisson (ZIP) model (see, for example, Greene, 1994) was found to give a good fit to the distributional properties of the observed series. The ZIP model requires that, if $y_1, y_2, \ldots, y_n$ is a series of counts, then for $k \in \{0, 1, \ldots\}$

$$\text{Prob}\,(Y_i = k) = \begin{cases} \alpha_i + (1-\alpha_i)p_0 & k = 0 \\ (1-\alpha_i)p_k & k > 0 \end{cases} \tag{1}$$

where Y_i denotes the i-th observation of the series of counts and p_k is the Poisson probability density function with intensity λ_i. When there are no excess zeros $\alpha_i = 0$ in equation (1) and the regression reduces to a primitive Poisson regression. While Greene (1994) suggests some stochastic formulations for α_i, these were found in many cases to give an inferior fit[1] to the case $\alpha_i = \alpha$. When $\alpha_i = \alpha$, the log-likelihood function is

$$\log L = \sum_{i=1}^{n} \log\left((1-\alpha)\frac{\exp(-\lambda_i)\lambda_i^{y_i}}{(y_i)!} + \alpha\, I_{\{y_i=0\}} \right) \tag{2}$$

which is maximised with respect to the parameters $\boldsymbol{\theta} = (\alpha, \boldsymbol{\beta})$.

The literature provides some idea of which variables may influence the inhomogeneity of the number of observed counts. It is clear from Knittel and Roberts (2005) and Escribano *et al.* (2002) that, when modelled as the sum of dummy variables, the intensity of exceedences displays strong seasonal, weekend and peak/off-peak intra-day effects. It is anticipated that these effects arise from the interplay of exogenous variables which themselves exhibit seasonal, weekday and peak/off-peak fluctuations. For example, it has been observed that load has a systematic diurnal and seasonal behaviour (Ramanathan *et al.*, 1997), and that weather-based variables exert a strong influence on electricity demand (Taylor and Buizza, 2003). These observations suggest that temperature and load provide two exogenous variables that potentially characterise the intensity of exceedences by contrast with the intrinsically less satisfactory approach of simply introducing dummy variables.

In this work daily temperature effects are modelled by $T_{\max}$, denoting the absolute deviation of the maximum daily temperature from the mean maximum for that day, and $T_{\min}$ denoting the absolute deviation of the minimum daily temperature from the mean minimum temperature for that day. The advantage in using the absolute deviation from the mean is that it corrects for the positive correlation between temperature and price in summer months and the negative correlation between temperature and price in winter months.[2] The third exogenous variable to be used here is daily peak load, which in this work was standardized by the previous years' data in order to preserve the stationarity of the load series. The exogenous variables $T_{\max}$, $T_{\min}$ and

[1] For example, in NSW, Qld and SA the value of the log-likelihood function with constant α was -2398.6, -4010.3 and -4787.8 whereas with $\alpha_i = \Phi(\tau\lambda_i)$ the respective corresponding values were -2396.0, -4090.9 and -4808.9 where Φ is the Gaussian cdf.

[2] This assumed relationship between temperature and price is not unreasonable. On a particularly warm summers day the demand for air conditioning will increase which in turn increases system stress causing the wholesale price of electricity to rise. On a particularly cold winters day the demand for electricity will likewise rise, but in this case to supply the call for heating.

Load were incorporated into the model by proposing that the daily values of λ in expression (2) are determined from the daily values of $T_{\max}$, $T_{\min}$ and Load by the formula

$$\log \lambda = \beta_1 + \beta_2 T_{\max} + \beta_3 T_{\min} + \beta_4 \text{Load}\,. \tag{3}$$

As the results in Table 5 show, the exogenous variables are strongly significant in all regions. Since these variables are time-varying, this is clear evidence that the occurrence of exceedences is time-inhomogeneous. In general, when a large deviation from average maximum temperature or average minimum temperature is experienced, this increases significantly the value of the intensity parameter which in turn corresponds to a higher probability of experiencing at least one exceedence. As has been noted previously, unseasonably high temperatures in summer or unseasonably low temperatures in winter can be expected to create an increased demand for electricity thereby increasing the stress on the system which in turn translates to a higher probability of experiencing an exceedence. Moreover, high peak load also has a significant and positive effect on the intensity parameter in most regions. The intra-day exceedence pattern observed in Table 3 is explained, in part, by this finding, since load is generally higher on working days due to commercial demand. When a test for consistency against an alternative of heterogeneity/overdispersion (Greene, 1994) is applied to this data, the zero-inflated model performs satisfactorily in all regions.

Variable	NSW	Qld	SA	Snowy	Vic
α	0.866*	0.781*	0.718*	0.877*	0.867*
	(0.006)	(0.008)	(0.008)	(0.006)	(0.007)
Constant	1.468*	1.568*	1.328*	1.078*	0.991*
	(0.012)	(0.009)	(0.009)	(0.019)	(0.019)
$T_{\max}$	0.057*	0.053*	0.085*	0.094*	0.081*
	(0.002)	(0.003)	(0.001)	(0.003)	(0.003)
$T_{\min}$	-0.019*	0.012*	0.035*	0.033*	0.066*
	(0.005)	(0.003)	(0.002)	(0.004)	(0.005)
Load	-0.009	0.285*	0.156*	-0.115*	0.179*
	(0.014)	(0.008)	(0.012)	(0.013)	(0.022)
$\log L$	-2597.3	-4155.9	-4638.1	-2108.3	-2210.2
p-value	0.499	0.497	0.486	0.496	0.493

Table 5: Results from zero-inflated Poisson regressions. Coefficients are listed with standard errors in parentheses and significance at the 5% level. The p-values from a consistency test for excess zeros in Greene (1994) are also shown.

To reinforce the claim of inhomogeneity of the occurrence of exceedences, Table 6 gives the values of the log-likelihood function evaluated at the maximum likelihood estimates for the zero-inflated model with a full complement of exogenous variables, and for the model with a constant intensity (primitive Poisson model), that is, the choice $\lambda = \exp(\beta_1)$ in expression (3). The inappropriateness of capturing the occurrence of exceedences using a primitive Poisson model is apparent from the likelihood ratio statistic, a result that further calls into doubt attempts to model exceedences either as a Poisson process (Weron *et al.*, 2004; Cartea and Figueroa, 2005) or as a Binomial process with constant intensity(Crespo Cuaresma *et al.*, 2004).

Model	NSW	Qld	SA	Snowy	Vic
Full model	-2597.3	-4155.9	-4638.1	-2108.3	-2210.2
Constant only	-2643.7	-4339.7	-5099.1	-2220.8	-2349.9
LR statistic	92.6	367.7	921.8	225.1	279.4

Table 6: Log-likelihood evaluated at MLEs for zero-inflated Poisson regressions for each state with full dummy variables and with an intercept only. Likelihood ratio statistics are also listed. The 99.5% quantile for the χ^2_3 distribution is 12.84.

The question now arises as to whether or not the intensity of exceedences can be explained adequately by a model based on exogenous factors alone or if this intensity also depends on the previous history of exceedences. In other words, do exceedences exhibit any persistence over and above the influences of load and temperature? Lindsay and Rosenberg (2007) propose the second order cumulant as an appropriate tool to identify significant correlations between events. The cumulant compares the distribution of lags between observed events to the distribution that would be obtained from a process where events occur independently. Further details are provided in the Appendix.

Consider the point process formed by partitioning time into days and associating an event with each day if at least one exceedence occurred during that day[3]. Figure 4 plots the cumulant for this point process for each region. It is apparent that there is a strong correlation between events separated by intervals of two months or less, an observation which suggests that the actual occurrence of exceedences is at odds with both homogenous and inhomogeneous Poisson processes if these assume independent counts in non-overlapping intervals of time.

At first sight, therefore, it appears that the exceedences occur with a significant degree of persistence, which suggests that their intensity cannot be modelled adequately using load and temperature variables alone. A simple consistency check (Breunig *et al.*, 2003) strengthens this point. A synthetic price series of roughly the same number of observations as the original series was simulated using the results of the Poisson regressions in Table 5. This allowed for a series with the same temperature and load effects as the original series to be simulated, with the added property of independence across time. Following this, exactly the same procedure for calculating the second-order cumulant was performed as was done for the original series of prices. It is clear from Figure 5 that the short-term memory exhibited by the true point process is absent from the synthetic series since nearly all peaks fall between the bounds of the confidence interval. The inevitable conclusion of this line of research must be that while exogenous variables such as load and temperature have some power to explain the observed pattern of exceedences, there is nevertheless a fundamental need to adopt a model of the underlying process which incorporates its history.

[3]Of the total number of days on which at least one exceedence occurred, 404 for NSW, 661 for Qld, 830 for SA, 347 for Snowy and 371 for Vic, the number of times the spot price exceeded \$100/MWh from one day into the next was 6, 0, 14, 6 and 5 respectively.

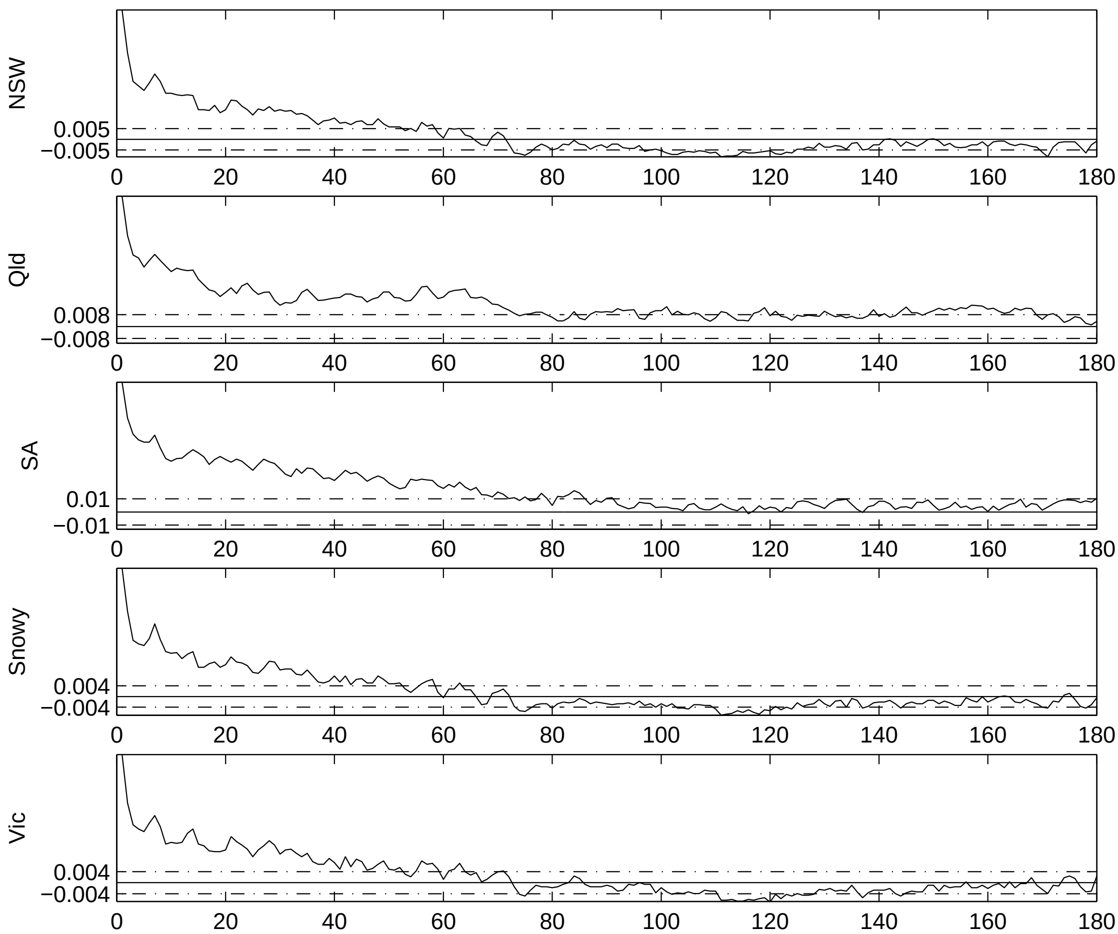

Figure 4: Second order cumulant for the point process formed from the actual data by assigning an event to each day in which at least one exceedence occurred. The corresponding 95% confidence intervals for the null model, a time-homogeneous Poisson process, are shown as dashed lines.

These results imply that the occurrence of spikes in Australian wholesale electricity prices is driven by an intensity parameter which depends on the history of these exceedences, in addition to obvious exogenous variables such as load and temperature. For the most part, existing attempts to model price have have failed to recognise this fact and do not incorporate the history of exceedences as a driving mechanism in the construction of a model. The next section describes a procedure to construct a model of prices that includes exogenous variables, such as load and temperature, and also accounts for short-term persistence in exceedences.

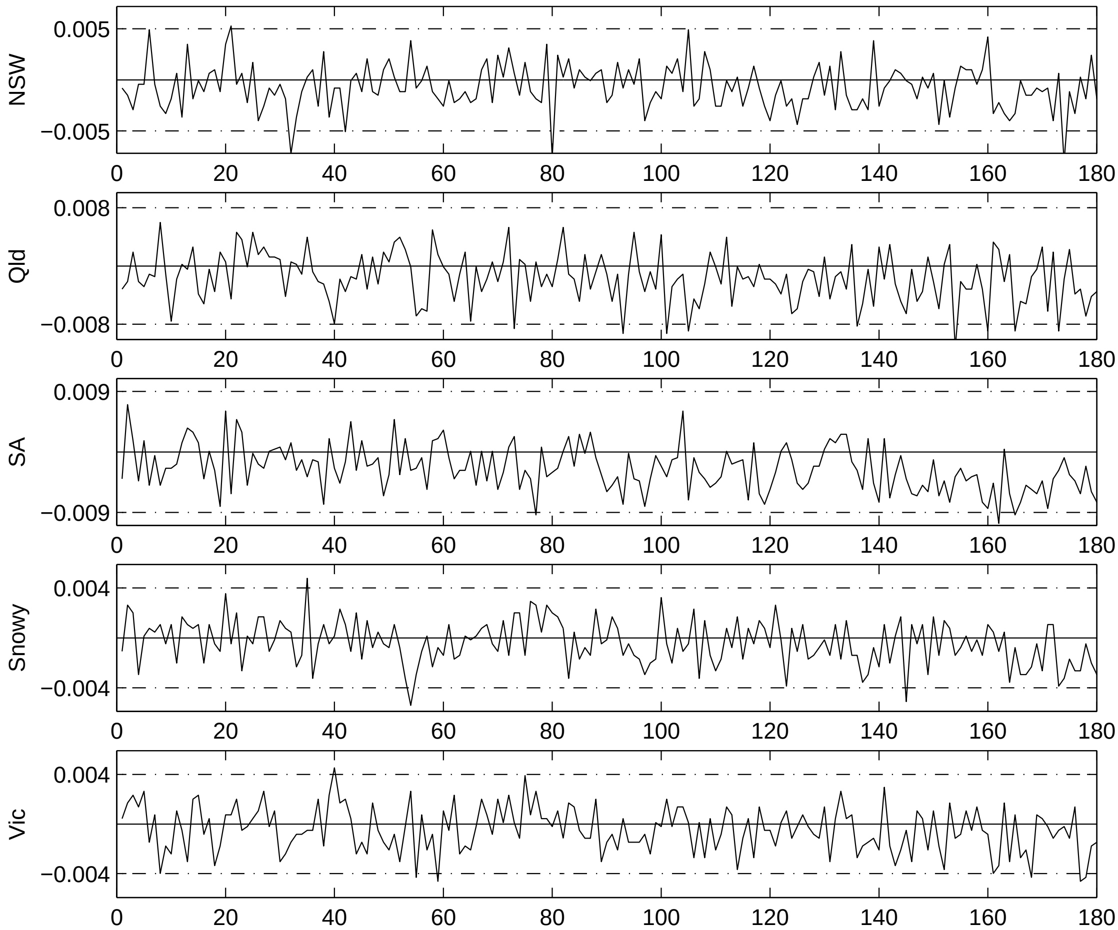

Figure 5: Second order cumulant for the synthetic data calculated in exactly the same as for the actual data. Confidence intervals are similarly calculated. The absence of memory in this process is clear since nearly all peaks lie within the bounds of the confidence interval, except for perhaps the odd random occurrence.

4 Modelling persistence

The analysis presented in the previous section supports the contention that price spikes in the wholesale electricity market cluster in time and behave as an inhomogeneous process driven in part by external stress to the system. This view is consistent with the mechanisms proposed by Mount *et al.* (2006) and Geman and Roncorni (2006) to explain price spikes alluded to in Section 2. Realistically, the occurrence of system stresses can be expected to behave as a time-varying stochastic process. On occasions the drivers of these stresses may persist for several days suggesting that some price spikes will be indicative of a critical failure within the generating infrastructure, and that this failure will in the short term increase the likelihood of fresh exceedences. This mechanistic argument goes some way to explaining why clusters of price spikes are observed in the wholesale price of electricity. The econometric treatment of a mechanism of this sort requires a stochastic model, in which the occurrence of a price spike is dependent both on the history of price spikes and on the usual exogenous factors. With this motivation in mind, the spiking process is assumed to be the combination of latent arrival and survival processes. The arrival process represents the advent of a period of stress and the survival process models its duration.

Although intuitively appealing, this explanation is complicated by the possibility of multiple events occurring concurrently. For example, a new stress could arrive prior to the resolution of an existing stress, and end before or after the original stress. However, the number of concurrent stresses cannot be observed: the only observable object is their net effect, namely, a price spike. So, if X_t is the number of stresses acting on the system on day t, the observed variable is Y_t, where

$$Y_t = \left[\begin{array}{ll} 0 & X_t = 0\,, \\ 1 & X_t > 0\,. \end{array} \right. \tag{4}$$

Thus Y_t is a binary random variable taking the value one if there is an exceedence on day t and zero otherwise. The arrival and resolution of a number of overlapping but separate stresses is manifest in the observed data as a single prolonged period of days on which exceedences are observed. This superposition of the arrival and survival processes makes their separation and direct estimation impossible. A further complication is that the values of X_t remain latent. However, these complications may be overcome in part by making the simplifying assumption that a maximum of one new stress can occur each day, independently of other days. Thus if e_t counts the number of new stresses to arrive on day t, then e_t takes the values zero and one, and the sequence of e_t is a sequence of independent Bernoulli random variables, with

$$\begin{aligned} \text{Prob}\,(e_t = 1) &= \lambda\,, \\ \text{Prob}\,(e_t = 0) &= 1 - \lambda\,. \end{aligned}$$

The reason for this restriction will become apparent after the model is fully described.

The survivals component of the process is dealt with by using a Poisson autoregressive (PAR) model originally introduced by Al-Osh and Alzaid (1987) and McKenzie (1988). Aspects of the PAR model have received renewed interest in recent years, with developments in its statistical analysis (Freeland and McCabe, 2004a) and use as a forecasting tool for low-count integer-valued time series (Freeland and McCabe, 2004b; McCabe and Martin, 2005). Let X_{t-1} be the known value of the series at time $(t-1)$. The PAR model posits that each count at time $(t-1)$ survives to time t with probability α, independently of every other count. Thus if X_{t-1} is known, then the value of the survival component at time t is a Binomial(X_{t-1}, α) random variable. This operation is called *Binomial Thinning* and is represented by $\alpha \circ X_{t-1}$. Values of $\alpha \circ X_{t-1} \,|\, X_{t-1}$ are assumed to be independent over time and of the arrivals process. In order to maintain consistency with the inhomogeneous nature of the problem, the quantities λ and α modelling respectively the arrival and survival of system stresses are replaced by time-varying functions λ_t and α_t to obtain the modified PAR model

$$X_t = \alpha_t \circ X_{t-1} + e_t \tag{5}$$

where e_t is a Bernoulli random variable with parameter λ_t. The evolution of the rates λ_t and α_t are specified by the ansatze

$$\begin{aligned} \lambda_t &= 1 - \exp\big(-\exp(\boldsymbol{z}'_{1t}\boldsymbol{\beta}_1)\big) \\ \alpha_t &= 1 - \exp\big(-\exp(\boldsymbol{z}'_{2t}\boldsymbol{\beta}_2)\big) \end{aligned} \tag{6}$$

in which $\boldsymbol{z}'_{1t}$ and $\boldsymbol{z}'_{2t}$ are sets of regressors for the arrival and survival processes respectively.[4]

The traditional form of the PAR model assumes that e_t is Poisson-distributed. While this is adequate when X_t is observed, it implies in this scenario that if $Y_t = 1$ is observed then X_t could have any positive integer value. The motivation for choosing e_t to be independent Bernoulli random variables is now apparent. By restricting the support of e_t to $\{0, 1\}$, the realisation $Y_{t-k} = 0, Y_{t-k+1} = 1, \ldots, Y_{t-1} = 1$ implies that X_t can take one of finitely many values $\{0, 1, \ldots, k\}$. Thus when conditioning on the history of the process $\Psi_t = \{Y_0, Y_1, \ldots, Y_t\}$, the probabilities $\text{Prob}\,(Y_t = 1 \,|\, \Psi_{t-1})$ and $\text{Prob}\,(Y_t = 0 \,|\, \Psi_{t-1})$ can be determined at each time step.

The procedure for calculating these probabilities is as follows. If $Y_{t-1} = 0$ then $X_t = 0$ or $X_t = 1$. Therefore

$$\begin{aligned}
\text{Prob}\,(Y_t = 1 \,|\, \Psi_{t-1}) &= \text{Prob}\,(e_t = 1) = \lambda_t \\
\text{Prob}\,(Y_t = 0 \,|\, \Psi_{t-1}) &= \text{Prob}\,(e_t = 0) = 1 - \lambda_t
\end{aligned}$$

However, if $Y_{t-2} = 0$ and $Y_{t-1} = 1$ then X_t can conceivably take the values 0, 1 or 2. This means that $\text{Prob}\,(Y_t = 0 \,|\, \Psi_{t-1})$ and $\text{Prob}\,(Y_t = 1 \,|\, \Psi_{t-1})$ must be calculated using

$$\Pr(X_t = m | X_{t-1} = n) = \begin{cases} 0 & m > n+1 \\ \alpha^n \lambda & m = n+1 \\ B(n, m, \alpha)(1-\lambda) + B(n, m-1, \alpha)\lambda & 1 \leq m \leq n \\ (1-\alpha)^n (1-\lambda) & m = 0 \end{cases} \tag{7}$$

where $B(n, m, p)$ is the probability of drawing m from a Binomial(n,p) distribution and the time dependence of α and λ has been suppressed. If $Y_t = 0$ is observed then $X_{t+1} = 0$ or $X_{t+1} = 1$. However, if $Y_t = 1$ is observed then X_{t+1} can take the values 0, 1, 2 or 3 and these probabilities are calculated conditionally upon the events $\{X_t = 1 \,|\, Y_t = 1\}$ and $\{X_t = 2 \,|\, Y_t = 1\}$ using equation (7), and from these $\text{Prob}\,(Y_{t+1} = 0 \,|\, \Psi_t)$ and $\text{Prob}\,(Y_{t+1} = 1 \,|\, \Psi_t)$ can be determined. This conditioning is continued iteratively until the next zero is observed thereby allowing $\text{Prob}\,(Y_t = 1 \,|\, \Psi_{t-1})$ to be calculated at each time step of the full sample. Simulation is less convoluted, since the series $X_1, X_2, \ldots, X_T$ is simulated directly and the sequence $Y_1, Y_2, \ldots, Y_T$ constructed from this simulation.

Let $p_t = \text{Prob}\,(Y_t = 1 \,|\, \Psi_{t-1})$ and let y_t denote the realisation of Y_t, then the log-likelihood function of the process Y is

$$\log L = \sum_{t=1}^{T} y_t \log p_t + (1 - y_t) \log(1 - p_t). \tag{8}$$

The log-likelihood function is maximised with respect to the parameter vector $\boldsymbol{\beta} = \boldsymbol{\beta}_1 \otimes \boldsymbol{\beta}_2$. Whilst $\log L$ is easy to compute, the advent of clusters of spikes in the data results in some of the probabilities p_t being conditioned upon many possible events. This makes a closed-form expression for (8) infeasible, and consequently parameter estimates and their standard errors were calculated numerically. Note also that since both the arrival and survival components of the latent process are independent over time and independent of each other, then $Y_t \,|\, \Psi_{t-1}$ is

[4] The analysis was found to be insensitive to the form of the ansatze for λ_t and α_t in (6). Similar results were obtained using the formulations $\frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\boldsymbol{x}'_t \boldsymbol{\beta}\right)$ and $\left(1 + \exp(-\boldsymbol{x}'_t \boldsymbol{\beta})\right)^{-1}$.

an independent process over time thereby avoiding the complications encountered when using constructed binary variables as regressands, as highlighted by Harding and Pagan (2006).

This model was estimated using constant intensities, and then again with the same temperature and load variables used in the Poisson regression analysis reported in Section 3. Use of these variables was again motivated by the findings of Ramanathan *et al.* (1997) and Taylor and Buizza (2003) and also ensured that the same information set is used for estimation and simulation as was used in the Poisson regressions. The three exogenous variables and constant were included in both $\boldsymbol{z}_{1t}'$ and $\boldsymbol{z}_{2t}'$ for two reasons: it is expected that load and temperature affect the occurrence and duration of a period of stress; and the rate at which stresses are relieved can be expected to depend on the type of the stress (weather, system failure, and so on) so these factors were used to proxy this effect.

Results from the model estimated using constant arrival and survival probabilities are exhibited in Table 7. In the top panel of Table 7, a higher coefficient value indicates a higher probability of the arrival of a stress, whereas, in the lower panel of this table, a higher coefficient indicates a higher probability that existing stresses will persist. The lower arrival parameter estimates for NSW, Snowy and Vic are consistent with fewer spikes observed in these regions relative to Qld and SA. Similarly, higher survival parameters for Qld and SA are consistent with these regions having comparatively longer periods of consecutive spikes. While these results are useful as a credibility check on the model, they are not themselves particularly insightful.

Variable	NSW	Qld	SA	Snowy	Vic
Constant	-2.669*	-2.155*	-1.880*	-2.797*	-2.708*
	(0.075)	(0.062)	(0.057)	(0.079)	(0.076)
λ_t	0.067	0.109	0.142	0.059	0.065
Constant	-0.305*	-0.254*	-0.252*	-0.360*	-0.386*
	(0.075)	(0.060)	(0.055)	(0.082)	(0.080)
α_t	0.522	0.540	0.540	0.502	0.493
$\log L$	-925.1	-1264.3	-1445.6	-846.1	-896.7

Table 7: Coefficient estimates and corresponding implied arrival probabilities (upper panel) and survival probabilities (lower panel) using constants only as drivers for the processes. An asterisk indicates significance at the 5% level. Standard errors are shown in parentheses.

The model was re-estimated using the full set of explanatory regressors, with results displayed in Table 8. It is clear from a comparison of the log-likelihoods that the explanatory variables are jointly very significant. Regarding arrival probabilities, it would be expected that all variables are significant with positive constants causing the probability of a price spike to increase in the presence of extreme temperatures or abnormally high load. This is the case for all variables in all regions, with two notable exceptions. *First*, extreme minimum temperatures are not significant in Qld. This observation can be reconciled with its comparatively warmer climate. *Second*, load is not significant in explaining arrivals in the Snowy region, a feature which may well be explained by its small load compared with other regions.

Survival rates are influenced less by extreme temperature and load events. All constants are strongly significant in all regions. However, different explanatory variables are significant in different regions. If no exogenous factors were significant in explaining the daily survival probabilities of a stress, this would suggest that stresses are resolved at the same rate. With the exception of the Snowy Mountains region, this is not what is observed.

Variable	NSW	Qld	SA	Snowy	Vic
Constant	-3.064*	-2.215*	-1.965*	-3.112*	-3.024*
	(0.102)	(0.066)	(0.063)	(0.102)	(0.100)
Tmax	0.341*	0.216*	0.181*	0.236*	0.173*
	(0.025)	(0.027)	(0.013)	(0.017)	(0.017)
Tmin	0.199*	0.025	0.116*	0.058*	0.111*
	(0.038)	(0.024)	(0.020)	(0.023)	(0.033)
Load	0.740*	0.363*	0.531*	0.053	0.770*
	(0.110)	(0.057)	(0.082)	(0.084)	(0.122)
λ_t	0.046	0.103	0.131	0.044	0.048
Constant	-0.776*	-0.423*	-0.510*	-0.512*	-0.816*
	(0.113)	(0.071)	(0.069)	(0.101)	(0.122)
Tmax	0.073	0.038	0.043*	0.024	0.116*
	(0.037)	(0.031)	(0.018)	(0.024)	(0.025)
Tmin	0.008	0.053*	-0.040	-0.015	-0.037
	(0.043)	(0.023)	(0.024)	(0.024)	(0.039)
Load	0.344*	0.121	0.351*	-0.020	0.431*
	(0.131)	(0.066)	(0.109)	(0.072)	(0.136)
α_t	0.369	0.481	0.451	0.451	0.357
Log-L	-790.2	-1205.2	-1279.1	-765.2	-781.3

Table 8: Coefficient estimates and corresponding implied arrival probabilities (upper panel) and survival probabilities (lower panel) at the sample mean of the exogenous factors using the full set of regressors as drivers for the processes. An asterisk indicates significance at the 5% level. Standard errors are shown in parentheses.

While estimation and analysis are important, there are at least two other considerations that must be satisfied by an accurate model. *First*, in the spirit of Breunig *et al.* (2003), series simulated by the model should have properties that are consistent with those of the observed data. *Second*, to be of any use at all the model should be capable of producing one-day-ahead forecasts of price spikes that are superior to those produced by naïve models.

One tool commonly used to capture the temporal coding residing in a sequence of events is the *correlation histogram*[5]. This is the histogram of all possible lags between pairs of events. For example, the correlation histogram of a primitive Poisson process is flat (within sample error) because all lags are equal likely, and therefore deviations from a flat profile conveys information about how the intensity of an event process depends on its history. Thus the

[5]The Appendix contains further technical details on the correlation histogram.

correlation histogram provides a means of assessing how well a simulated process mimics the temporal coding of events in the observed data. Clearly a comparison of correlation histograms is sensitive to both the number of events in a sample and to the temporal coding of these events. The number of events controls the total area under the correlation histogram while the temporal coding is embedded in the relative size of the bars of the histogram at each lag.

Figure 6 (left panel) illustrates the correlation histogram for exceedences in the NSW data whereas Figure 6 (right panel) illustrates the correlation histogram of simulated data for the same region. The results from the electricity markets in the other regions are typical of those presented in Figure 6 for NSW. The model appears to characterise the process reasonably well, especially with respect to the differing behaviour at short and long lags. The ability of the Poisson autoregressive model to capture persistence in the spiking process has therefore contributed significantly to the efficacy of the model, and provides a marked improvement over the model presented in Section 3, in which the intensity parameter of the Poisson process is linked to diurnal and seasonal factors only. Of course, it is to be expected that a more accurate representation of the process could be developed by incorporating into the model additional factors such as, for example, reserve margins or scheduled generator outages. This line of inquiry is an important avenue for future research.

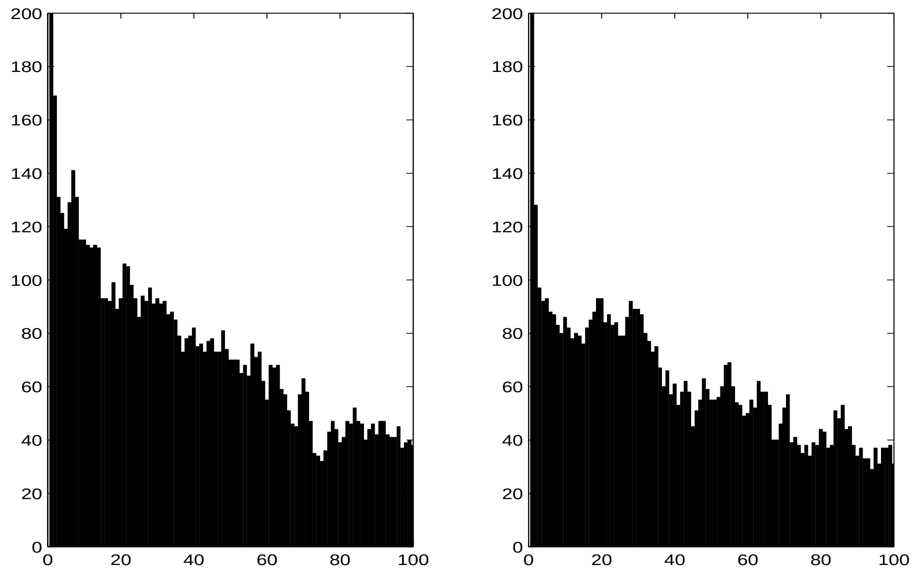

Figure 6: Correlation histogram for exceedences in the observed data (left panel) and simulated data (right panel) for NSW. The x-axis plots the lag between events and the y-axis plots the frequency of that lag.

A naïve model for spike generation is that adopted by Escribano *et al.* (2002) and Knittel and Roberts (2005), namely, that exceedences are driven solely by weekday and month effects. Therefore, the probabilities of an exceedence occurring on a particular weekday-month combination is just the rate observed for that combination in the sample. These probabilities were estimated and compared to those from the full model for 10 samples of length 90 days, drawn randomly from the full series without replacement. Two measures of accuracy were applied to the set of 10 samples in order to assess the degree to which the day-ahead probabilities differ from 0 if there is no exceedence on the next day, or 1 if there is an exceedence on the next day. The first criterion used was mean absolute error (MAE) but this metric is a symmetric one, in that failure to predict a price spike is penalised in exactly the same way as predicting a price spike that subsequently fails to materialise. From the perspective of electricity market participants, however, it is reasonable to surmise that the forecasting problem is an asymmetric one. On the other hand, it is likely to be less costly to hedge unnecessarily against price spikes that to bear the losses incurred when an unexpected exceedence is encountered. Therefore, a measure of forecast accuracy reflecting this asymmetry is required. If $t_1^S, t_2^S, \ldots, t_n^S$ are the days in the sample on which a price spike occurs, $t_1^N, t_2^N, \ldots, t_m^N$ are the days in the sample on which no price spike occurs, and p_t is the forecast probability of a spike on day t, then let

$$\text{PERR} = \sum_{i=1}^{n} (1 - p_{t_i^S})^{1/2} + \sum_{i=1}^{m} p_{t_i^N}.$$

This criterion penalises the under-prediction of a price spike more than the over-prediction of no price spike. Table 9 contains the measures of forecast error for the autoregressive model both with the full set of exogenous variables and with constant arrival and survival probabilities, and the measures of forecast error for the naïve model. It is not surprising, given the ability of the full model to provide realistic simulations of the spiking series, that it yields estimates of the probability of a price spike that are superior to those estimates implied under a naïve model on both symmetric and asymmetric criteria. What is remarkable, however, is that even if the arrival and survival probabilities are held constant, the autoregressive model still outperforms the naïve model on both criteria. The forecasting superiority of the model introduced in this paper, therefore, is taken to be a direct result of its ability to capture the persistence in the spiking process.

Model	Measure	NSW	Qld	SA	Snowy	Vic
Autoregressive –	MAE	0.186	0.249	0.246	0.130	0.125
full model	PERR	0.214	0.286	0.288	0.145	0.136
Autoregressive –	MAE	0.210	0.261	0.292	0.143	0.140
constant rates	PERR	0.235	0.297	0.332	0.156	0.150
Naïve	MAE	0.254	0.328	0.366	0.168	0.177
	PERR	0.268	0.352	0.400	0.177	0.184

Table 9: Measures of forecast error for the autoregressive model with probabilities determined by deterministic variables (upper panel), for the autoregressive model with fixed arrival and survival probabilities (middle panel), and for a naïve estimate of the probability of a price spike (lower panel). A lower error value for each criterion indicates superior performance.

5 Conclusion

Time series of electricity prices exhibit a number of idiosyncratic features making the task of modelling them both fascinating and complex. Of particular interest in the current research is the problem of the accurate characterisation of the intensity with which large spikes in electricity prices are observed. Modelling the spiking process is important, since price spikes are particularly detrimental to electricity retailers who, because of regulatory constraints, are unable to pass the price risk on to consumers. Despite this commercial imperative, the treatment of the spiking process in the current literature remains simplistic.

This paper has adopted the novel approach of treating price spikes in electricity markets as discrete events by contrast with the continuous approaches used in previous work on this topic. One advantage of this approach is that exceedences themselves become the focus of the analysis rather than the price of electricity from which, of course, the occurrence of exceedences can be extracted. One important finding of this work is that the intensity of price spikes is driven by the previous history of price spikes, in the respect that these proxy failures in the supply infrastructure, in addition to exogenous factors such as temperature and system load.

Modelling the spiking process as the superposition of stresses which arrive and persist at varying rates has an air of realism and in this work is seen to capture the persistence of price spikes in the wholesale market price of electricity. A Poisson autoregressive framework for integer-valued time series was adapted to account for the number of simultaneous stresses remaining latent, and provided a model that could be estimated by maximum likelihood. The arrival and survival rates of price spikes were found to be dependent upon extreme temperature events and peak load. However, it was the model's ability to capture the intrinsic persistence in price spikes that enabled it, first, to generate simulated electricity prices that were characteristically similar to those observed in practice, and second, to forecast the probability of a price spike with better accuracy than simpler models.

Appendix

Let $\mathcal{N}$ be a stationary point process in which $N(t)$ counts the number of events to have occurred in the interval $[0, t)$ and $\Delta N(t)$ denotes the number of events to have occurred in the interval $[t, t+\Delta t)$, that is, $\Delta N(t) = N(t+\Delta t) - N(t)$. The process $\mathcal{N}$ is said to be *orderly* if

$$\text{Prob}\,(N(t+\Delta t) - N(t) > 1) = o(\Delta t)$$

and is said to satisfy a *mixing* condition if

$$\text{Prob}\,(\Delta N(t+u) > 1 \,|\, \Delta N(t)) = \text{Prob}\,(\Delta N(t+u) > 1)$$

as $u \to \infty$ where is understood that $\Delta N(t+u) = N(t+u+\Delta t) - N(t+u)$.

Consider a sample of an orderly point process in which events occur at the strictly increasing sequence of times $t_1 < t_2 < \ldots t_n \cdots$. Under the assumptions of stationarity and orderliness, the intensity of the process $\mathcal{N}$ and its correlation histogram (also known as the second-order intensity) are defined respectively by

$$\begin{aligned} p_1 &= \lim_{\Delta t \to 0^+} \frac{\text{Prob}\,(\Delta N(t) > 0)}{\Delta t} \\ p_2(u) &= \lim_{\Delta t \to 0^+} \frac{\text{Prob}\,(\Delta N(t+u) > 0 \text{ and } \Delta N(t) > 0)}{(\Delta t)^2}\,. \end{aligned} \tag{9}$$

In particular, the mixing property of the process $\mathcal{N}$ means that $p_2(u) \to p_1^2$ as $u \to \infty$. In the case of a primitive Poisson process the numbers of events occurring in non-overlapping intervals are independent and therefore $p_2(u) = p_1^2$. However if the rate of the process is conditioned by its history then the independence assumption is invalid and the correlation histogram $p_2(u)$ now becomes a non-constant function of lag u.

Suppose that $t_1 < t_2 < \ldots < t_N$ are the ordered times of events in a sample occupying $[0, T]$ and let $u_{i,j} = t_j - t_i$ for $1 \le i \le j \le N$. The correlation histogram is constructed from the lags $u_{i,j}$ by first choosing a bin width, say β, and then constructing a family of bins with centres $u_k = k\beta$. The $N(N+1)/2$ lags are now distributed among the bins such that the lag $u_{i,j}$ falls within the k-th bin provided $u_{i,j} \in [u_k - \beta/2, u_k + \beta/2)$. Let $J(u_m)$ be the number of lags falling within bin $[u_m - \beta/2, u_m + \beta/2)$ then the columns of the correlation histogram take the values $J(u_1)$, $J(u_2)$, $\ldots$, $J(u_n)$. It is natural to take $\widehat{p}_1 = \frac{N(T)}{T}$, that is, the sample estimate of the intensity of the process is the number of events to occur in the interval $[0, T)$ divided by T, the duration of the interval. Cox and Lewis (1972) show that an unbiased estimate of $p_2(u)$ is asymptotically

$$\widehat{p}_2(u_m) = \frac{J(u_m)}{\beta T}.$$

Moreover, Brillinger (1975) shows that the sample variance of $\sqrt{\widehat{p}_2(u_m)}$ is $(4\beta T)^{-1}$ independent of u and $J(u)$.

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