Peer Evaluation and Social Networks

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Abstract

Peer evaluation, as a new alternative to traditional performance appraisal, has recently become very popular. This paper studies how the structure of a social network shapes the aggregate outcome and affects individuals' contributions while individuals' payoffs depend on a peer evaluation system. We design two allocation rules appropriate for different purposes to weigh individuals' opinions: simple weighted allocation rule (SWAR) and Myerson weighted allocation rule (MWAR), and analyze the equilibrium outcomes under these rules. We find that peer effects operate in a heterogeneous manner.

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1 Introduction

Peer effect is the influence of a social group on an individual. This subject can be found in many fields, especially in a workplace environment. An important character of peer effect is that individuals voluntarily form a group and decide whom they want to work with. Thus the peer members of the group and the relationships among peers should be decided endogenously. Many researches point out that an organization can be more efficient under the influence of peer effect because peer pressure may eliminate the "free-rider" problem, and make individuals collect information from multiple sources of a variety of organizational level. Kandel and Lazear (1992) modeled this feature into a theoretical analysis. In their model they showed organizations with a significant size could really suffer a great loss, mainly because of the free-rider problem, if lack of existence of evidence for peer pressure. Many business study reports also showed that by introducing peer monitoring, such as partnership arrangements, employers could actually enhance the achievement of employees in workplaces. Barron and Gjerde (1997) constructed an agency model of peer policing, by the introduction of mutual monitoring, and claimed an organization would easily get rid of the principal-agent problems. More extensions and applications to peer effect were widely studied in business and management researches. For example, feedback of colleagues could improve individuals' work skills. One type of feedback that has become popular in recent years is known as 360-degree feedback. It is a process of using multiple sources among the organization to evaluate workers (Greenberg and Baron, 2003). Peer rating system is another application of peer effect which often makes a positive result in an organization (Bernardin, Hagan and Kane, 1995). Dominick, Reilly and Mcgourty (1997) illustrated that peer effect raises productivity. Falk and Ichino (2006) showed a similar result; moreover, they found low-productivity workers were more sensitive to the behavior of peers. Other empirical evidences like Sacerdote (2001) showed that peer effect has an impact on college students' grades and the will to take part in the social group. Calvo-Armengol, Patacchini and Zenou (2005) showed more empirical evidences of peer effect through individuals' relationship.[1] However, In management view, there exists a tendency that group members exert less efforts on task as the size of group increases. This has been explained by **social impact theory (**Latane and Nida, 1980**):** The larger the society, the less each member is influenced by the social force acting on the society.

Among those applications of peer effects, peer evaluation, as a new alternative to traditional performance appraisal, has recently become very popular. Peiperl (1999) concluded several factors and conditions correlated with a successful peer evaluation. Peer evaluation is the concretization of peer

[1] The utility function in their model was

$$u_i(y, g) = ay_i - \frac{1}{2}cy_i^2 + d\sum_{j=1}^{n} g_{ij}y_iy_j.$$

where a, c, d > 0. The lastest term of untility fuction was the benefit (or loss) of peer effect.

effect. It captures two main concepts, the first is that information is exposed adequately; the second is the existence of rewards and punishments. This two hidden concepts are the fundamental of peer effect, and can not be abandoned for any issue related to it. Though the topics about peer effect or peer pressure were well documented in economics, psychology and management literatures, it still hardly to find a discussion to explore how these effects are generated. Some have described peer evaluation's growing use and its related psychological mechanisms, or its potentially high validity. Those studies have assumed the effects were exogenously determined and focused on the influence of the effects. In this research project we will propose a study on peer evaluation through a view of social networks. We aim to create a new model which can well interpret the characters of peer evaluation. Furthermore, we will study how a peer evaluation system is endogenously formed by the force of social networks.

Consider a society where its aggregate outcome, $f(\mathbf{e})$, depends on individuals efforts. Let $\mathbf{e}$ be a vector of individuals' efforts. Since efforts exerted are different among players, the story is how to fairly divide the outcome to each individual. One may suggest dividing the outcome according to each individual's effort. Indeed, this is the best method since it satisfies the condition of efficiency. However, it could be difficult to achieve due to the asymmetric information problem. An individual may have difficulty to observe others' efforts when she is not socially connected to those people. A free-ride problem would really damage the productivity of the society unless a monitoring system with extra cost is introduced. In our model a peer evaluation is treated as the peer effect for members of a social network. We study how a peer evaluation system may improve the efficiency. We develop two allocation rules in which an agent's evaluation is weighted by her position in the social network. Our model is allowed to explain many figures of equilibrium phenomenon happened in an organization. For instance, why the peer effect may be non-monotonic as the number of participants or links increases? Does the peer effect influences symmetrically or asymmetrically when a new member joins in or a new relationship is formed?

The main idea of position-weighted allocation rule comes from the spirit of self-monitoring and self-management. It follows a basic principal that an individual should be given more power when she is allowed to evaluate more people. We use the word *power* to describe the scale of resources that individuals have to allocate to others. Note that we give more "power" but not more "payoff" to an individual who evaluate many others. The purpose to design such a platform is to solve the asymmetric information problem[2].

This paper is organized as follows. In the next section, we present a peer evaluation model with exogenous social networks, and in Section 3, we study the Nash equilibrium and equilibrium outcomes. In Section 4, we consider a network formation game by using the concept of pairwise stability, and in Section 5 we give several extensions and discussions by changing the model's

[2]Coleman (1988) had proposed this idea in his paper. It is the first discussion of the role of social networks on free riders and altruism issues.

specifications. Section 6 concludes.

2 The model

Let $N = \{1, \ldots, n\}$ be a set of players and let i and j be two typical members of this set. $e_i \in [0, 1]$ denotes the player $i's$ level of effort and $\mathbf{e} = (e_1, \ldots, e_n)$ denotes an effort profile of all agents. Players are located in an arbitrary social network, which we present as a graph g. An interpersonal relationship between player i and player j exists if $g_{ij} = 1$, $g_{ij} = 0$ otherwise.[3] We say player j is player i's neighbor if $g_{ij} = 1$. Suppose each player chooses her own effort and the aggregate output of the society is a function of players' effort profiles $f(\mathbf{e})$. Agent $i's$ indirect utility function u_i is expressed as following,

$$u_i(\mathbf{e}; g) = f(\mathbf{e}) \cdot S_i(\mathbf{e}; g) - c(e_i), \tag{1}$$

where $S_i(\mathbf{e}; g)$ is a share of aggregate output for player i, and $c(e_i)$ is the cost of effort exerted by agent i. To simplify our model, we assume that

$$f(\mathbf{e}) = \sum_{i=1}^{n} e_i,$$

$$c(e_i) = \frac{1}{2} e_i^2.$$

The interpretation of a player's utility is quite intuitively. Every player

[3] g_{ij} and ij stand for the same thing in the rest of the paper.

gets her reward from the society according to her share of the total output and the amount of total output. A player also pays a cost to exert effort. We suppose that the marginal cost of effort increases as the effort continue to be exerted.

2.1 Allocation rules

First, we define player $i's$ share ratio $S_i(\mathbf{e}; g)$. S_i is a parameter to represent the social evaluation for player i. A proper evaluation parameter should consist of two sections: how is player $i's$ performance comparing to others, and how important the evaluator's opinion is placed. In our social network model, we focus on the latter question and develop a system to allocate an endowment power to each player. Let Y_i denote the positional weight of player i, and then share ratio can be expressed as

$$S_i(\mathbf{e}; g) = \sum_{j \in N_i} \left(Y_j(g) \cdot \frac{e_i}{\sum\limits_{k \in N_j} e_k} \right), \tag{2}$$

where Y_j presents a degree of power that player j is endowed, N_i denotes the set of player $i's$ neighbors, $N_i = \{j \mid g_{ij} \in g,\ j \in N\}$. Let $\sum_{i=1}^{n} Y_i(g) = 1$ and $\sum_{i=1}^{n} S_i(g) = 1$. Note the power a player endowed is irrelevant to her effort; it only depends on the player's position. We assume the proportion of player's effort can be observed by all of her neighbors. We use this share ratio system as a mechanism of peer evaluation to allocate the resources. The only question left is how we determine the endowment of power for

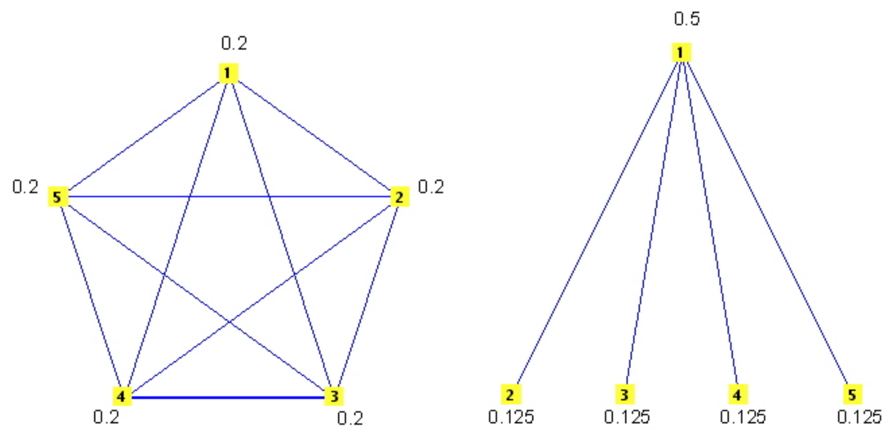

Figure 1: Case N=5. Numbers report the weight of players under simple weighted allocation rule.

players regarding to their social positions in the network. Next we provide two different approaches to endow the power for players.

2.1.1 Simple Weighted Allocation Rule (SWAR)

A simple method to decide the endowment distribution is to count the interpersonal links among players and claim that a player who has more connections should receive a larger endowment. There is a reason for justifying our setting. We think the more persons a player has to evaluate, the more power this player should be endowed. A simple way to catch our idea is to consider the width of interpersonal relationship of an individual. More specifically, to count how many neighbors an individual has. Express it as a ratio

$$Y_i^S = \frac{N_i^{\#}}{\sum_{i=1}^{n} N_i^{\#}}, \tag{3}$$

where N_i denote a set of neighbors that player i has, $A^{\#}$ denotes the cardinal of the set A. Therefore, the position weight of players in a network is simply determined by how many players that they direct link to. See figure 1 for an example.

Substitute (2) into (1), then utility functions for each player is

$$u_i(\mathbf{e}, g) = \left(\sum_{i=1}^{n} e_i\right) \cdot \sum_{j \in N_i} \left(Y_j^S \cdot \frac{e_i}{\sum_{k \in N_j} e_k} \right) - \frac{1}{2} e_i^2, \tag{4}$$

where Y_j^S is the power of player j is endowed, N_j denotes the set of players who are linked to player j, and N_i denote the neighbors of player i. Moreover, u_i is a strictly concave function for $e_i > 0$.

2.1.2 Myerson-Weighted Allocation Rule (MWAR)

The allocation rule in our model is rooted in Myerson (1976) and refined by Aumann and Myerson (1988). Originally, the idea in their studies was to compute the marginal contribution of a player who joined a network, under this computation we could well define each person's value in the networks, in other words, we could identify who among a group was more important to form a network by this value called **Myerson Value**. Since Myerson value stands for the benefit of the members contributed to a network, we use

Myerson value to be a description of weight of importance of the members of a network.[4] We will discuss this in detail in a later section.

Definition 1 *A path $p(ij)$ between players i and j in a network $g \in G$ is a sequence of players $i_1, ..., i_k$ such that $g_{i_k i_{k+1}} \in g$ for each $k \in \{1, ..., K-1\}$, with $i_1 = i$ and $i_k = j$.*

A path $p(ij)$ represents a possible social contact between player i and player j such that $i, j \in g$ and $i \neq j$. For example, the total number of paths in the network $g = \{12, 23\}$ is three, and the set of paths is $\{12, 23, 123\}$. We use this approach to sketch how much information flow is contained in a given network. This approach has another useful property, compare a network $g = \{12, 23, 13\}$ to $g' = \{12, 23\}$, without any computation readers can intuitively realize that the first network contain more information flow than the second one, further, player 2 is more important in the first network than the second one, since without player 2 the second network cannot create any information flow. Now we are going to define the importance in a network.

Definition 2 *A value function $v(g|s) = P(g|s)^{\#}$ is the number of total paths in network $g|s$, where $s \subset N$ and $P(g|s)$ is the set of paths in the network $g|s$, $P(g|s) = \{\ p(ij)|p(ij)$ is a path of $g|s\}$.*

We set a special allocation rule that each player has some resources to allocate to her neighbors. The resources, or the power, a player can have

[4] An alternative measurement was called *Bonacich Centrality*. By giving an exogenous information discount rate *Bonacich Centrality* measures who is the key person in a social network. For more discussions about this measurement, see Bonacich (1987); Ballester, Calvo-Armengol and Zenou (2006).

is determined by her importance in the social network. That is, the more important the agent in the network, the more resources he has to give to the others. We use Myerson value as an index of importance, recall that

$$y_i^{MV} = \sum_{s \subset N \setminus \{i\}} (v(g|s \cup \{i\}) - v(g|s)) \frac{s^{\#}!(n - s^{\#} - 1)!}{n!}. \quad (5)$$

The weight of each agents are

$$Y_i^{MV} \equiv y_i^{MV} / \textit{ total paths in } g.$$

Definition 3 *An agent in a network is "important" if she has a large Myerson value. That is, a network structure may lost many paths while the important agent is removed from this network.*

Note that in this rule, the most dedicative agent may not be the most important guy. The allocation law is as follow: while agents receive some resources, they review the effort exerted by their neighbor and give the resources to the one who is more hard-working. For instance, given a network $g = \{12, 23\}$, one may expect agent 2 will contribute zero effort, since he will certainly receive the resources from agent 1 and 3. Note that a player's payoff depends on two sections: neighbors' evaluation and the aggregate output. Next we can rewrite the payoff function as follows.

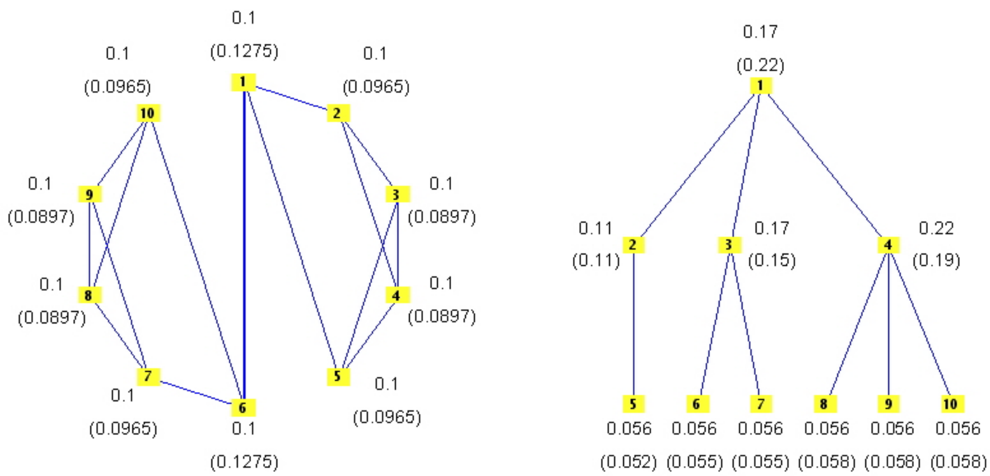

Figure 2: Case N=10. This case show the different weight of each players under two distinguishing rules.

$$u_i(g,e) = \sum_i e_i \cdot \sum_{j \in N_i} \left(Y_j^{MV} \cdot \frac{e_i}{\sum\limits_{k \in N_j} e_k} \right) - \frac{1}{2} e_i^2, \tag{6}$$

where Y_j^{MV} is the power endowed to player j who is a neighbor of i. In order to easy our discussion, we call a model based on simple weighted allocation rule SWAR, while call a model based on Myerson-weighted allocation rule MWAR in the rest of paper.

Compare this two allocation rules, they all have a common property: Players get more power if they direct link to more players. From a social point of view, this property is reasonable. A society may not want to endow too much power to an unsociable individual who has lesser information relatively. On the other hand, the difference between this two allocation rules need to more justify. Consider the case in figure 2 first,[5] each number denotes the weight of players under SWAR; number in parenthesis denote the weight of players under MWAR. In this case each player has three neighbors, hence every players yield the same weight under the setting of SWAR. However, under the setting of MWAR, player $\{1, 6\}$ are relatively important than others, player $\{2, 5, 7, 10\}$ are at the secondary position, then player $\{3, 4, 8, 9\}$ are at an unimportant position relatively. The factor of causing this difference is that Myerson Value emphasizes not only the relationship among players but what position players are at. Combine this two features we shall know another property under the setting of MWAR: Players get more power if they link to a powerful one. In figure 2 player $\{1, 6\}$ stand for a critical point since this organization would loss most information flow without the linkage between player 1 and 6. The power endowments for player $2, 5, 7, 10$ are larger than the endowments for player $3, 4, 8, 9$ because player $2, 5, 7, 10$ are linked to player 1 and 6 respectively. Sometimes this difference can yield the equilibrium results very dissimilarly.

It seems MWAR is more reasonable, however, a reasonable rule does not necessarily lead to an efficient equilibrium. Note that a player may dislike to be endowed a large share of power because the more power a player has, the lesser resources received from others possibly. Consider an individual who is endowed the entire power to allocate the resources, she then collects zero

[5]Thanks for the progress of computer science, we develope a program to deal with these annoying computation. For examples, the networks showing in figure 2 contains 633 paths; a network $g = (g^c - ij)$ with $N = 10$ contains 4055236 paths!

benefit from her neighbors. Thus there always exists a negative force against linkage.

3 Equilibrium efforts in peer evaluation networks

We analyze the characters of Nash Equilibria in a given network. We prove that the equilibrium effort profile is unique. Recall the payoff functions, each player solves the optimal problem:

$$\underset{e_i}{Max} : u_i(\mathbf{e}, g) = f(\mathbf{e}) \cdot S_i(\mathbf{e}; g) - c(e_i), \tag{7}$$

and evaluate the marginal benefit and marginal cost. An optimal profile $\mathbf{e}^*$ is a Nash Equilibrium if and only if $\mathbf{e}^*$ satisfies (7) for all i.

Consider a case that a player i is a member in a certain organizational structure, like a sport team or a project group. Player i can choose to be a free rider, or to work hard to improve her performance. Our purpose is to eliminate the incentive of idleness, and enforce members to contribute more. Consider S_i is any ratio irrelevant to g, say $S_i = \frac{1}{n}$, and simply set $u_i(e) = \frac{1}{N} \cdot \sum e_i - \frac{1}{2} e_i^2$, the optimal problem show players has no motivation to exert at a higher level, moreover, the optimal effort would goes toward zero while N is large. Or we could create a share ratio not necessarily equal among players, for instance, $S = \frac{e_i}{\sum e_i}$. The optimal problem shows that players have no incentive to contribute more. The point is that the punishment from team members is very weak, members could not punish the lazy workers through an official law, and the free-rider problem still could occur even in an organization introduced profit-sharing scheme.

Kandel and Lazear (1992) considered the peer pressure into the workers' optimal problem, and yielded a result that equilibrium effort was higher than it would be without peer pressure. Our pattern would not go beyond their studies, but we refined the past ideas into a computable platform, not just an abstract concept.

3.1 Networks and effort profiles

In our design players not only participate in production scheme but also participate in profit-sharing plan. The power of determining how much profit a worker should receive is belong to everyone. Hence, the mechanism of punishment is quite clear, once someone try to shirk, he would be punished by their neighbors seriously. In other words, this kind of platform provides an environment in which has strong force to encourage players to contribute more.

Players consider their position in this game and take it as a variable of utility. Since the payoff is relied on their effort as well as the neighbors' power, it is normal the effects of peer are heterogeneous among players. For example, player A is aside by a powerful player while player B is not, obviously player A has more incentive to work harder than player B. In this

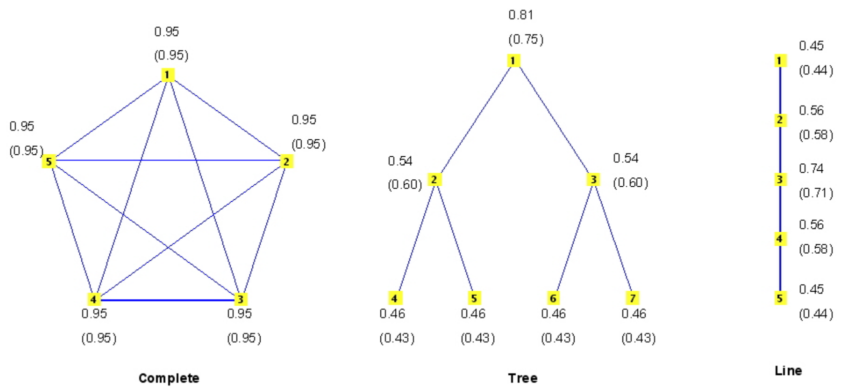

Figure 3: Equilibrium in different structures. Each number denote the equilibrium effort under SWAR; numbers in parenthesis denote the equilibrium effort under MWAR.

part we analyze the relationship between graph and equilibrium effort, under the convex setting one graph can only determine one set of equilibrium effort, it's convenient for our study.

Let's start with several cases. Figure 3 demonstrate the examples of equilibrium, in complete network SWAR and MWAR yield the same equilibrium since the weights are indifferent under these two rules. However, in tree and line structures the results are distinct when the allocation rule switched. Switch the allocation rule means impose another set of weight on the players, therefore players respond to this change and choose another set of effort to maximize their payoff.

We found in star networks structures, a player in center position was tend to exert more effort relatively than the peripheral under MWAR, further, this tendency was intense when the number of players increased.

Proposition 1 *Consider a star network g. Let S be the center player, E be a peripheral player. Under MWAR, S's equilibrium effort increases as the number of peripheral players increases, while E's equilibrium effort decreases as the number of peripheral players increases.*

Proof. Use the facts that in any star networks, Y_S^{MV} and Y_E^{MV} are decreased as n increases. From (6), the first derivation of S's payoff function with respect to Y_S^{MV} yield

$$e_s = (1 - Y_S^{MV}). \tag{8}$$

The above equation shows that agent S's effort increases as n increases. Similarly, the first derivation of $E's$ payoff function with respect to Y_S^{MV} yield

$$e_E = e_S \cdot Y_S^{MV} \cdot \frac{\sum\limits_{k \in N_S \setminus \{i\}} e_k}{\left(\sum\limits_{k \in N_S} e_k\right)^2} + Y_S^{MV} \tag{9}$$

Take the general derivation to (9), then we can find $\frac{de_E}{dY_S^{MV}} > 0$. Therefore, E's effort would decreased as n increases. ■

Star network structures are common in everywhere, this proposition reflects an important fact. Usually we do not wish to see an agent (worker, unit, etc.) at a hinge has not to engage in honest work because of its externality. Although our model ignore the externality, once concern this effect, the property we show would be very useful.

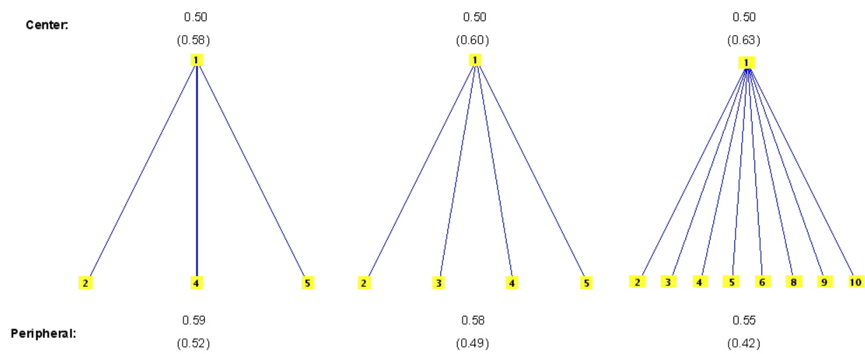

Figure 4: This example sketchs the property in star structures. Each number denotes the equilibrium effort under SWAR; number in parenthesis denote the equilibrium effort under MWAR.

Under SWAR the result was quite different, the center would exert $\frac{1}{2}$ and the peripheral exerted a level higher than center. The aggregate output under SWAR may be larger than the outcome under MWAR. Since we do not count the effect of externality in therefore this comparison is not absolute.

Sociologists and politicians may have an interest in if there is any possible that a group of agents would dedicate all selflessly, for any purpose. Complete networks provide the best environment to examine this debate, since in which contain the most information flows so that any active would be observed by everyone as well as the strongest punishment. Our model show that even in complete networks agents still try to shirk, but this phenomenon would be eliminated gradually as the size of network goes to infinity.

Proposition 2 *The equilibrium effort profile in complete networks converges to the maximum effort profile while* $n \to \infty$.

Proof. Given any complete network g^c, agent i's payoff can express as

$$u_i = \sum \left(e_j \cdot Y_j^{MV} \cdot \frac{e_i}{\sum e_k} \right) + \sum Y_j^{Mv} \cdot e_i - \frac{1}{2} e_i^2,$$

since in a complete network $Y_i^{MV} = Y = \frac{1}{n}$, the first order condition with respect to e are

$$e_i = \frac{n^2 - n - 1}{n^2 - n}.$$

Hence, no one would exert to his extreme unless n goes to infinity in a complete network structure. ■

Peer pressure is generated from two components, one is the punishment within the group, and the second is interpersonal position. Agents tend to shirk when they aside by the unimportant agents, and on the contrary, agents tend to work hard. An inter-linked star network has more productivity than other structures is expected because every agent is arranged aside by an important agent in a star network.

We suggest the force of punishment should be distinguished among players, and this force must be correlated with the position. For instance, Let us consider a star network with five agents. If $Y_i = \frac{1}{5}$ for all player i regardless to their positions, then the equilibrium output is 1. It is strictly smaller than the outcome when SWAR or MWAR is imposed. By the differentiation of network positions the different influences was done for individuals. It is obvious that links to an agent with high power is beneficial as well as means more

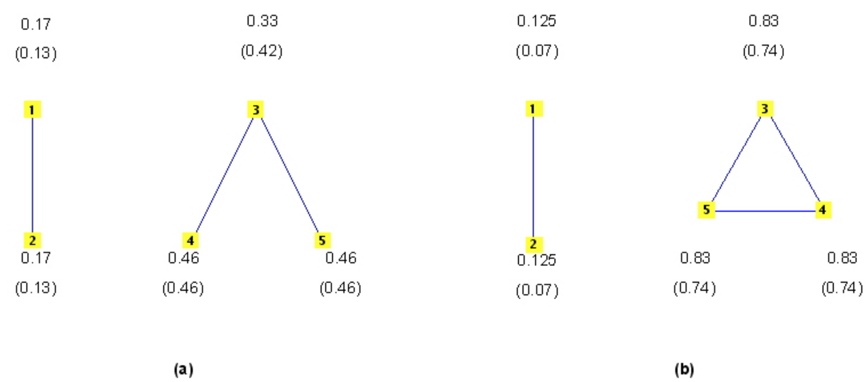

Figure 5: Equilibrium in different structures. Each number denote the equilibrium effort under SWAR; numbers in parenthesis denote the equilibrium effort under MWAR.

competition, then the competition would enhance the productivity. On the other side, a person stands aside by a low power one may lose some potential profit but faces a low level of competition. Thus there exists a trade-off between whether players want to form a link to a powerful one or not, if we lose the restriction of forming structures.

Divide a group into two components, one is competitive, another is not. In Figure 5, graph (a) shows players $\{1, 2\}$ in a less competitive component exert less effort than players $\{3, 4, 5\}$ in a more competitive component, in equilibrium. When the competition is enhanced in one of component, graph (b) shows the effect of this behavior done to players. As our discussion, players who face more competition tend to be more productivity, whereas the remainders shirk more.

3.2 Decomposition of effort

One of difficulties for studying peer effect is how to identify it. Rewrite the original model into

$$u_i = \sum_i a_{ij} Y_j \frac{e_i}{\sum\limits_{k=1}^{N} a_{jk} \cdot e_k} \cdot f(e) - \frac{1}{2} e_i^2,$$

if j is an unit of agent i's neighbors, or $j \in N_i$; if k is an unit of agent j's neighbors, or $j \in N_j$, then $a_{ij} = 1$; $a_{jk} = 1$. Otherwise, 0. Then the general form of optimal solutions are

$$e_i^* = \sum_j a_{ij} Y_j + \sum_j \left[a_{ij} Y_j \cdot \left(f(e) - \sum_k a_{jk} \cdot e_k \right) \cdot \frac{\left(\sum\limits_k a_{jk} \cdot e_k \right) - e_i}{\left(\sum\limits_k a_{jk} \cdot e_k \right)^2} \right].$$

That means the equilibrium effort can be decomposed into two components. The first item is irrelevant to effort profile but only relevant to g. The second item is the result after the interaction among the agents. Therefore, only the second item in which agents compete with their peers is the peer effect. We may call the first item the direct effect of peer evaluation, and call the second item the feedback effect generated from peer evaluation.

3.3 Existence and uniqueness of Nash equilibrium

Typically, the strategic set $\{\mathbf{e}\}$ is a nonempty, convex, compact set, and v is continuous in g, u_i is a concave function for e_i and is continuous in e_i. Therefore, by Kakutani's Fixed Point Theorem, the game $G = \{N, \mathbf{e}, \langle u_i \rangle\}$ has a Nash equilibrium. Rosen (1965) also forced another approach to prove that an N-person game with strictly concave payoff functions has an equilibrium point.

We also show the game G has a unique equilibrium point by Rosen's approach. In his notation, let $\varphi_i(\mathbf{e})$ be a payoff function of player i, where $\mathbf{e}$ is the strategic profile, and let $\mathbf{e}^0$ denote an equilibrium that satisfies

$$\varphi(\mathbf{e}^0) = \max(\varphi(\mathbf{e})|\mathbf{e} \in R). \tag{10}$$

Suppose a nonnegative mixed function $\sigma(\mathbf{e}, r) = \sum r_i \varphi_i(\mathbf{e})$, and $\theta(\mathbf{e}, r)$ is the gradients of $\sigma(\mathbf{e}, r)$. By definition a function $\sigma(\mathbf{e}, r)$ will be called diagonally strictly concave for $\mathbf{e} \in R^n$ and $r = \overline{r} > 0$, if for every $\mathbf{e}^0, \mathbf{e}^1 \in R$

$$(\mathbf{e}^1 - \mathbf{e}^0)'\theta(\mathbf{e}^0, r) + (\mathbf{e}^0 - \mathbf{e}^1)'\theta(\mathbf{e}^1, r) > 0. \tag{11}$$

Proposition 3 *There exists a unique Nash equilibrium $\mathbf{e}^0$ in the peer evaluation game if $\sigma(\mathbf{e}, r)$ is diagonally strictly concave and $\mathbf{e}^0$ satisfying* (11).

Proof. let $\mathbf{e}^0$ be an equilibrium strategic vector, then

1. $\theta(\mathbf{e}^0, r) = 0.$

2. If $\mathbf{e}^1$ is any other non-equilibrium strategic vector, since $u_i(\mathbf{e})$ is strictly concave for $\mathbf{e} \in R^n$, then $(\mathbf{e}^0 - \mathbf{e}^1)'\theta(\mathbf{e}^1, r) > 0$ is true. because

$$\begin{bmatrix} \triangle \mathbf{e}_1^0 & \triangle \mathbf{e}_2^0 & \dots & \triangle \mathbf{e}_n^0 \end{bmatrix} \begin{bmatrix} r_1 \nabla_1 u_1(\mathbf{e}_1^1) \\ r_2 \nabla_2 u_2(\mathbf{e}_2^1) \\ \dots \\ r_n \nabla_n u_n(\mathbf{e}_n^1) \end{bmatrix} > 0. \quad (12)$$

3. If $\mathbf{e}^1$ is another equilibrium strategic vector, then $\theta(\mathbf{e}^1, r) = 0$.

   Combine 1. 2. and 3 then we know that inequation (11) can not be satisfied if $\mathbf{e}^1$ be another equilibrium point, $\mathbf{e}^0$ is a unique equilibrium point in our model.

■

4 Network formation

This part is a discussion of network formation and stability. Following the definition of pairwise stable networks, Jackson and Wolinsky (1996) proposed that any network were called pairwise stable network if it satisfied these two features: (i) no any individual has an incentive to sever a link, and (ii) no pair of agents have an incentive to form a new link. That is, given a set of utility functions u, a network g is pairwise stable if

(i) for all $i, j \in g$, $u_i(g) \geq u_i(g - ij)$, and

(ii) for all $ij \notin g$, if $u_i(g+ij) > u_i(g)$ then $u_j(g+ij) < u_j(g)$.

Indeed, pairwise stability is a definition of stable networks but not the only one. In our model agents' consideration are limited so that they consider whether to form a link or sever a link at a time. This assumption may not be necessary.

4.1 Pairwise stability with costless links

Given a particular network agents can solve a set of effort and a set of payoff from the model. The dynamic process of network change can be thought as a series of comparative statistic. Once every agent could not improve their payoff by adjusting their network structure, we say this network is stable.

The incentive of forming links with others is that it is a channel to receiving profit. For more specifically, it is a channel to exhibiting how hard a player works and how much contribution she made. According to her contribution, a player will be rewarded differently. The drawback of forming links is that agents could be too powerful to receive more profit. Another drawback accompanies with its advantage, since the linkages play a role of information channel, therefore the cost of shirk is high and this pressure would push agents to exert at a higher level, so the cost of effort could be a heavy burden.

If agents freely choose their interpersonal relationship, what kind of network could be shaped? Let's go through some examples. Figure 6 exhibits a process of network formation from star network to complete one. Peripheral agents have improved their profit if they raised a new relationship with the others, in this case, they gain a 0.24 margin if they do so. Repeat this consideration we see all agents would stick together and all agents share the profit equally. In other words, no one would like to deviate from the complete network. Observe the payoff change in figure 6, there is no exist a pattern that every agents can improve simultaneously. That means, once someone benefited by the change of networks, someone must be loss at the same time, regardless the total profit is increased or not. The sense tell us that peer effect improve the performance by competition, rather than by coordination. Through the strategies thinking agents raise a new partnership or deviate from current relationship, and by this reallocate the resources. Peer effect provides another way to competitions, creates a high pressure environment and punish not only the people who shirk their duty but also person who isolate from the society. These phenomena are showed in every level of a social group, from schools to workplaces, and it often cause numerous problems and issues, since the allocation of resources are distorted twice, one is distorted by the difference of ability; and the second is distorted by the difference of relationship.

The property of complete networks is quite useful for improving the performance and avoids the drawback mentioned above since it's symmetric. Another property which is further important is stability. That means there is no any agent would like to break this structure, deviation is not a beneficial behavior, so that a group can maintain a well productivity and an acceptable

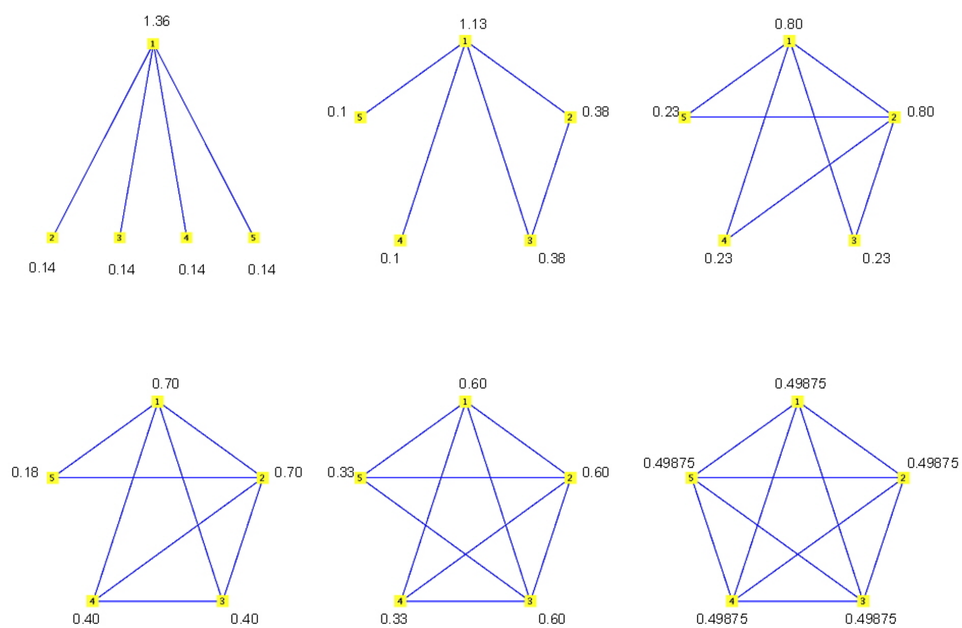

Figure 6: The examples of network formation. Each numbers denote the payoff under the MWAR.

distribution. Next we show the stability of complete networks in our model, which is under MWAR.[6]

Proposition 4 *A complete network is pairwise stable if utility function satisfy* (6)*.*

Proof. Consider a network $g = (g^c - ij)$. The benefit of agent i collects from j when $ij = 1$ is

$$e_i \cdot Y_j^{MV} \cdot \frac{e_i}{\sum e_k} + Y_j^{MV} \cdot e_i \doteqdot \frac{1}{n} + \frac{1}{n(n-1)}$$

since $e_i \to 1$ while $n \to \infty$.

On the other hand, the benefit collects from $J \notin \{i, j\}$ are

$$\left(\sum e_J \cdot Y_J^{MV} \cdot \frac{e_i}{\sum e_k} + e_i \cdot \sum Y_J^{MV}\right)_g \lesseqgtr \left(\sum e_J \cdot Y_J^{MV} \cdot \frac{e_i}{\sum e_k} + e_i \cdot \sum Y_J^{MV}\right)_{g^c}.$$

When n is large enough, the right hand side of the inequation can be expressed as $\frac{n-2}{n-1}$. Then assume $Y_J^{MV} = \frac{1}{n-1}$ and $\left(\frac{e_i}{\sum e_k}\right)_g \to \left(\frac{e_i}{\sum e_k}\right)_{g^c} = \frac{1}{n-1}$[7], the left hand side can be expressed as $\frac{n^2-n-2}{n(n-1)}$. Under these assumptions, the value of LHS is strictly greater than the value of RHS. That is

$$\frac{n^2 - n - 2}{n(n-1)} - \frac{n-2}{n-1} = \frac{n-2}{n(n-1)} < \frac{1}{n} < \frac{1}{n} + \frac{1}{n(n-1)}.$$

[6] The proof under Rule I is same as Rule II, so we skip it.

[7] Actually, $\left(\frac{e_i}{\sum e_k}\right)_g$ is slightly smaller than $\left(\frac{e_i}{\sum e_k}\right)_{g^c}$.

In the inequation above we see even under these strict assumptions agent i's benefit actually loss while he disconnects to j. Therefore, we show that complete network is a pairwise stable structure for all N. ■

Please note that complete network may not be the only stable one. Agents consider the trade-off of linkage, wider relationship may bring more accesses to receive rewards, but the competitive pressure also rises immediately. In a real world, not every person can take stress well. Some may decide to avoid the excess works and would rather to earn a lower pay. Heterogeneity of individuals' personality has not been measured in our model, but we can assume that individuals may have different maintain costs on their social links. An unsociable person should encounter a higher maintain cost, while a sociable will encounter a lower cost.

4.2 Stability in Peer Evaluation Networks (with cost of linkage)

In a lot of places or situations participators construct a relationship are not free, environment influence is a valuable issue to study. Any environment exist the barriers among the participators, some places are friendly to interact with others but some places are not. For example, enterprise culture that encourages cooperation, team work and knowledge sharing may provide a nice environment for interaction, whereas enterprise culture that emphasizes discipline and law may damp employee's zest to cooperate with others.

Besides, different organizational are designed to suit different purposes. A flat hierarchy tends to encourage individuals to work with peers, and a tall hierarchy may tend to emphasize personal performances.

In order to show this story, rewriting the cost function to

$$c(e_i, k) = \frac{1}{2}e_i + k \cdot N_i^{\#}$$

where k denote the cost of a link. Thus, k is an important parameter for a planner to adjust the network structures. Since k is irrelevant to $\mathbf{e}$, once the network is given, parameter k would not change agents' optimal decisions. In the consideration of forming or severing a link at a time, agents should compare the marginal benefit from forming or severing a link to marginal cost of a link, k. For instance, agent i's payoff in current network g with m neighbors is $u_i(g) - k \cdot m$, now he has an opportunity to build a new relationship with j, if he does it, the new payoff that he faces is $u_i(g + ij) - k \cdot (m + 1)$, hence, unless $u_i(g + ij) - u_i(g) - k > 0$, he would not build up this relationship. We define the a network with cost of linkage if

(i) for all $i, j \in g$, $u_i(g) - u_i(g - ij) \geq k$, and

(ii) for all $ij \notin g$, if $u_i(g + ij) - u_i(g) > k$ then $u_j(g + ij) - u_j(g) < k$.

Once we impose the cost of linkage to the model, the stable state would be changed according to the cost. Figure 7 demonstrates the stable area in complete networks. Low bound means the marginal benefit of an additional

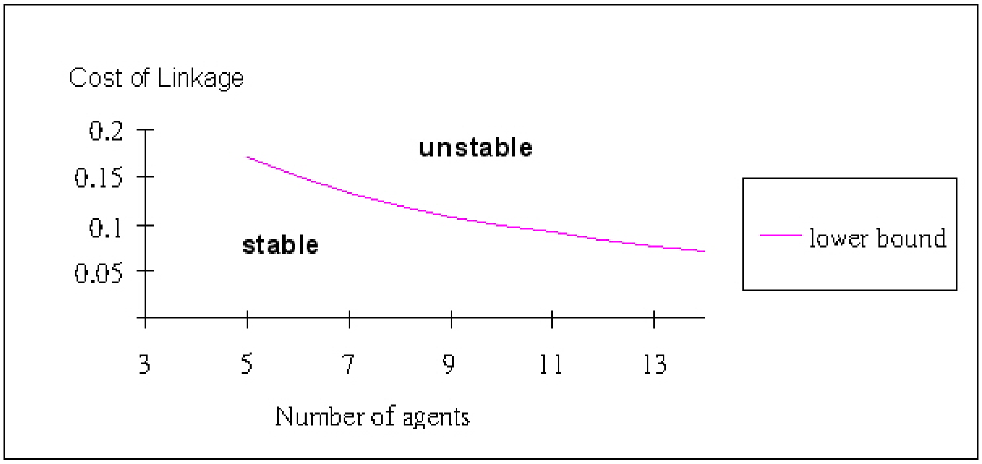

Figure 7: Stable area in complete network.

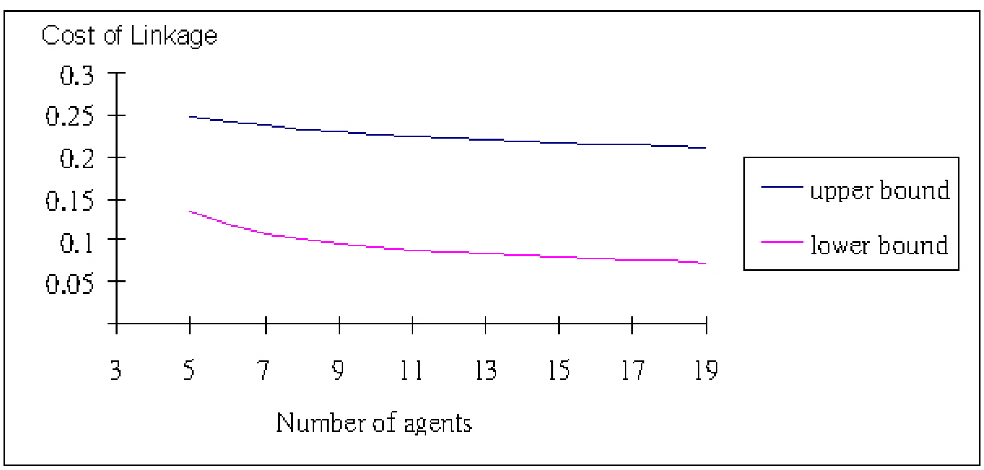

Figure 8: Stable area in Star network.

link to be a complete network, hence the cost of linkage higher than this margin would not sustain a complete network. But there is a possible situation that a network structure could not be stable no matter the cost of linkage. As Figure 8 shows, star network is an obvious example. Additional link between a pair of peripheral agents bring much benefit, but higher cost of linkage would damp their will to build up this relationship. If the cost of linkage higher than the benefit that generate from link, they would rather to stay in the current state. On the other hand, peripheral agents and center agent could not sustain their relationship if the cost of linkage is too high. Since peripheral agents face zero profit once they sever their link with center, the lower bound cost could not higher than their net profit at the current state. Therefore, there does not exist any cost of linkage that can sustain star network structure.

However, inter-linked star networks are sustained with some cost of linkage. Additional link between peripheral agents can enlarge much power for them, and yield more profit share. They could be viewed as a secondary center probably. But in inter-linked star networks there are already have too many center agents, benefits from additional link are more smaller then it would be in single star networks. That would damp the incentive to form an additional link. Figure 9 exhibits the stable area of inter-linked star networks with two cores. Another interesting result found in figure 9 is that, the multiple equilibria are possible. In the gray area complete networks and inter-linked star networks are both equilibrium structures. This property

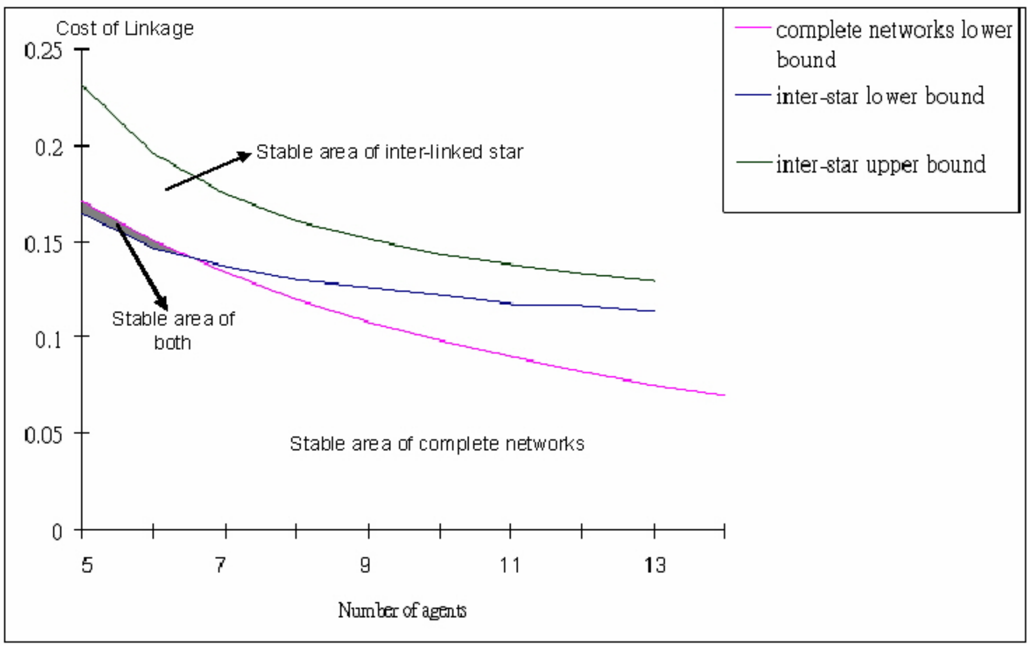

Figure 9: Stable area of multiple structures. Gray area stands for the stable areas of complete networks and inter-linked star networks that are overlaped.

very enriches our explanation of real world, many different structures exist in practice may due to their cost of linkage.

In addition, group size also is a natural binding of networks formation. In some cases, a graph is stable with cost of linkage within certain group size. Multiple stars structure are the good examples. In figure 10, we show a multiple stars structure with 2 cores is stable within $N = 10$ with cost ranged between 0.16 and 0.24, and unstable if group size goes beyond 10 agents. The same arguments show in 3 and 4 cores structures, their limits of size are 10 and 11 agents, respectively.

5 Discussion

5.1 Alternative allocation rule

Many arguments are about human's recognition about the knowing of the entire world. A usual discussion is that we human's perceptivity is very limited so that our knowing about the world is very partially. In the practical level, evaluation system faces the same problem, too. Evaluate all group members may not be a wise method. The relative problem is "who can evaluate whose performance?" Should it be someone who is socially connected to the object, or someone who is just a member of the society? Our model takes a local evaluative system in which agents can only evaluate their neighbors' performance. Oppositely, agents can evaluate all members' performance in a group no matter the relationship is called global evaluative system. Now we still

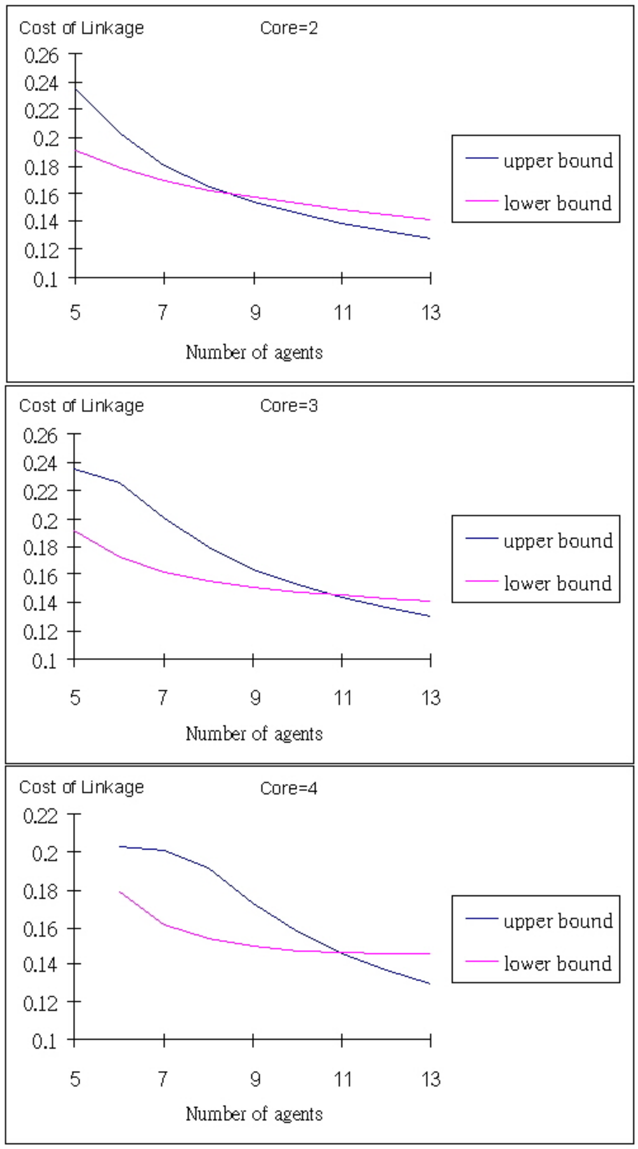

Figure 10: Stability in multiple stars networks with core= 2, 3,and 4.

follow the local evaluative system but release agents' knowing to the whole organization.

$$u_i(g,e) = \sum_i e_i \cdot \sum_{j \in N_i} \left(Y_j \cdot \frac{e_i}{\sum_i e_i} \right) - \frac{1}{2} e_i^2, \tag{13}$$

all symbols are same as before. In addition, assume all agents' effort can be revealed for everyone. In this setting agents consider their neighbors' contribution to the total output then agents' optimal decisions are

$$e_i = \sum_{j \in N_i} Y_j,$$

just equal to the sum value of their neighbors' power, and the net profit are

$$u_i = \frac{1}{2} \cdot \left(\sum_{j \in N_i} Y_j \right).$$

These values are smaller than the before values that were solved from ordinary model.[8] This result implies agents in this evaluative system would compete with all members, and reduce their motivation to exert effort. What is the intuition of this mathematical result? The reason is that force of punishment of shirk becomes weaker. Agents punish the slacker through share

[8]Since

$$e_i = \sum_j a_{ij} Y_j + \sum_j \left[a_{ij} Y_j \cdot \left(f(e) - \sum_k a_{jk} \cdot e_k \right) \cdot \frac{\left(\sum_k a_{jk} \cdot e_k \right) - e_i}{\left(\sum_k a_{jk} \cdot e_k \right)^2} \right] > \sum_j a_{ij} Y_j, i \neq j.$$

ratio, by giving a lower share ratio to add the shirk cost they prevent neighbors from dawdle. Once the system switches its platform to (13), the standard of evaluation transfers to the contribution to the total output instead of the contribution to the local output, therefore, the influential strength of share ratio is diminished so that agents would more tend to shirk.

Under the assumption of perfect information the model show that knowing too much to be good, that's why we prefer a local evaluative system. Practically, local systems are more executable and maintainable without perfect information since it's hard to believe that everyone's performance can be well revealed to everyone.

5.2 Logrolling

Peer evaluation is an evaluation system designed to improve performance that is different from traditional up-to-down evaluation system. However, logrolling can be a potential problem accompanying with this evaluation. Colleagues may make some agreements prior, giving favors among each others. In our model we prevent this problem through the share ratio. Since payment is according to appraising rate which is a proportion, a diligent worker could hardly make this agreement because his payoff will be shared by his colleagues who are more lazy than him.

Another similar problem is coalition. A part of workers may stick together to crowd out certain colleagues, by giving the imbalance rating. This is a typical issue reminded in many social studies. However, the power of profit distribution is belong to workers themselves in our design, any unfair behavior may generate revenge so that this problem isn't so serious probably. Of cause we cannot prevent coalition completely, since the power of checks and balances is relative to agents' position weight, obviously agents at the outlying location will lack ability against unfair behavior. Although it exists some kind of misgiving, the defence mechanism against logrolling in this model is still superior than usual peer evaluation system.

6 Conclusion

Empirical evidence shows that the peer effect has a strong impact on the society.[9] Crime, epidemic, discrimination, etc., are often associated with the peer effect. A key question for these complex interactions is: who is the most influential person? Our model allows us to define and explain the meaning of a powerful man.

We propose a peer evaluating platform based on the interpersonal relationship, in which agents are given the appraisal right as well as the allocation right. In traditional evaluating system agents are told to appraisal but actually the right of profit distribution is not belong to agents who really participant in job plan. Therefore we redesign it, in our model agents are told not only to appraisal but to determine profit distribution at the same time. The results show that peer effect is stronger and function well so that

[9] See Calvo-Armengol and Zenou (2004); Evans, Oates and Schwab (1996); Calvo-Armengol, Patacchini, and Zenou (2005).

the complete networks are stable. In addition, influence of peer effect is heterogeneous among agents with the different network structures, the outcomes that social group affects individuals can be described more precise.

More future works allow to extend. Because of the maneuverability of our model, we should confirm its practicability by doing experiments. Further, many settings in the model can be challenged. Heterogeneous agents analysis is worth further study, since networks approaches are very useful to investigate these problems, this paper could probably be a extendable tool for someone who investigate these topics.

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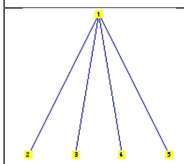
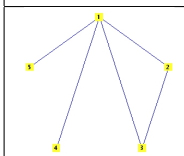
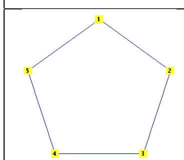
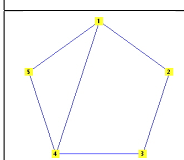
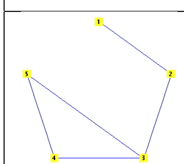
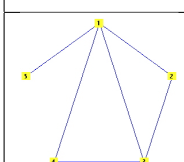
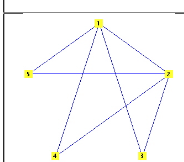
A Appendix: Case N=5.

Graph	Equilibrium Effort	Payoff
	{0.60,0.49,0.49,0.49,0.49}	{1.36,0.14,0.14,0.14,0.14}
	{0.87,0.75,0.75,0.43,0.43}	{1.13,0.38,0.38,0.10,0.10}
	{0.46,0.46,0.46,0.46,0.46}	{0.7,0.7,0.7,0.7,0.7}
	{0.90,0.67,0.67,0.90,0.69}	{0.68,0.33,0.33,0.68,0.31}
	{0.41,0.53,0.89,0.73,0.73}	{0.12,0.45,0.79,0.39,0.39}
	{0.98,0.70,0.91,0.70,0.36}	{0.95,0.29,0.59,0.29,0.08}
	{1.00,1.00,0.65,0.65,0.65}	{0.80,0.80,0.23,0.23,0.23}