The effects of a minimum wage increase in a model with multiple unemployment equilibria

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Abstract

We introduce the heterogeneity of labor in a simple imperfectly competitive aggregate labor market model “à la Manning (1990)” in order to analyze the effects of an exogenous rise of the legal minimum wage on the unemployment equilibrium, the wage dispersion and the general price level. We assume the presence of "knowledge spillovers" in the individual production function leading to increasing returns to scale at the aggregate level. This assumption involves the possibility of multiple equilibria in the model. Then, thanks to a static comparative exercise, we show that a rise in the legal minimum wage has no impact on the unemployment equilibrium (wherever the economy is stands), reduces the wage dispersion and increases the general price level. We also find that the larger the proportion of unskilled workers paid at the minimum wage in the total employment, the higher the increase in the general price level is. These results are broadly consistent with Card-Krueger's findings (1995).

Keywords: multiple unemployment equilibria, minimum wage, general price level.

JEL Classification: D43, E24, J24, J31.

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Introduction

The problem of minimum wage hikes has been actively discussed among economists. The primary goal of such government's intervention is to improve the welfare of low paid workers. However, many are those who think that an increase in the minimum wage leads also to employment losses for the workers at the bottom of the wage distribution as young workers or low-skilled workers. In their book, Card and Krueger (1995) argue that the data on the fast-food industry in U.S. states give no support to this idea. Machin and Manning (1994) analyze also the effect of the minimum wage cuts on employment in the U.K. and reach to the same conclusions. With these works, the debate has been revived as well in the academic area as in the political area.

At the theoretical level, the effect of minimum wage on employment has been analyzed especially with models like the efficiency wage model (Rebitzer and Taylor, 1995) or the monopsony model (Card and Krueger, 1995), i.e. models where the firm has a high power on wage determination. As mentioned by Cahuc and al. (2001), only few papers rely on models using wage bargaining with trade union; even though this way of wage determination appears as an important feature of labor markets. Furthermore, for several years, models with multiple equilibria have been used in order to explain the performance of labor markets. For example, Manning (1992) explains the British unemployment experience in the 1980s by the shift from one equilibrium to another.

According to these reports, the purpose of our paper is to examine the effects of minimum wage increase on labor market's performance in a multiple equilibria model where the wage negotiation takes place between a firm and a trade union. The model used in the paper is essentially the imperfectly competitive model of Manning (1990) in which we introduce the heterogeneity of labor and the presence of "knowledge spillovers" in the individual production technology. Indeed, considering two types of workers is essential when we want focus the analysis on the minimum wage, so the labor force employed by the firm consists of low-skilled workers who are paid at the minimum wage and high-skilled workers who are paid at a negotiated wage. In the Manning's model, the possibility of multiple equilibria is due to the presence of increasing returns to scale in the firm's production function, although this model's assumption has no strong theoretical justification. The assumption of the presence of "knowledge spillovers" is a convenient way to produce increasing returns to scale at the aggregate level while having constant returns to scale at the firm level. Indeed, for several years, knowledge has been considered as a fundamental source of increasing returns to scale and a determinant of the persistence of productivity and income differentials across economic agents of production (Romer, 1986). In our model, this "knowledge spillovers" pass by the average level of high-skilled labor employed in the economy due to the fact that the high-skilled labor is both the engine and the carrier of knowledge.

The paper is organized as follows. In the first section, we present the model (the price behavior of the firm and the wage determination) and the general symmetric equilibrium. Then, we explain under which condition the economy exhibits multiple equilibria and we prove the existence of them. In a third section, we run a static comparative analysis and we show that a minimum wage increase has no effect on the unemployment equilibrium, increases general price level and reduces wage dispersion. Finally, we compare our findings with those of Card and Krueger (1995).

1 The model

We use the Manning's model (1990) broadly based on Layard-Nickell (1985, 1986)[1], in which we introduce the heterogeneity of labor.

1.1 The price behavior of the firm and the demand of labor

The economy is made up of F identical imperfectly competitive firms and the firm i has a production function of the form[2]:

$$y_i = An_{1i}^{\alpha}n_{2i}^{\beta} \qquad \textit{with} \qquad \alpha + \beta = 1 \tag{1}$$

where n_{1i} represents its employment of high-skilled labor, n_{2i} its employment of low-skilled labor and A an efficiency parameter of labor taken as given by the firm i which is assumed to be a function of the average level of employment of high-skilled labor in the economy n_1:

$$A = n_1^{\alpha\sigma} \tag{2}$$

where $\alpha\sigma > 0$ represents the size of the knowledge spillovers and $\sigma > 0$ the degree of externalities. Thus, we assume that the average level of high-skilled labor used in the economy affects the output of firms. Given the fact that knowledge is mainly produced and spread by high-skilled labor force, considering that the labor efficiency depends on the average use of it in the economy is a relevant assumption. Given $\alpha + \beta = 1$, we have constant returns to scale at the firm level.

We can write the firm i's total demand of labor as:

$$n_i = n_{1i} + n_{2i} \tag{3}$$

Let $\gamma_i = n_{2i}/n_i \in \,]0;1[$ the proportion of low-skilled labor employment in the total employment for the firm i and $\theta_i = w_{2i}/w_{1i} \in \,]0;1[$ a measure of the wage dispersion between the high-skilled and low-skilled labor. The lower θ_i is, the

[1] Layard, R. and Nickell, S.J. (1985), "The cause of British unemployment", National Institute Economic Review, February, pp. 62-85.
Layard, R. and Nickell, S.J. (1986), "Unemployment in Britain", Economica, Vol. 53, pp. S121-70.

[2] As in Manning (1990), the number of firm is assumed fixed and the capital is excluded for simplicity.

larger the gap between the low-skilled worker's wage and high-skilled worker's wage is. Consequently, the total wage cost of the firm i can be written as a function of the high skilled worker's wage w_{1i}, the total labor demand and the two parameters defined above :

$$w_i n_i = [1 + (\theta_i - 1)\gamma_i]\, w_{1i} n_i = C(\gamma_i, \theta_i) w_{1i} n_i \qquad , C(\gamma_i, \theta_i) \in\,]0;1[\tag{4}$$

and the production of the firm i as a function of n_i and γ_i:

$$y_i = AB(\gamma_i) n_i \qquad\qquad where \qquad B(\gamma_i) = (1-\gamma_i)^{\alpha} \gamma_i^{\beta} \tag{5}$$

The demand of the firm i's output is assumed to be given by:

$$y_i^d = \left(Y^d/F\right)(p_i/P)^{-s}, \qquad s > 1 \tag{6}$$

where Y^d is the total aggregate demand, p_i the firm i's price, P the general price level and s the demand elasticity in the good produced by the firm i[3].

The real profit of the firm i can be written as:

$$\pi_i/P = (p_i/P)\, y_i - C(\gamma_i, \theta_i)(w_{1i}/P) n_i \tag{7}$$

Each firm i chooses p_i in order to maximize its real profit. We assume also that it treats the real wage of high-skilled workers as given, produces exactly the amount of demanded product and supposes that it is too small to affect the aggregate price level and the average employment of high-skilled workers. Thus, it solves the following program:

$$\begin{array}{ll} \underset{pi/P}{Max} & \pi_i/P \\ & \\ s.t & y_i = AB(\gamma_i) n_i \\ & y_i^d = \left(Y^d/F\right)(p_i/P)^{-s} \\ & y_i = y_i^d \\ & w_{1i}/P \qquad given \end{array}$$

The first order condition of this program gives us the following partial equilibrium pricing equation:

$$(p_i/P) = \left(\frac{s}{s-1}\right) \frac{C(\gamma_i, \theta_i)}{AB(\gamma_i)} \left(\frac{w_{1i}}{P}\right) \tag{8}$$

Thus, we can see that the price fixed by the firm is an increasing function of labor cost. Then, we use the equations (5) and (6) in (8) to obtain an employment equation corresponding at the labor demand of the firm i:

$$n_i^* = \left[\left(\frac{s}{s-1}\right) C(\gamma_i, \theta_i)\right]^{-s} [B(\gamma_i)]^{s-1} \left(Y^d/F\right) \left(\frac{w_{1i}}{P}\right)^{-s} \tag{9}$$

It is downward sloping curve in the high-skilled labor real wage-employment space. Thus, it has the common features found in the literature.

[3] This specification of the demand function is derived from CES preferences (see Blanchard and Kiyotaki (1987), and Julien and Sanz (2007) for detailed calculations).

1.2 The wage determination at the firm level

The wage negotiation is decentralized. The low-skilled labor wage corresponds to the legal minimum wage. It is fixed by law and not negotiated between the firm and the trade union. The negotiation concerns just the high-skilled labor wage w_{1i}.

The trade union has no insider's behavior. It takes into account the welfare of all employees in the firm even the low-skilled employees for which the wage is fixed exogenously. Its utility function is given by:

$$\begin{aligned} V\left(w_i/P, n_i\right) &= \left[\left(\frac{w_i}{P}\right) - w_R\right] n_i \\ &= \left[C(\gamma_i, \theta_i)\left(\frac{w_{1i}}{P}\right) - w_R\right] n_i \end{aligned} \tag{10}$$

where w_R is the reservation wage which is exogenous at the firm level. The firm utility is represented by its real profits.

The high-skilled labor wage is negotiated in order to solve the following Nash bargaining program:

$$\begin{array}{ll} \underset{(w_{1i}/P)}{Max} & [\pi_i/P]^{(1-\rho)}\left[\left\{C(\gamma_i,\theta_i)\left(\frac{w_{1i}}{P}\right) - w_R\right\} n_i\right]^{\rho} \\ s.t & n_i = n_i^* \\ & w_R \quad given \end{array}$$

Where $\rho(1-\rho) \in]0;1[$ represents the negotiation power of the union (of the firm). The first order condition of this program give us the high-skilled labor real wage that results from this negotiation:

$$\left(\frac{w_{1i}}{P}\right) = \frac{\chi\left(\rho, s\right)}{C(\gamma_i, \theta_i)} w_R \qquad with\ \chi\left(\rho, s\right) = \left(\frac{\rho}{s-1}\right) + 1 \geq 1 \tag{11}$$

Where χ is the mark-up. This wage equation is traditional and says that the real wage is marked-up over the reservation wage. We now turn to the general equilibrium of this economy.

1.3 The general equilibrium

At the general symmetric equilibrium, all the firms are identical. Thus, we have $p_i{=}P,\ w_{1i}{=}w_1,\ n_{1i}{=}n_1,\ n_{2i}{=}n_2,\ n_i{=}n,\ y_i{=}y{=}Y/F{=}Y^d/F\ldots$

The aggregate production becomes:

$$Y = Fy = F(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta} n^{(1+\alpha\sigma)} \tag{12}$$

At the aggregate level, we have increasing returns to scale of labor whatever the size of knowledge spillovers in the economy. These increasing returns to scale are completely external to the firm but internal to the economy. In our case, they are created by the high-skilled labor employment of other firms. Thus, we have introduced strategic complementarities between firms in the form of a positive high-skilled labor externality.

Given the definition of unemployment rate $u = 1 - (Fn/\overline{N})$ where $\overline{N}$ represents the total active population in the economy, we insert (12) in (9) and we obtain the aggregate pricing equation (PS for price setting) which relates the aggregate real wage of high-skilled labor to the unemployment rate u and other variables:

$$\left(\frac{w_1}{P}\right)_{PS} = \frac{\left[(1-u)\,\overline{N}/F\right]^{\alpha\sigma}}{\frac{s}{s-1}\frac{C(\gamma,\theta)}{(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}}} \tag{13}$$

In order to determine the aggregate real wage equation, we need to model the reservation wage which is exogenous at the firm level. A convenient specification of it can be:

$$\begin{aligned} w_R &= u(B/P) + (1-u)(w/P) \\ &= u(B/P) + (1-u)C(\gamma,\theta)(w_1/P) \end{aligned} \tag{14}$$

Where u is the unemployment rate and B/P the real unemployment allocations which are assumed to be the same for all the workers and lower than minimum wage. We introduce (14) in (11) and we obtain the following aggregate real wage equation (WS for wage setting) which relates the real high-skilled labor wage to the unemployment rate and exogenous variables as the real unemployment allocations:

$$\left(\frac{w_1}{P}\right)_{WS} = \frac{\chi\left(\rho,s\right)(B/P)u}{\left[1-\chi\left(\rho,s\right)(1-u)\right]C(\gamma,\theta)} \tag{15}$$

We can see that this expression admits a vertical asymptote in the space $(u,\frac{w_1}{P})$ when the unemployment rate gets close to $\underline{u} = \frac{\chi\left(\rho,s\right)-1}{\chi\left(\rho,s\right)} < 1$. This implies an inferior bound to the definition interval of the unemployment rate. We see that the larger the mark-up on the reservation wage, the higher the inferior bound of unemployment rate is.

2 Multiple equilibria

In order to show the existence of multiple equilibria in this economy, we first analyze the properties and shape of both aggregate equations. Secondly, we show graphically the different possible cases. Then, we prove the existence of multiple unemployment equilibria.

2.1 Equation's properties

For both equilibrium equations we calculate the first and second order derivatives to determine their shape, and theirs limits towards the bounds of the unemployment rate's definition interval.

2.1.1 The aggregate pricing equation

The first and second order derivatives of the aggregate pricing equation give us:

$$\frac{\partial\left(\frac{w_1}{P}\right)_{PS}}{\partial u}=(-\alpha\sigma)\frac{(\overline{N}/F)^{\alpha\sigma}}{\frac{s}{s-1}\frac{C(\gamma,\theta)}{(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}}}(1-u)^{(\alpha\sigma-1)} \tag{16}$$

$$\frac{\partial^2\left(\frac{w_1}{P}\right)_{PS}}{\partial u^2}=(-\alpha\sigma)(1-\alpha\sigma)\frac{(\overline{N}/F)^{\alpha\sigma}}{\frac{s}{s-1}\frac{C(\gamma,\theta)}{(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}}}(1-u)^{(\alpha\sigma-2)} \tag{17}$$

Proposition 1 *The aggregate pricing curve is always downward sloping in the space $(u,\dfrac{w_1}{P})$ whatever the size of the knowledge spillovers in the economy. If $\alpha\sigma\in\left]0;1\right[$ $(\alpha\sigma=1)$, the aggregate pricing curve is concave (linear) in the space $(u,\dfrac{w_1}{P})$; but if $\alpha\sigma>1$ it is convex in the space $(u,\dfrac{w_1}{P})$.*

Proof. We verify that the expression (16) is always negative when $\alpha\sigma>0$, and the expression (17) is negative (null, positive) when $\alpha\sigma\in\left]0;1\right[$ $(\alpha\sigma=1$, $\alpha\sigma>1)$. ■

Now, we investigate the limits towards the bounds of the definition interval of the unemployment rate. We have:

$\lim\limits_{u\to\underline{u}^+}\left(\dfrac{w_1}{P}\right)_{PS}\to\left(\dfrac{\overline{N}/F}{\chi(\rho,s)}\right)^{\alpha\sigma}\left[\dfrac{s}{s-1}\dfrac{C(\gamma,\theta)}{(1-\gamma)^{\alpha(1+\sigma)}\gamma^{\beta}}\right]^{-1}>0$

$\lim\limits_{u\to1^-}\left(\dfrac{w_1}{P}\right)_{PS}\to 0.$

which remains true whatever the size of knowledge spillovers in the economy

2.1.2 The aggregate wage equation

The first and second order derivatives of the aggregate wage equation give us:

$$\frac{\partial\left(\frac{w_1}{P}\right)_{WS}}{\partial u}=\frac{[1-\chi(\rho,s)]}{[1-\chi(\rho,s)(1-u)]\,u}\left(\frac{w_1}{P}\right)_{WS} \tag{18}$$

$$\frac{\partial^2\left(\frac{w_1}{P}\right)_{WS}}{\partial u^2}=\frac{-2\chi(\rho,s)}{[1-\chi(\rho,s)(1-u)]}\frac{\partial\left(\frac{w_1}{P}\right)_{WS}}{\partial u} \tag{19}$$

Proposition 2 *Since $\chi(\rho,s)>1$ and $u\in\left]\underline{u};1\right[$, the aggregate wage curve is downward sloping and convex in the space $(u,\dfrac{w_1}{P})$ whatever the size of the knowledge spillovers in the economy.*

Proof. We verify that the expression (18) is always negative and the expression (19) is always positive since $\chi(\rho,s)>1$ and $u\in\left]\underline{u};1\right[$. ■

For its limit's calculations, the size of knowledge spillovers has no effect.

We find that:

$$\lim_{u \to \underline{u}^+} \left(\frac{w_1}{P}\right)_{WS} \to +\infty \quad and \quad \lim_{u \to 1^-} \left(\frac{w_1}{P}\right)_{WS} \to \frac{\chi\left(\rho, s\right)\left(B/P\right)}{C(\gamma, \theta)} > 0.$$

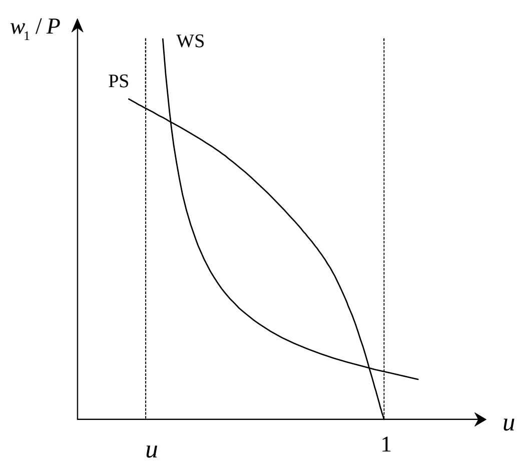
Thus, we have the following interesting result for the position of the two curves when u gets close to the bounds of it definition interval:

- When $u \to \underline{u}^+$, $\left(\frac{w_1}{P}\right)_{PS} < \left(\frac{w_1}{P}\right)_{WS}$: the PS curve is below the WS curve.
- When $u \to 1^-$, $\left(\frac{w_1}{P}\right)_{PS} < \left(\frac{w_1}{P}\right)_{WS}$: the PS curve is below the WS curve.

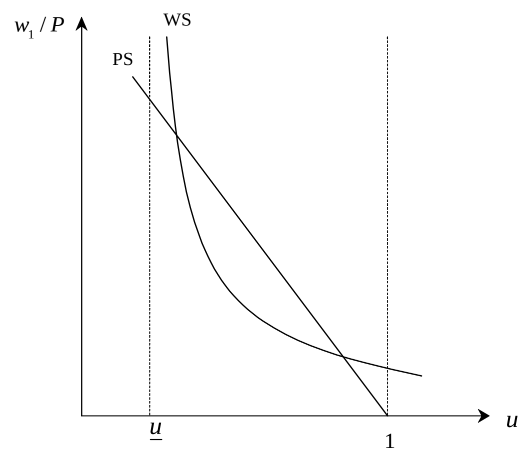
2.2 The different possible cases

Following the shape and limits of the curves towards the bounds of the definition interval of the unemployment rate, we can conclude that three different cases can occur in our model according to the size of the knowledge spillovers in the economy. In the three cases, the aggregate pricing curve stands always below the aggregate wage curve as we get close to the bounds of the definition interval. These different cases can be graphically represented in this way:

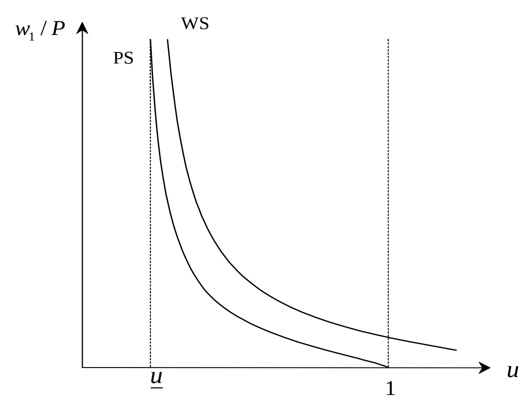
- If $\alpha\sigma \in]0;1[$: Diagram 1



- If $\alpha\sigma = 1$: Diagram 2



- If $\alpha\sigma > 1$: Diagram 3



The diagrams show us that it exists two equilibria if and only if $\alpha\sigma \in]0; 1]$, i.e. if the size of knowledge spillovers is not too large, more specifically if the returns to scale at the aggregate level are not above 2. If the size of knowledge spillovers is too important ($\alpha\sigma > 1$) and the returns to scale too large, no equilibrium exists. Consequently, in what follows we assume that the condition $\alpha\sigma \in]0; 1]$ is always satisfied and the case with no equilibrium is moved aside from the analysis.

2.3 Multiple unemployment equilibria

As we can see on the diagrams 1 and 2, both equilibria correspond to the intersections between the PS and WS curves.

Let $\Gamma(u) = \left(\frac{w_1}{P}\right)_{PS} - \left(\frac{w_1}{P}\right)_{WS}$, both equilibria correspond to the roots of this expression.

Proposition 3 *When $\alpha\sigma \in]0;1]$ the economy exhibits two distinct unemployment equilibria which belong to the interval $u \in]\underline{u};1[$.*

We demonstrate easily that the expression $\Gamma(u)$ is concave on the interval $u \in]\underline{u};1[$ when $\alpha\sigma \in]0;1]$. It reaches its maximum above the abscise axis and it cuts this axis twice in the interval $u \in]\underline{u};1[$ (for more detailed calculations see annex A).

According to the diagrams 1 and 2, we can characterize the two equilibria as follow: one named the "low equilibrium" with a high unemployment rate and a low high-skillde labor real wage $(u_L, \left(\frac{w_1}{P}\right)_L)$ and another named the "high equilibrium" with a low unemployment rate and a high high-skilled labor real wage $(u_H, \left(\frac{w_1}{P}\right)_H)$.

3 Effects of an exogenous increase in the minimum wage

In this section, we first analyze the effects of an exogenous increase in the minimum wage on the unemployment equilibrium and the high-skilled labor real wage thanks to a static comparative exercise. Then, we find that a policy that would raise the minimum wage would have no effect on the unemployment equilibrium and reduces the purchasing power of the high-skilled workers and this whatever the nature of the equilibrium. Later, we compare these findings with those of Card and Krueger (1995) and we find some similarities. The problem of equilibrium selection is not discussed.

3.1 Static comparative analysis

A policy that implements a rise in the nominal minimum wage is translated in our model in an increase in the parameter θ.

Proposition 4 *Whatever the nature of the equilibrium, an increase in the nominal minimum wage has no effect on the unemployment equilibrium. Conversely, it involves a decrease in the high-skilled labor real wage equilibrium that is larger when the economy is at the high equilibrium than when it is at the low equilibrium.*

We demonstrate using Cramer's rule on the equilibrium system composed of PS and WS curves that $\frac{du_L}{d\theta} = \frac{du_H}{d\theta} = 0$ and $\frac{d\left(\frac{w_1}{P}\right)_H}{d\theta} < \frac{d\left(\frac{w_1}{P}\right)_L}{d\theta} < 0$ (See the annex B for more details) .

To give an intuition to these results, we may say things in this way: The firm responds only to this exogenous increase in production cost by a rise in its own price. Indeed, the partial derivative of expression (8) shows us that the behavior of the firm following a such increase is to raise its price proportionally about its low-skilled employment γ_i[4]. Given the fact that the trade union weights in the same manner the welfare of both types of workers, it consents to a decrease in the real wage of workers who don't take advantage of this nominal wage rise in order to keep the employment level unchanged, so the negotiations lead to a lower high-skilled labor real wage. In this way, the labor demand of the firm remains unchanged as well as the unemployment equilibrium at the aggregate level.

We can also analyze this effect on the general price level by considering the shift of both curves. The partial derivatives of PS and WS curves give us:

$$\frac{\partial\left(\frac{w_1}{P}\right)_{PS}}{\partial\theta} = -\left(\frac{w_1}{P}\right)_{PS}\frac{\gamma}{C(\gamma,\theta)} < 0 \tag{20}$$

$$\frac{\partial\left(\frac{w_1}{P}\right)_{WS}}{\partial\theta} = -\left(\frac{w_1}{P}\right)_{WS}\frac{\gamma}{C(\gamma,\theta)} < 0 \tag{21}$$

Since $\left(\frac{w_1}{P}\right)_{PS} = \left(\frac{w_1}{P}\right)_{WS}$ at the equilibrium , both curves shift in the same extent and in the same direction. This is the result of an increase in the price set by the firms. The firm increases its own price which leads to a higher general price level at the general equilibrium and at a lower real wage for the high-skilled workers. The expressions (20) and (21) show also that the variation of the general price level depends on the proportion of low-skilled labor employment in the total employment γ, the larger this proportion, the more the increase of the general price level is. Thus, we can conclude that the increase in the price is large enough to cover the new production cost following an increase in the minimum wage.

An interesting feature of these results is also the fact that the implications of such policy on unemployment are the same whatever the equilibrium of the economy. Indeed, we have the same static comparative result at the low and high equilibrium. Consequently, the selection equilibrium analysis is not required here.

[4] The partial derivative of (8) gives us $\frac{\partial\left(\frac{p_i}{P}\right)}{\partial\theta_i} = \left(\frac{s}{s-1}\right)\frac{\gamma_i}{AB(\gamma_i)}\left(\frac{w_{1i}}{P}\right) > 0$

3.2 Comparisons with Card and Krueger's findings

Card and Krueger (1995) analyze the effects of such policy by comparing the labor market performances of the 50 US states[5] before and after the 1990 and 1991 increases in the federal minimum wage.

They identify different employment outcomes on the concerned group of workers affected by this rise in the minimum wage in different time periods and regions of country. There is compelling evidence that the estimated employment effects are insignificantly different from zero. These rises have no negative employment effect and even sometimes a positive employment effect on the concerned group of workers (Card and Krueger, 1995, pp-389). According this evidence, the rise in the minimum wage does not imply a necessary rise on unemployment as most models suggest.

They analyze also the effects of a higher minimum wage on prices in the fast-food restaurants industry which is the leading employer of low-wage workers. Their results show that *"the price increases of about the magnitude required to cover the higher cost of labor associated with the rise in the minimum wage"* (Card and Krueger, 1995, pp-390).

Another set of their empirical results is about the effect of a higher minimum wage on the distribution of hourly wages. They find that *"these increases in the federal minimum wage led to significant increases in wages for workers at the bottom of the wage distribution, and to a reduction in overall wage dispersion"* (Card and Krueger, 1995, pp-391).

The two first Card and Krueger's results support strongly our model's results. As for the effect of a higher minimum wage on the distribution of hourly wages, the similarities with our model are more intuitive. Indeed, we cannot show clearly that the rise in minimum wage increases the wage of workers at the bottom of wage distribution due to the discrete types of labor assumption. Nevertheless, our model allows us to conclude that such policy leads to a reduction in real wage dispersion since the real wage of high-skilled labor decreases and the real wage of low-skilled labor increases or remains unchanged[6].

The most important discrepancy between these empirical evidences and the common theory concerns the employment effect of a higher minimum wage. Therefore, Card and Krueger (1995) attempt to give a theoretical explanation of their findings by considering alternative models of labor market from the "textbook" model. They consider models where the wage is either taken by the firm (variants of the "textbook" model) or set by the firm (monopsony model[7]), but never models where the wage of workers paid more than the minimum wage results from a negotiation between the firm and a trade union. Machin and Manning (1994), who examine the impacts of a minimum wage decline on

[5] They divide the states into three groups: two where the wages are high and this increase has little or no effect and one where the wages are low and this increase has important effect.

[6] The effect of price increase on real minimum wage depend on the extent of this increase, but in all case it cannot be superior to the extent of nominal minimum wage increase.

[7] Machin and Manning (1994): *"A monopsonistic labor market is one in which an employer possesses some market power in setting wages, so that the supply of labor to the firm is a positive function of the wage paid."*

employment in U.K., find similar empirical evidences and attempt also to explain theoretically these effects by a monopsony model. Dickens and al. (1999) present a general theoretical model whereby employers have some degree of monopsony and in which minimum wage increases can have positive, neutral or negative effects on employment. Thus, our model can be also considered as an additional theoretical explanation of the effects of minimum wage increase or decrease on the unemployment.

Conclusion

In this paper, we have presented an imperfectly competitive model in which multiple unemployment equilibria can occur if the aggregate returns to scale of labor are not too high. Then, we show that a minimum wage increase has no effect on unemployment equilibrium but raises the general price level as a result of cost push inflation. Since these results come out irrespective to the nature of the equilibrium, the selection problem is not treated in this paper. Moreover, our analysis makes no allowance to variations in the stock of capital which would be certainly an agenda for further researches.

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ANNEX

Annex A. Proof of proposition 3

Let $\Gamma(u) = \left(\frac{w_1}{P}\right)_{PS} - \left(\frac{w_1}{P}\right)_{WS}$ defined on $u \in]\underline{u}, 1[$, with $\Gamma(u) \in C^2$. Then, we must show that $\Gamma(u)$ admits two distinct positive roots in the interval u when $\alpha\sigma \in]0;1]$ i.e. that $\Gamma(u)$ cuts twice the abscissa axis in the interval of $u \in]\underline{u};1[$ in the space $(u, \frac{w_1}{P})$.

$\Gamma(u)$ is a concave function on the interval $u \in]\underline{u};1[$ when $\alpha\sigma \in]0;1]$. Indeed, we have:

$$\frac{\partial^2\Gamma(u)}{\partial u^2} = \frac{\partial^2 \left(\frac{w_1}{P}\right)_{PS}}{\partial u^2} - \frac{\partial^2 \left(\frac{w_1}{P}\right)_{WS}}{\partial u^2} < 0$$

Furthermore, we have:

$$\lim_{u\to\underline{u}} \Gamma(u) \to -\infty \quad > \quad \lim_{u\to 1^-} \Gamma(u) \to -\frac{\chi(\rho,s)(B/P)}{C(\gamma,\theta)}$$

Now, we just verify that a part of the graph of the function $\Gamma(u)$ is above the abscissa axis in the interval of $u \in]\underline{u};1[$ when $\alpha\sigma \in]0;1]$, so we show that its maximum is positive and belongs to the interval of $u \in]\underline{u};1[$.

The polynom $\Gamma_u(u) = \frac{\partial \left(\frac{w_1}{P}\right)_{PS}}{\partial u} - \frac{\partial \left(\frac{w_1}{P}\right)_{WS}}{\partial u}$ is null when $\left[\frac{-\alpha\sigma}{(1-u)} - \frac{(1-\chi(\rho,s))}{[1-\chi(\rho,s)(1-u)]\,u}\right]\frac{w_1}{P}$ is null. This polynom has one positive root $\widetilde{u} = \frac{(1-\alpha\sigma)[\chi(\rho,s)-1] - \sqrt{\Delta}}{-2\alpha\sigma\chi(\rho,s)}$, with $\Delta = (\alpha\sigma - 1)^2 [\chi(\rho, s) - 1]^2 + 4\alpha\sigma\chi(\rho, s) [\chi(\rho, s) - 1] > 0$, which belongs to the interval $u \in]\underline{u}; 1[$. In addition, we have $\Gamma(\widetilde{u}) > 0$ when $\alpha\sigma \in]0; 1]$ and $\chi(\rho, s) > 1$. Thus, $\Gamma(u)$ admits two distinct positive roots in the interval u when $\alpha\sigma \in]0; 1]$ and $\chi(\rho, s) > 1$ and we have two distinct unemployment equilibria.

Annex B. Proof of proposition 4

Let $\left(\frac{w_1}{P}\right)_{PS} = \Psi(u, \theta)$ and $\left(\frac{w_1}{P}\right)_{WS} = \Phi(u, \theta)$, the high equilibrium $(u_H, \left(\dfrac{w_1}{P}\right)_H)$ is solution of this system:

$$\begin{cases} \Psi(u_H, \theta) - \left(\frac{w_1}{P}\right)_H = 0 \\ \Phi(u_H, \theta) - \left(\frac{w_1}{P}\right)_H = 0 \end{cases}$$

After total differentiation and arrangements, we obtain this system:

$$\begin{cases} \dfrac{\partial\Psi(u_H, \theta)}{\partial u_H} \dfrac{du_H}{d\theta} + \dfrac{\partial\Psi(u_H, \theta)}{\partial\theta} - \dfrac{d\left(\frac{w_1}{P}\right)_H}{d\theta} = 0 \\ \dfrac{\partial\Phi(u_H, \theta)}{\partial u_H} \dfrac{du_H}{d\theta} + \dfrac{\partial\Phi(u_H, \theta)}{\partial\theta} - \dfrac{d\left(\frac{w_1}{P}\right)_H}{d\theta} = 0 \end{cases}$$

The application of Cramer's rule and the properties of the PS & WS curves give us:

$$\frac{du_H}{d\theta} = \frac{\dfrac{\partial\Psi(u_H, \theta)}{\partial\theta} - \dfrac{\partial\Phi(u_H, \theta)}{\partial\theta}}{\dfrac{\partial\Psi(u_H, \theta)}{\partial u_H} - \dfrac{\partial\Phi(u_H, \theta)}{\partial u_H}} = \frac{[\Phi(u_H, \theta) - \Psi(u_H, \theta)] \frac{\gamma}{C(\gamma, \theta)}}{\dfrac{\partial\Psi(u_H, \theta)}{\partial u_H} - \dfrac{\partial\Phi(u_H, \theta)}{\partial u_H}}$$

$$\frac{d\left(\frac{w_1}{P}\right)_H}{d\theta} = \frac{\dfrac{\partial\Phi(u_H, \theta)}{\partial u_H} \dfrac{\partial\Psi(u_H, \theta)}{\partial\theta} - \dfrac{\partial\Psi(u_H, \theta)}{\partial u_H} \dfrac{\partial\Phi(u_H, \theta)}{\partial\theta}}{\dfrac{\partial\Psi(u_H, \theta)}{\partial u_H} - \dfrac{\partial\Phi(u_H, \theta)}{\partial u_H}} = \Phi(u_H, \theta) \frac{-\gamma}{C(\gamma, \theta)} < 0$$

At the equilibrium u_H, we have $\Phi(u_H, \theta) = \Psi(u_H, \theta)$. Therefore, an increase of minimum wage has no effect on unemployment equilibrium as $\dfrac{du_H}{d\theta} = 0$. The same argument give us the same result at the low equilibrium $(u_L, \left(\dfrac{w_1}{P}\right)_L)$.

Inversely, an increase in minimum wage leads to a decrease of $\left(\frac{w_1}{P}\right)_H$ which is more important when the proportion of low skilled worker γ is high. In the same manner, such policy leads to a decrease of $\left(\frac{w_1}{P}\right)_L$ when the economy is at the low equilibrium, but as $\Phi(u_H, \theta) > \Phi(u_L, \theta)$ this decrease is larger at the high equilibrium than at the low equilibrium.