Practical Monetary Policies

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Abstract

This paper investigates the theoretical implications of targeting average inflation or following a speed limit policy in a dynamic backward-looking model where monetary policy works with lags. Our findings reveal that the target horizon for expected inflation in the target rule must be correctly specified for the monetary policy strategies to achieve best results. Average inflation targeting dominates a speed limit policy for plausible values of society's relative aversion to inflation variability. The efficiency loss associated with average inflation targeting relative to optimal policy is very small if society values output stability. A speed limit policy becomes attractive only if society places great emphasis on inflation stability.

Over the past two decades, monetary policy in OECD countries has been spectacularly successful in reducing inflation from high and variable rates to relatively stable levels of around one to four per cent. The containment of inflation reflects the emergence of a general consensus that the ultimate goal of monetary policy is to achieve price stability. What price stability means in practice is open to interpretation, however. Some central banks like the Reserve Bank of New Zealand have a formal legislated mandate to keep inflation, defined as the percentage change in the CPI, within an announced target band. Other central banks such as the European Central Bank and the Bank of England aim at a specific target level for the percentage change in the price level, typically two percent or below. In the United States, the mission of the Federal Reserve Board is to achieve price stability without being bound by a formal inflation target.

There are also marked differences among central banks in the definition of the inflation target that monetary policy seeks to attain. A number of central banks, notably the Reserve Banks of Australia and New Zealand, define the inflation objective as maintaining low *average* inflation over the cycle or medium term.[1] The Bank of Canada seeks to maintain low average inflation over longer horizons but aims to keep inflation at two percent annually. The term "average inflation" does not appear in the definition of the policy objectives of other central banks such as the European Central Bank, the Bank of England or the Federal Reserve Board.[2]

Doubts about a consensus on the target variables of monetary policy have also been expressed by Walsh (2003). He questions whether the Federal Reserve's target variable - other than the rate of inflation - is the output gap proper. According to his interpretation of recent Fed policy, the Fed has pursued a *speed* limit policy, i.e. focused on the growth rate of actual output relative to growth of potential output in the design of monetary policy. Analyzing such a speed limit policy in a forward-looking model that also allows for output and inflation persistence, he finds that the

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[1] The Policy Target Agreement 2007 between the Minister of Finance and the Governor of the Reserve Bank of New Zealand stipulates that "[f]or the purpose of this agreement, the policy target shall be to keep future CPI inflation outcomes between 1 and 3 per cent on *average* over the medium term." In the description of its monetary policy framework, the Reserve Bank of Australia reports that "[i]n the Third Statement on the Conduct of Monetary Policy, issued in 2006, the Governor and the Treasurer agreed that the appropriate target for monetary policy is to achieve an inflation rate of 2-3 per cent on *average*, over the cycle, ….".

[2] The designers of the monetary policy strategy of the European Central Bank do emphasize, however, that the chief objective of policy is to maintain a rate of inflation close to (and preferably below) two percent over the *medium term* (Issing, Gaspar, Angeloni, and Tristani (2001)).

focus on the change in the output gap in the design of monetary policy is warranted. A speed limit policy dominates flexible single-period inflation targeting unless agents are predominantly backward-looking. Nessén and Vestin (2005) assess the performance of average inflation targeting in a model similar to Walsh's and find that it is superior to flexible single-period inflation targeting. The reason that both types of policies work well in the forward-looking set-up is self-evident. Optimal policy dictates that monetary policy be history-dependent in the sense that past information is essential for setting current policy. Both a speed limit policy and average inflation targeting introduce a dynamic element into the policy-setting process which in turn establishes a conduit through which the monetary authorities can affect the forward-looking inflationary expectations formed by agents. This expectations channel is not operative under flexible single-period inflation targeting. Söderström (2005) finds that a speed limit policy delivers a better stabilization performance than average inflation targeting in the standard forward-looking model upon which the current literature predominantly relies to analyze monetary policy strategies.

The workhorse model employed by Walsh (2003), Söderström (2005), and Nessén and Vestin (2005) is based on sound microeconomic underpinnings but has one potentially serious weakness. The model does not account for the lags in the transmission process of monetary policy.[3] The existence of these lags and their importance in the transmission process of monetary policy is widely acknowledged by central bankers, however. According to conventional wisdom, a change in monetary policy affects output after 12 to 15 months and inflation after 18 to 24 months. Thus in practice, the effect of a change in the policy instrument on the target variables of monetary policy is not contemporaneous, and the change in policy affects the real economy before it impacts on inflation.

Ball (1999) investigates the implications of such lags in the effect of monetary policy in the context of a simple backward-looking model.[4] According to his results, nominal income growth targeting, which is a special case of a speed limit policy, is a

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[3] The absence of transmission lags in the forward-looking model enables the central bank to exercise perfect control over aggregate demand. A disturbance in the goods market that displaces output can be totally offset by the appropriate adjustment of the nominal rate of interest.
[4] The literature on monetary policy of the 1950s and 1960 was acutely aware of the difficulties posed for policymakers by lags in the transmission process of monetary policy. See Culbertson (1960) and Friedman (1961). Svensson (1997) derives the target rules that underpin flexible and strict inflation targeting strategies in a backward-looking model that is essentially the same as Ball's (1999). Taylor (1994) employs a simpler variant of the backward-looking model, one where a change in the real rate of interest affects the output gap in the same period.

problematic monetary policy strategy as it causes the rate of inflation and the output gap to become excessively volatile. In sharp contrast, inflation-oriented policy strategies such as gradual and strict inflation targeting are efficient policies as they generate variances of the rate of inflation and the output gap on the policy frontier.

The current paper addresses two basic issues. First, it evaluates the performance of average inflation targeting and a speed limit policy – two candidate rules for monetary policy that describe actual policymaking - in a model similar to Ball's where the effects of policy take time. A change in monetary policy affects the output gap with a one-period lag and the rate of inflation with a two-period lag. The paper thus goes beyond merely embedding a dynamic monetary policy strategy into a framework where monetary policy effects occur contemporaneously, which is the norm in forward-looking New Keynesian models. Second, the paper focuses on the specification of target rules. A central finding is that the target rule underlying average inflation targeting and a speed limit policy, respectively, must conform to the policy lag structure imposed by the model. Choosing the appropriate target horizon for expected inflation in the target rule is absolutely essential for ensuring that both strategies live up to their potential to stabilize the economy and control inflation. This result stands in marked contrast to the benchmark case of optimal policy where the specification of the target horizon for expected (single-period) inflation is immaterial for the behavior of the target variables.

The performance of average inflation targeting and a speed limit policy, respectively, is analyzed from society's perspective. Society's welfare under each strategy is measured by the output-inflation variability trade-off and a simple numerical evaluation of expected losses. The paper finds that average inflation targeting *is* and a speed limit policy *can* be a sound strategy of monetary policy if agents are backward-looking. The undesirable consequences associated with a conventional speed limit policy – huge swings in the output gap and the rate of inflation - can be avoided if the target rule underlying a speed limit policy adheres to the two period lag between the policy instrument and the rate of inflation. Average inflation targeting does exceedingly well compared to a speed limit policy if society values stability of output. As society's aversion to inflation variability increases, however, average inflation targeting becomes less attractive. The exact opposite result holds for a speed limit policy. Thus society's relative aversion to inflation variability

is a critical element in determining the performance of the two strategies of monetary policy.

The remainder of the paper is as follows. Section 2 introduces a simple backward-looking model and discusses society's preferences. Section 3 analyzes optimal policy from society's perspective. Section 4 analyzes average inflation targeting and Section 5 discusses in detail a speed limit policy in the backward-looking model. The relative performance of either strategy vis-à-vis optimal policy is evaluated in Section 6. Section 7 concludes.

2. The Backward-Looking Model and the Policymaker's Preferences

The simple backward-looking model consists of two equations that describe the dynamic behaviour of the output gap and the rate of inflation:

$$y_t = -b\,r_{t-1} + l\,y_{t-1} + e_t \tag{1}$$

$$p_t = g\,p_{t-1} + a\,y_{t-1} + h_t \tag{2}$$

$$b > 0,\ 0 \le l\ < 1 \qquad a > 0,\ 0 \le g \le 1$$

y = the output gap (the difference between real output and its potential)

r = the real rate of interest

π = the rate of inflation

ε and η = white noise shocks with constant variances s_e^2 and s_h^2, respectively.

The backward-looking IS relation of equation (1) has two prominent features. First, the output gap exhibits persistence, with λ measuring the degree of persistence. Second, the output gap responds to a change in the real rate of interest with a one-period lag. Persistence and a lagged response are also critical elements in the Phillips Curve. According to equation (2), the current rate of inflation depends on the previous period's rate and reacts to the output gap with a one period lag.[5]

This lag structure of the model gives rise to the following relationship between the policy instrument and the two key variables of the model:

$$r_t \rightarrow y_{t+1} \rightarrow p_{t+2}$$

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[5] In Ball (1999) the parameter $g = 1$. As shown in the appendix, $g = 1$ results in an accelerating Phillips Curve which in turn accounts for the instability of nominal income growth targeting

The effect of policy on the economy and the rate of inflation takes time, with policy affecting the output gap sooner than the rate of inflation. Time is measured in years. This distinctive lag structure provides the model with a dose of realism. As pointed out in the introduction, there is general agreement in policy circles that the effects of changes in monetary policy are not instantaneous but set in with delay. The existence of transmission lags in turn has profound implications for the efficient operation of monetary policy strategies such as average inflation targeting and a speed limit policy. The target rules underlying both strategies need to conform to the notion that policy affects inflation with a two-period lag.

Society is concerned about the variability of the output gap and the rate of inflation in period t:

$$E[L_t] = V(y_t) + \mu V(\pi_t) \tag{3}$$

The objective is to minimize the above expected loss function where μ is society's aversion to inflation relative to output gap variability. Equation (3) represents society's loss function, which the policymaker minimizes under optimal policy.

3. Optimal Policy

Given the quadratic objective function, the policymaker follows a linear policy rule in the conduct of policy. This policy rule provides for a systematic relationship between the two targets of monetary policy. As policy works with lags, in the current period the policymaker chooses the expected output gap next period and treats the rate of inflation as predetermined:[6]

$$\theta E_t y_{t+1} + E_t \pi_{t+1} = 0 \tag{4}$$

The target rule embodied by equation (4) assumes that the target value for the output gap and the rate of inflation is zero, respectively. The parameter θ represents the weight that the policymaker places on the output gap relative to the rate of inflation when setting policy.

Combining the target rule in (4) with the IS and PC relations yields the policymaker's reaction function:

$$r_t = \frac{\gamma}{\theta\beta}\pi_t + \left(\frac{\alpha}{\theta\beta} + \frac{1}{\beta}\right)y_t \tag{5}$$

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[6] Svensson (1997) invokes dynamic programming to derive an equivalent linear target rule.

The reaction function specifies how the policymaker adjusts the policy instrument, the real rate of interest, in the wake of deviations of the output gap and the rate of inflation from target.[7] In the event of a one-percentage point rise (fall) in the rate of inflation, the policymaker raises (lowers) interest rates by $\gamma/\theta\beta$ percentage points. Similarly, following a one percentage rise (fall) in the output gap, the policymaker responds by raising (lowering) the interest rate by $\alpha/\theta\beta+\lambda/\beta$ percentage points.

Computing the variances of the target variables requires a few steps. First, backdate equation (5) by one period and insert it into the IS equation to obtain the reduced form equation of the output gap:

$$y_t = -\frac{g}{q}p_{t-1} - \frac{a}{q}y_{t-1} + e_t \tag{6}$$

Along with the Phillips curve, equation (6) can be set up in matrix form to calculate the variances of the rate of inflation and the output gap:[8]

$$V(y_t) = \frac{\left(2agq + q^2(1-g^2)\right)s_e^2 + g^2 s_h^2}{q^2 - (gq-a)^2} \tag{7}$$

$$V(p_t) = \frac{a^2 q^2 s_e^2 - \left(a^2 - q(2ag+q)\right)s_h^2}{q^2 - (gq-a)^2} \tag{8}$$

The variability of the output gap and the rate of inflation, respectively, does not depend exclusively on the extent of uncertainty on the supply side of the economy. Due to the one-period transmission lag between the policy instrument and output, the variance of the IS disturbance also affects the variance of both target variables. Inserting (7) and (8) into (3) and minimizing the loss function with respect to θ yields the optimal value of the policy parameter, the weight on the output gap in the target rule:

$$q^* = \frac{1-g^2 + a^2 m + \sqrt{4a^2 g^2 m + \left(-1+g^2-a^2 m\right)^2}}{2agm} \tag{9}$$

The optimal policy parameter θ^* depends on two parameters from the Phillips curve: a (the responsiveness of inflation to the lagged output gap) and g (the degree

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[7] As Ball (1999) we assume that the policymaker has complete control over the real rate of interest. This assumption is necessary to implement a given policy strategy successfully. Distinguishing between the nominal and real rate of interest will not provide any additional insights in the current paper.
[8] For further details see Hendry (1995, pp.111-112), Ball (1999) or Chapter 12 of Froyen and Guender (2007).

of persistence of inflation). θ^* also depends on society's preference parameter m. The higher the aversion to inflation variability, the lower θ^* is.[9]

By choosing values for $0 \leq m \leq \infty$ and picking representative values for a and g as well as the variances of the shocks, we can trace out the optimal policy frontier which depicts the trade-off between the variance of inflation and the variance of the output gap.[10]

Figure 1 underscores the importance of the degree of inflation persistence in determining the location and shape of the tradeoff between output and inflation variability.[11] For $g = 0.6$ the policy frontier is more compact and closer to the origin than for $g = 0.9$.

In the following two sections, we examine the properties of average inflation targeting and a speed limit policy. Both are considered plausible strategies for policymaking in practice. Our objective is twofold. First, we wish to assess whether a policy that focuses in part on past inflation is superior to a policy that focuses in part on the lagged output gap. Second, we want to determine how either policy compares to optimal policy. Specifically, we want to measure the efficiency loss that average inflation targeting and a speed limit policy impose on society.

4. Average Inflation Targeting

This section examines the performance of average inflation targeting (AIT). Average inflation is defined over two periods:

$$\overline{p_t} = \tfrac{1}{2}(p_t + p_{t-1}) \tag{10}$$

The policymaker's objective is to minimize the unconditional variances of both average inflation and the output gap, with m^{AIT} denoting the policymaker's relative aversion to average inflation variability:

$$E[L_t]^{AIT} = V(y_t) + m^{AIT} V(\overline{p}_t) \tag{11}$$

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[9] As $m \to \infty, q \to a/g$. Thus even if the policymaker cares only about the variability of inflation, he still puts some positive weight on the output gap in setting policy. The weight on the output gap remains positive because the policymaker can affect the output gap, which affects the rate of inflation, sooner than inflation proper. In the opposite case where the policymaker cares only about output variability the weight on the output gap in the target rule becomes infinitely large: $q \to \infty$ as $m \to 0$.
[10] The parameter values $\alpha = 0.4$, $\sigma_\varepsilon^2 = 1$ and $\sigma_\eta^2 = 1$ are taken from Ball (1999). In addition, we choose $\gamma = 0.9$.
[11] Both policy frontiers are based on values of the preference parameter m that range from 0.01 to 60.

The target rule underlying average inflation targeting is:

$$q^{AIT} E_t y_{t+1} + E_t \overline{p_{t+2}} = 0 \qquad (12)$$

Given the two-period lag between the interest rate in period t and the rate of inflation, the target for average inflation is expected average inflation in period $t+2$ rather than period $t+1$. Grounding policy on expected average inflation two periods into the future avoids the inclusion of the current rate of inflation in the target rule. As a rule, the inclusion of contemporaneous information in the form of current inflation in the target rule (or the current output gap in the case of a speed limit policy) complicates the determination of the optimal policy parameter.[12]

Combining the above rule with the IS and Phillips curve equations and making use of equation (10) yields the response of the policy instrument to the rate of inflation and the output gap under average inflation targeting:

$$r_t = \frac{g(1+g)}{(a+2q^{AIT})b} p_t + \left(\frac{a(1+g)}{(a+2q^{AIT})b} + \frac{l}{b} \right) y_t \qquad (13)$$

Substituting (13) back into the IS relation and following the aforementioned solution procedure yields the variances of the output gap, the rate of inflation and average inflation:[13]

$$V(y_t^{AIT}) = [1 + \frac{a^2(1+g)}{4q^{AIT}(a+q^{AIT}(1-g))}]s_e^2 + \frac{g^2(1+g)}{4q^{AIT}(a+q^{AIT}(1-g))}s_h^2 \qquad (14)$$

$$V(p_t^{AIT}) = \frac{a^2(a^2+4q^{AIT}(a+q^{AIT}))s_e^2 + (a^2g^2+4q^{AIT}(a(1+g(1+g))+q^{AIT}))s_h^2}{4(1+g)q^{AIT}(a+q^{AIT}(1-g))}$$

$$(15)$$

$$V(\overline{p}_t^{AIT}) = [\frac{a^2(a+2q^{AIT})}{4(a+q^{AIT}(1-g))}]s_e^2 + \frac{[2(a(1+g)+q^{AIT})+ag^2]}{4(a+q^{AIT}(1-g))}s_h^2 \qquad (16)$$

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[12] The target rules presented in this and the following section target the inflation rate at time $t+2$ because they dominate alternative specifications of the target rules from a welfare maximizing perspective. The inferior target rules, which contain the expected rate of inflation in period $t+1$, are briefly discussed in the appendix. The appendix also shows that the evolution of expected inflation becomes more complex if contemporaneous information enters the target rule. The added complexity makes the derivation of the optimal policy parameter more difficult.
[13] The acronym AIT is added as a superscript to emphasize that equations (14) – (16) represent the variances of the output gap, the rate of inflation, and average inflation under average inflation targeting.

Inserting (14) and (16) into (11) and minimizing the loss function with respect to θ^{AIT} results in the optimal policy parameter under average inflation targeting:

$$q^{AIT} = \frac{1-g+\sqrt{(1-g)^2+a^2 m^{AIT}}}{am^{AIT}} \qquad (17)$$

Given the solution for q^{AIT}, we can trace out the policy frontier ground out by average inflation targeting. How this policy frontier compares to the policy frontier under optimal policy or a speed limit policy will be discussed further in Section 6.

5. Speed Limit Policy

As stated in the introduction, the speed limit is shorthand for the change in the output gap, and is defined as $y_t - y_{t-1}$. Accordingly, under a speed limit policy (SL) the policymaker's objective is to minimize the weighted sum of the unconditional variances of the change in the output gap and the rate of inflation:

$$E[L_t]^{SL} = V(y_t - y_{t-1}) + m^{SL}V(p_t) \qquad (18)$$

m^{SL} = the weight the policymaker accords to the variance of inflation relative to the output gap in the objective function.

Again taking proper account of the transmission lag, we specify the target rule for a speed limit policy as:

$$q^{SL}[E_t y_{t+1} - y_t] + E_t p_{t+2} = 0 \qquad (19)$$

Combining the IS and Phillips Curve equations with the target rule (19) yields the response of the policy instrument to the rate of inflation and the output gap under a speed limit policy:

$$r_t = \frac{g^2}{(a+q^{SL})b}p_t + \left(\frac{ag-q^{SL}}{(a+q^{SL})b} + \frac{1}{b}\right)y_t \qquad (20)$$

Comparing equation (20) with equation (5), the reaction function under optimal policy, we find that the coefficients on π_t and y_t in (20) are most likely smaller.[14] Indeed the coefficient on the output gap in (20) is not unambiguously positive. Its sign depends on the size of the model parameters α and g and the policy parameter θ^{SL}.

Backdating equation (20) by one period and inserting it into the IS equation (1) reveals the behaviour of the output gap under a speed limit policy:

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[14] A direct comparison is not possible as $q^* \neq q^{SL}$ A definitive answer can be given if numerical values are assigned to the parameters and variances of the shocks. This is done later in the paper.

$$y_t = -\frac{g^2}{a+q^{SL}}p_{t-1} - \frac{(ag-q^{SL})}{a+q^{SL}}y_{t-1} + e_t \qquad (21)$$

Combining this equation with the Phillips curve (2), we can calculate the variances of the output gap (y_t), inflation (π_t), and the change in the output gap $(y_t - y_{t-1})$:

$$V\left(y_t^{SL}\right) = \frac{(a+q^{SL})^2[a(1+g^2)+q^{SL}(1-g(1+g-g^2))]s_e^2 + g^4(q^{SL}(1+g)+a)s_h^2}{aD} \qquad (22)$$

$$V\left(p_t^{SL}\right) = \frac{a(q^{SL}+a)^2(a+q^{SL}(1+g))s_e^2 + [(1+g^2)(a(a+q^{SL}(3+g)))+2q^{SL^2}]s_h^2}{D} \qquad (23)$$

$$V\left((y_t-y_{t-1})^{SL}\right) = [1+\frac{a(a(1+2g(1+g))+q^{SL}(1+g))}{D}]s_e^2 + \frac{2g^4 s_h^2}{D} \qquad (24)$$

$$D = (a+q^{SL}(1-g))(a+2q^{SL}(1+g))$$

Inserting (23) and (24) into (18) and minimizing the expected loss function with respect to θ^{SL} yields the optimal value of the policy parameter under a speed limit policy. The analytical solution turns out to be rather complex and unwieldy. However, its general form is as follows:

$$q^{SL} = f(a,\ g,\ m^{SL}, s_e^2, s_h^2). \qquad (25)$$
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The (-) (+) sign denotes the effect of an increase in the size of the parameter or variance on the size of the policy parameter.

The distinctive feature of θ^{SL} is that it depends on the variances of demand and cost-push shocks (σ_ε^2 and σ_η^2) while θ^* and θ^{AIT} do not. Thus, under a speed limit policy the origin of the disturbance influences the setting of the policy parameter. Additionally, the policy-setting process becomes more complicated because not only does a speed limit policy depend on contemporaneous information (as the current output gap appears in the definition of the speed limit in (19)) but the policymaker also relies on less information: the expected *change* in the output gap – and not the expected output gap proper - guides a speed limit policy. This point is discussed further in the next section.

6. Performance of Strategies: An Assessment

Of central concern in this section is the attractiveness of average inflation targeting and a speed limit policy, respectively, vis-à-vis optimal policy. More precisely, to what extent does a policymaker who implements a monetary policy rule that differs from the one society finds most desirable achieve optimal results?[15]

We approach this question from two different angles, using graphical and quantitative performance measures. The first performance criterion is the output-inflation variability tradeoff while the second is a quantitative evaluation of society's welfare losses under average inflation targeting and a speed limit policy, respectively, relative to optimal policy.

Figure 2 clearly establishes the fact that average inflation targeting dominates a speed limit policy in the backward-looking framework over a sizeable range of the preference parameters.[16] A speed limit policy does very poorly in case the policymaker cares very little about inflation stability. For low values of m^{SL} a speed limit policy keeps fluctuations in the *change* of the output gap at bay but only at the expense of large fluctuations in both the output gap and the rate of inflation. In sharp contrast average inflation targeting is far more efficient for low relative weights on the variance of average inflation.[17] The policy frontier traced out by average inflation targeting lies below the policy frontier under a speed limit policy unless the policymaker places almost exclusive emphasis on minimizing inflation variability. Indeed, the policy frontier under average inflation is virtually identical to the optimal policy frontier over a plausible range for the preference parameter m^{AIT} ($0 < \mathsf{m}^{AIT} < 25$). Notice, however, that unlike the policy frontiers associated with optimal policy and a speed limit policy, the policy frontier under average inflation targeting bends upward eventually. For values of $\mathsf{m}^{AIT} > 25$, average inflation

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[15] Rogoff's (1985) observation that a central bank's objectives may differ from society's is relevant here.

[16] The parameters m^{AIT} , m^{SL} , and m vary from a low of *0.01* to a high of *1000* to cover the extreme cases of low and high relative aversion to inflation variability. For each strategy, the output-variability tradeoff is measured in terms of the variances of the (single-period) rate of inflation and the output gap.

[17] Preference parameters are deemed low if they are less than unity. To be concrete, for $\mathsf{m}^{AIT} = \mathsf{m}^{SL} = 0.1$, $V(\mathsf{p}_t^{AIT}) = 4.36$ and $V(y_t^{AIT}) = 1.06$ while $V(\mathsf{p}_t^{SL}) = 6.21$ and $V(y_t^{SL}) = 3.88$. Interestingly, a speed limit policy leads to continuously decreasing variances of both the output gap and inflation but increasing variances of the *change* in the output gap for $\mathsf{m}^{SL} < 1$

targeting produces hugely inefficient outcomes relative to optimal policy and even a speed limit policy.

Intuitively, the reason for the inferior performance of a speed limit policy for plausible values of the preference parameter is that the target rule of this policy strategy relies on less information than average inflation targeting. Some important information (whether real output is above or below potential) is lost. It is this information that is essential for providing an effective response to future inflation, because inflation in the next period depends on the current output gap. This is illustrated graphically in Figure 3. At points A and B, the output gap is negative and positive respectively, while at both points the speed limit is positive. By taking account of the speed limit only, the policymaker responds in a similar manner at both points (tightening monetary policy), even though at point A the economy is below potential.[18]

Figure 3: The Output Gap
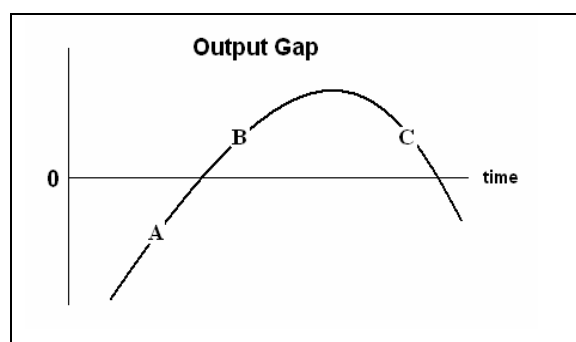
That a speed limit policy produces an inferior policy response can also be seen by comparing the reaction of the policymaker at points B and C. At both points the output gap is the same and positive so that a tightening of monetary policy is called for under optimal policy. The change in the output gap is positive at B so that policy tightens under a speed limit policy. However, at point C, the change in the output gap is negative which prompts the policymaker to ease the stance of policy even though a tightening is warranted.

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[18] Based on the parameter values in Ball (1999), a comparison of the reaction functions associated with optimal policy (equation (5)) and a speed limit policy (equation (20)) reveals that the coefficient on the output gap is much smaller under a speed limit policy than under optimal policy. This indicates that the response of the policy instrument to the output gap is too weak under a speed limit policy compared to optimal policy.

The results of a quantitative evaluation of the performance of average inflation targeting and a speed limit polity are set out in Tables 1 and 2. A simple two-stage procedure is followed. Initially, we determine what values of m society must have for average inflation targeting to generate the same welfare losses for both the policymaker and society. We then compare society's welfare losses under average inflation targeting relative to optimal policy for those given values of m. This two-stage procedure is repeated for a speed limit policy.

The last column of Table 1 shows that average inflation targeting is almost as efficient as optimal policy, especially for rather low values of m. For $m < 4$ average inflation is only slightly less efficient than optimal policy, with the efficiency loss well below one percent. For $m < 8$ the relative welfare loss associated with average inflation targeting still amounts to less than two percent. The performance of average inflation relative to optimal policy steadily worsens as the size of m increases with the efficiency loss approaching *12* percent for m around *60*.

In contrast, a speed limit policy is vastly inferior to optimal policy if society is as much concerned about output gap variability as it is about inflation variability. The efficiency loss associated with a speed limit policy declines as the size of m increases but still hovers around *6.5* percent for $m = 10.58$. The relative welfare loss under a speed limit policy ranges from a maximum of approximately *62* percent when society cares less about inflation than output gap variability (m ; *0.67*) to a minimum of nearly zero when society is far more concerned about inflation variability than output gap variability (m ; *103*).

Taken altogether, society's concern about inflation relative to output gap variability is the critical factor in determining the performance of average inflation targeting and a speed limit policy, respectively, vis-à-vis optimal policy.[19] If society values output stability, then average inflation targeting is superior to a speed limit policy. However, if society shows far greater, i.e. almost exclusive concern for inflation stability, then a speed limit policy dominates average inflation targeting. In such a scenario, a welfare-maximizing policymaker all but ignores the expected change in the output gap when setting policy and pays almost exclusive attention to

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[19] Here we assume that the parameters of the model and the variances of the shocks remain constant while society's preference parameter takes on different values.

the expected rate of inflation. The target rule is essentially the same as under strict inflation targeting which is an extreme form of optimal policy and hence efficient.

7. Summary and Conclusion

The Reserve Banks of Australia and New Zealand state explicitly that their objective for monetary policy is to control average inflation over the medium term or cycle. Federal Reserve policy has recently been interpreted as an attempt to follow a speed limit policy. Both strategies of monetary policy have been analyzed from a theoretical perspective in forward-looking New Keynesian models where the effect of policy on the target variables occurs within the same period. A speed limit policy tends to dominate average inflation targeting in such forward-looking models. How robust is the superior performance of a speed limit policy to a change in the modeling framework?

This paper evaluates the performance of average inflation targeting and a speed limit policy in a backward-looking model. The study is motivated by the observance that sound policymaking in practice requires central bankers to take account of lags in the effect of monetary policy on the target variables. It is widely accepted that a change in monetary policy has a delayed effect on output and inflation, with the impact on output occurring sooner than on inflation. A backward-looking model featuring this lag pattern serves as the framework within which the efficiency losses of average inflation targeting and a speed limit policy vis-à-vis optimal policy are investigated. Particular attention is paid to the specification of the target rules that underlie both strategies of monetary policy in light of Ball's (1999) claim that nominal income growth targeting, which is a special case of a speed limit policy, has undesirable properties.

A central finding of this paper is that policymakers must pay heed to the transmission lag in monetary policy in designing target rules for average inflation targeting and a speed limit policy. A speed limit policy can be very inefficient if the target rule is not properly specified in the sense that the target horizon for expected inflation does not conform to the two-period lag imposed by the structure of the model. The choice of a target horizon for expected average inflation is less crucial under average inflation targeting but choosing a shorter target horizon results in some welfare loss for society.

Our findings suggest further that average inflation targeting imposes relatively little cost on society if the target rule is correctly specified. Its performance relative to optimal policy is extremely good, especially if society values output stability. By comparison, a properly specified speed limit policy is considerably less efficient if society values output stability. Average inflation targeting becomes less attractive while a speed limit policy becomes more attractive as society becomes increasingly concerned about inflation variability relative to output gap variability. When society has little or no concern for output fluctuations, a speed limit policy dominates average inflation targeting. Under these circumstances, a speed limit policy approaches strict inflation targeting which is a special case of optimal policy.

To sum up, barring extreme aversion to inflation variability, a speed limit policy, which dominates average inflation targeting in the forward-looking framework, may prove to be an inferior policy strategy in the backward-looking model where policy lags matter. Given that the attractiveness of the two policy strategies is largely model-specific, the question of which policy is superior remains unsettled. Further analysis of the performance of average inflation targeting and a speed limit policy across a wider spectrum of modeling frameworks is warranted.

The current paper has not addressed an acute problem that policymakers face in the real world. It assumes that the policymaker has perfect information about the target variables when setting policy. Recent evidence suggests that "real-time" policymaking must rely on very imprecise measures of the output gap.[20] Owing to this imperfection, the existing literature advises that the policymaker ought to exercise greater caution in implementing policy. The precise implications of output gap uncertainty for policy analysis in the current framework are left for future research.

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[20] See, for instance, Orphanides and van Norden (2002) or Orphanides (2003). The fact that there is a measurement problem associated with potential output is often cited in support of a speed limit policy.

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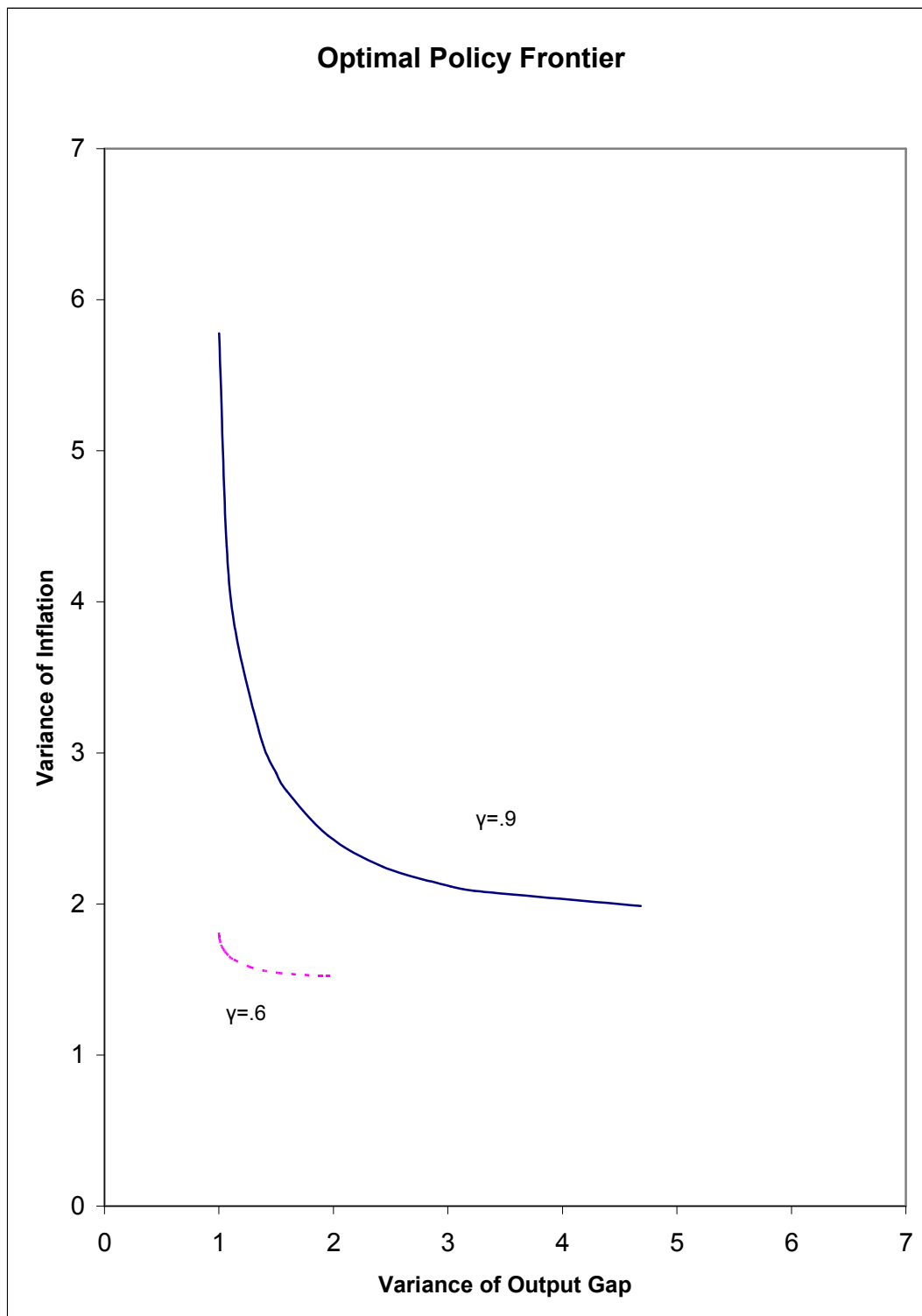
*Figure 1***:** The Optimal Policy Frontier

Optimal Policy Frontier

γ=.9

γ=.6

Variance of Inflation

Variance of Output Gap

Figure 2: A Comparison of the Output-Inflation Variability Trade-Off
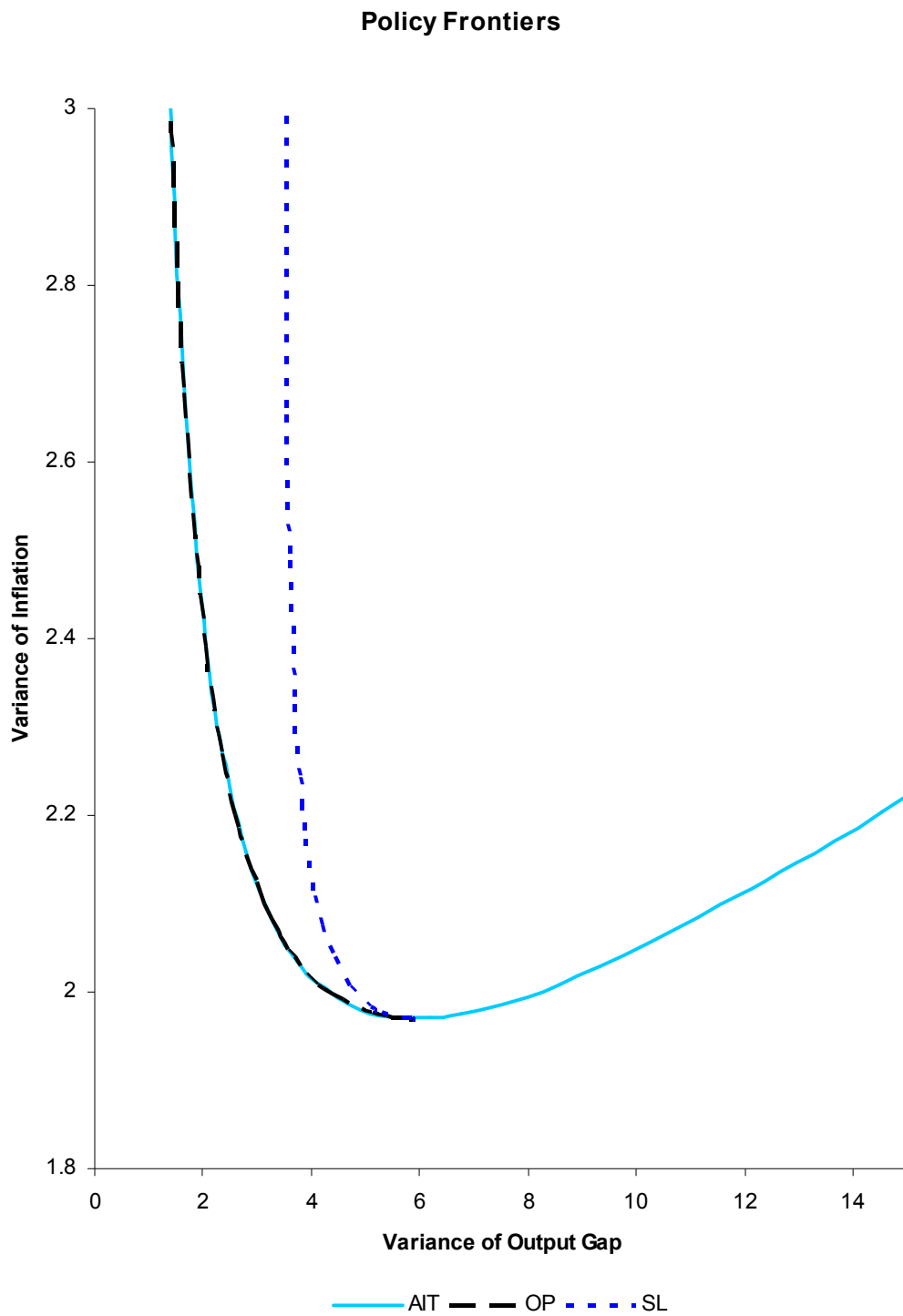
Policy Frontiers

Table 1: Society's Welfare under Average Inflation Targeting Relative to Optimal Policy

Policymaker's aversion to inflation variability	Society's aversion to inflation variability	Society's welfare loss under AIT	Society's welfare loss under optimal policy	Relative Loss: loss under AIT relative to loss under optimal policy (%)
m^{AIT}	m	$E[L_t]_{AIT} = V(y_t^{AIT}) + \mathsf{m}V(\mathsf{p}_t^{AIT})$	$E[L_t] = V(y_t) + \mathsf{m}V(\mathsf{p}_t)$	$\dfrac{E[L_t]_{AIT} - E[L_t]}{E[L_t]}$
1	0.870	3.991	3.983	0.002
2	1.690	6.117	6.095	0.003
4	3.267	9.837	9.774	0.006
8	6.280	16.505	16.311	0.011
10	7.739	19.656	19.373	0.014
50	33.730	76.349	71.667	0.065
100	61.949	142.718	127.529	0.119

Note: m^{AIT} is the weight the policymaker places on average inflation in his objective function. The first column lists plausible values for m^{AIT}. The second column reports the values for m that society would have to have for average inflation targeting to generate the same welfare losses for both the policymaker and society: $E[L_t]^{AIT} = E[L_t]_{AIT}$ where $E[L_t]^{AIT} = V(y_t^{AIT}) + \mathsf{m}^{AIT}V(\bar{\mathsf{p}}_t^{AIT})$ and $E[L_t]_{AIT} = V(y_t^{AIT}) + \mathsf{m}V(\mathsf{p}_t^{AIT})$.

Table 2: Society's Welfare under a Speed Limit Policy Relative to Optimal Policy

Policymaker's aversion to inflation variability	Society's aversion to inflation variability	Society's welfare loss under speed limit	Society's welfare loss under optimal policy	Relative Loss: loss under SL relative to loss under optimal policy (%)
$\mathfrak{m}^{SL}$	$\mathfrak{m}$	$E[L_t]_{SL} = V(y_t^{SL}) + \mathfrak{m}V(\mathfrak{p}_t^{SL})$	$E[L_t] = V(y_t) + \mathfrak{m}V(\mathfrak{p}_t)$	$Rel.\ Loss = \dfrac{E[L_t]_{SL} - E[L_t]}{E[L_t]}$
1	0.676	5.577	3.439	0.621
2	1.824	8.390	6.424	0.305
4	4.072	13.388	11.562	0.157
8	8.441	22.535	20.835	0.081
10	10.588	26.921	25.261	0.065
50	51.972	109.019	107.806	0.011
100	102.597	208.661	207.758	0.004

Note: $\mathfrak{m}^{SL}$ is the weight the policymaker places on the rate of inflation in his objective function. The first column lists plausible values for $\mathfrak{m}^{SL}$. The second column reports the values for $\mathfrak{m}$ that society would have to have for a speed limit policy to generate the same welfare losses for both the policymaker and society: $E[L_t]^{SL} = E[L_t]_{SL}$ where $E[L_t]^{SL} = V((y_t - y_{t-1})^{SL}) + \mathfrak{m}^{SL}V(\mathfrak{p}_t^{SL})$ and $E[L_t]_{SL} = V(y_t^{SL}) + \mathfrak{m}V(\mathfrak{p}_t^{SL})$.

Appendix:

The alternative specification of the target rule for a speed limit policy takes the following form:

$$q^{SL}\left[E_t y_{t+1} - y_t\right] + E_t p_{t+1} = 0 \tag{A1}$$

Because of the two-period transmission lag expected inflation next period is predetermined. This target rule is equivalent to nominal income growth targeting (where the target growth rate has been normalized to zero) if $q^{SL} = 1$.

Combining the above target rule with the model equations determines the output gap under a speed limit policy:

$$y_t = -\frac{g}{q^{SL}} p_{t-1} + \frac{q^{SL} - a}{q^{SL}} y_{t-1} + e_t \tag{A2}$$

Along with the Phillips curve, equation (A2) gives rise to the following variances of the output gap (y_t), inflation (π_t), and the change in the output gap $(y_t - y_{t-1})$:

$$V\left(y_t^{SL}\right) = \frac{q^{SL}[2ag + q^{SL}(1 - g(1 + g - g^2))]s_e^2 + g^2(1+g)s_h^2}{(1-g)a(2q^{SL}(1+g)-a)} \tag{A3}$$

$$V\left(p_t^{SL}\right) = \frac{a(q^{SL})^2(1+g)s_e^2 + \left(2q^{SL} - a(1-g)\right)s_h^2}{(1-g)(2q^{SL}(1+g)-a)} \tag{A4}$$

$$V\left((y_t - y_{t-1})^{SL}\right) = \frac{2[q^{SL}(ag + q^{SL}(1-g^2))s_e^2 + g^2 s_h^2]}{(1-g)q^{SL}(2q^{SL}(1+g)-a)} \tag{A5}$$

Here we see that all three variances are well defined provided that $g \neq 1$, i.e. that the Phillips Curve is not of the accelerating type.

Inserting (A4) and (A5) into (18) and minimizing the expected loss function with respect to θ^{SL} yields the optimal value of the policy parameter under a speed limit policy. The optimal policy parameter is again a function of the parameters of the Phillips Curve, the preference parameter of the policymaker, and the variances of the shocks of the model.

Figure A1 shows that the alternative specification of the target rule under a speed limit policy leads to average inflation targeting strictly dominating a speed limit policy. The policy frontier traced out by average inflation targeting lies below the policy frontier ground out by a speed limit policy. Average inflation targeting

generates a more favorable output-inflation variability trade-off than a speed limit policy.

Figure A1: The Output-Inflation Variability Trade-Off under the Alternative Specification of a Speed Limit Policy
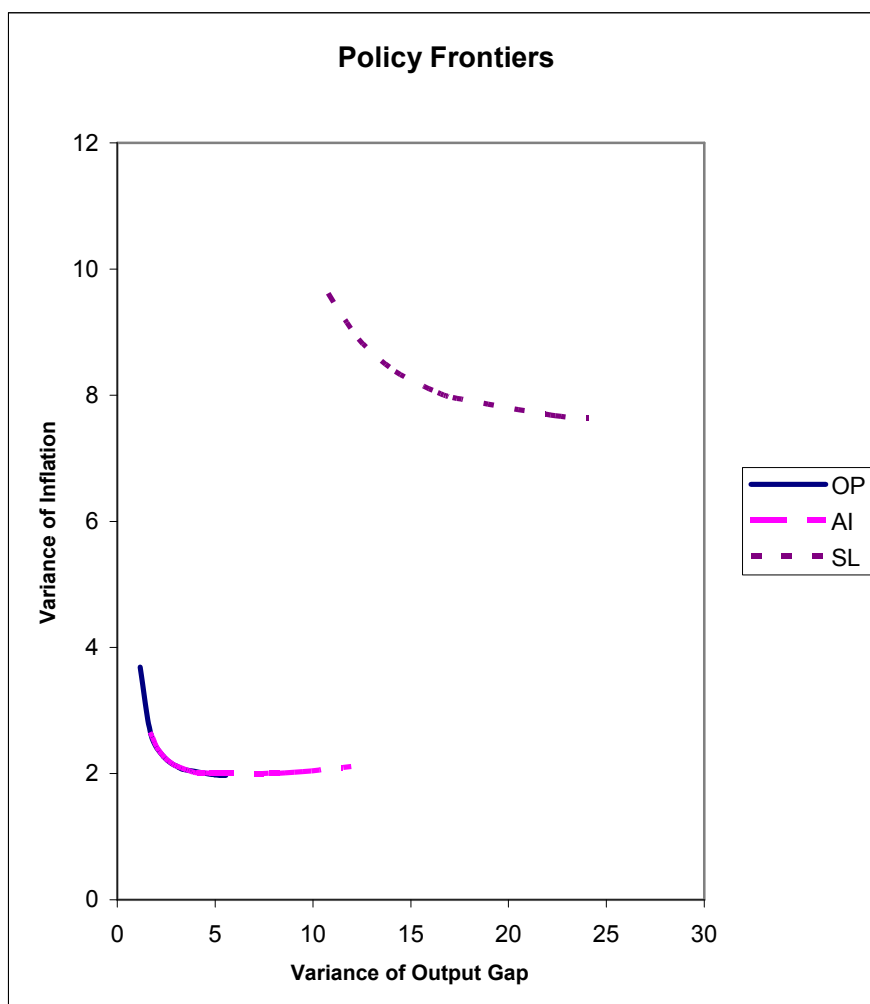
The target rule underlying average inflation targeting could also be specified as:

$$q^{AIT} E_t y_{t+1} + E_t \overline{p}_{t+1} = 0 \qquad (A6)$$

Basing policy on this target rule causes the variability of the output gap to be lower but the variability of average inflation and single-period inflation to increase compared to the target rule of Section 4. Expected losses under the above target rule exceed those under the target rule discussed in the paper. In addition, there is no simple closed-form solution for the optimal policy parameter q^{AIT}.

<u>Evolution of Expected Inflation:</u>

This part of the appendix discusses the factors that drive expected inflation in period *t+2*. In each of the four cases considered, the target rule is combined with the Phillips Curve to determine the inflation forecast.

1. Target Rule under Average Inflation: $q^{AIT} E_t y_{t+1} + E_t \overline{p_{t+2}} = 0$

    The resulting rate of inflation expected in period *t+2* evolves gradually and is tied only to the expected rate of inflation in period *t+1*:

    $$E_t p_{t+2} = \frac{2q^{AIT}g - a}{2q^{AIT} + a} E_t p_{t+1} \qquad (A7)$$

2. Target Rule under a Speed Limit Policy: $q^{SL}\left[E_t y_{t+1} - y_t\right] + E_t p_{t+2} = 0$

    Under a speed limit policy, contemporaneous information enters the target rule. This complicates the policy-setting process. The evolution of expected inflation depends not only on expected inflation in period *t+1* but also on the current output gap:

    $$E_t p_{t+2} = \frac{q^{SL}}{q^{SL} + a}(g\, E_t p_{t+1} + a\, y_t) \qquad (A8)$$

The two remaining target rules describe less efficient policy outcomes under average inflation targeting and a speed limit policy.

3. Target Rule under Average Inflation: $q^{AIT} E_t y_{t+1} + E_t \overline{p}_{t+1} = 0$

    Notice that the current rate of inflation appears in the definition of $E_t \overline{p}_{t+1}$. As a result, expected inflation in period *t+2* depends on both expected inflation in period *t+1* and the current rate of inflation:

    $$E_t p_{t+2} = \frac{1}{2q^{AIT}}((2q^{AIT}g - a)E_t p_{t+1} - a\, p_t) \qquad (A8)$$

    Compared to the first case, the policy-setting process becomes more complicated.[21]

4. Target Rule under a Speed Limit Policy: $q^{SL}\left[E_t y_{t+1} - y_t\right] + E_t p_{t+1} = 0$

    Expected inflation two periods into the future depends on both expected inflation in period *t+1* and the current output gap:

    $$E_t p_{t+2} = \frac{g q^{SL} - a}{q^{SL}} E_t p_{t+1} + a\, y_t \qquad (A9)$$

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[21] Equation (A8) is akin to a second-order difference equation.

The current output gap carries a greater weight in determining expected inflation in period *t+2* compared to the other specification of a speed limit policy.