K.I.T.T.: Kiwi Inflation Targeting Technology

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Preface

This book details KITT (Kiwi Inflation Targeting Technology) the Reserve Bank of New Zealand's new DSGE model. The guide describes a the macroeconomic structure of the model is derived from a consistent set of assumptions regarding the micro-founded interactions between between firms, households and other agents in the model. The guide shows how this structure is able to replicate the key dynamic features of the New Zealand economy. The multi-sector production structure facilitates distinct dynamics for non-tradable and tradable inflation, and allows a role for the housing sector in determining inflation. We estimate the model using Bayesian methods and show the dimensions along which the data are informative, before presenting the match of the model to the data. Impulse responses are used to demonstrate the behaviour of the model. We also show how the model might be used in the the policy environment at the Reserve Bank of New Zealand. More specifically, we show how to add judgment to the model forecasts, how to treat uncertainty and how the forecasts from the model can be deconstructed into their key drivers.

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Chapter 1

Introduction

1.1 The role of models at the Reserve Bank

The Reserve Bank of New Zealand (RBNZ) is tasked with flexible inflation targeting and operates within a small open economy that is subject to shocks that originate both at home and abroad.[1] Setting monetary policy is made difficult by uncertainty about how these shocks are transmitted throughout the economy, and the lags inherent in how movements in interest rates affect key macroeconomic variables. To help formulate the appropriate policy response to economic conditions, the Reserve Bank aims to understand the current state of the economy as well as how it is expected to evolve over the future. As part of this process, the RBNZ employs a range of macroeconomic models and uses the FPS (Forecast and Policy System) as a core model to help inform and organise model based monetary policy advice.

The RBNZ actually has a surprisingly long history of macroeconomic modelling (see Spencer and Özer Karagedikli, 2006, for an overview) that precedes inflation targeting. The RBNZ's first macroeconomic model, Deane

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[1] In particular, New Zealand's inflation targeting objectives are encapsulated in the Policy Targets Agreement between the Minister of Finance and the Governor of the Reserve Bank of New Zealand. This specifies that the RBNZ: "keep future CPI inflation outcomes between 1 per cent and 3 per cent, on average over the medium term," and "...implement monetary policy in a sustainable, consistent and transparent manner and shall seek to avoid unnecessary instability in output, interest rates and the exchange rate."

(1971) had a simple new Keynesian structure and was later updated to include an inflation-expectations augmented Phillips curve by Spencer et al. (1979).

These early models were based on aggregate demand driven frameworks and struggled to deal with the succession of supply shocks in the 1970s. Subsequent modelling efforts across the 1980s largely focussed on introducing cointegrating models. Wells and Evans (1985) documented the impact of traded goods in a VAR framework but later modelling efforts had to confront limited data confounded macroeconomic data that contained structural breaks introduced by a series of reforms to the New Zealand economy that occurred in the late 1980s (Spencer and Özer Karagedikli, 2006).

These breaks led to a temporary decline in the use of formal models at the RBNZ for forecasting and policy analysis. However, confronted with a new inflation targeting remit in February 1990, the RBNZ developed the Forecasting and Policy System (FPS) model in the mid 1990s to provide forecasts and analysis to help set policy to meet the targets specified in the Policy Targets Agreement. The Reserve Bank's current core FPS model, documented in Black et al. (1997), has served the RBNZ fairly well for over a decade. The model is calibrated (a legacy of the paucity of data in the mid-1990s), and is characterised by the use of optimising microeconomic behaviour to pin down the long-run steady-state. Short-run dynamics are driven by Keynesian constructs that are not derived from the microeconomic behaviour of firms and households. In particular, in the current model vintage, non-tradables inflation is determined by both the output gap and surveyed measures of inflation expectations that are themselves largely determined by lag of consumer price inflation (Hargreaves et al., 2006, see). The FPS model introduced in 1997 has undergone many changes and contains a different structure to the FPS currently in use today (see Delbrück et al., 2008, for an overview).

The FPS model forms the key organisational construct for producing the macroeconomic forecasts published in the Bank's quarterly *Monetary Policy Statement*. These forecasts are adjusted to include the judgments of both the Economics Department and the Monetary Policy Committee. In fact, non-judgmentally adjusted forecasts are rarely presented to the Monetary Policy Committee.

The strong history of using models and the role of the FPS model in the current policy environment has generated a supporting environment for the introduction of a DSGE model. Policymakers and staff generally do not have to be convinced about the value of using models to assist the conduct of monetary policy.

1.2 Why a DSGE model?

The RBNZ had been strongly influenced by the success of large-scale DSGE models completed or under development at a number of central banks (see Harrison et al., 2005; Medina and Soto, 2006; Murchison and Rennison, 2006; Adolfson et al., 2007b). The RBNZ saw in these developments macroeconomic models that could address a number of issues important to the forecasting and policy environment. Importantly, the DSGE framework could provide consistency in terms of a macroeconomic story developed from the microfounded interactions between firms and households. Over 2005 and 2006, in-house research at the RBNZ (see Kam et al., 2009; Matheson, 2006) had a focus on learning Bayesian techniques (from a stream of international visitors from both central banks and academia). Staff applied the Bayesian techniques to estimating small DSGE models. Based on the perceived advantages of DSGE models and the development of some human capital, in the middle of 2006, the RBNZ decided to devote resources to developing a Dynamic Stochastic General Equilibrium (DSGE) model, with the explicit aim of replacing the existing Forecasting and Policy System (FPS) model.

Relative to earlier RBC and DSGE models, DSGE models employed as core models at cental banks, reached a scale that offered the degree of richness required to capture most of the important features of the New Zealand business cycle. These models were multi-sector models with explicit production functions for tradable goods, non-tradable goods and a separate export sector.[2] Further, the general equilibrium approach at the heart of DSGE models exploits specific market clearing conditions in each sector of the economy to generate a description of how prices, and more broadly, the economy, evolves. This facilitates a description of the economy based on the shocks to underlying factors of production and consumr preferences.

The advent of Bayesian techniques for estimating DSGE models offered the RBNZ the opportunity to move from a calibrated model to a model more strongly, and more formally, informed by the data. Computational power and estimation algorithms had also improved to the point that small- and medium-sized DSGE models, can be taken to the data using Bayesian methods. Moreover, within the RBNZ, there was a strong sense that the almost twenty years of data since the start of inflation targeting could provide the opportunity for estimating a macroeconomic model that was not present when the FPS model was developed over ten years earlier. An estimated model would also allow staff to better distinguish between competing hypotheses about the drivers of current economic conditions.

Estimated DSGE models also appeared to offer greater opportunity to address the uncertainty inherent in macroeconomic forecasting and policy analysis. Some literature indicated DSGE models could produce competitive forecasting performance relative to small empirical models, at least at longer horizons Smets and Wouters (see for example 2003); Adolfson et al. (see for example 2007). Furthermore, combining DSGE models with statistical models, predominantly Bayesian VARs, had been shown to produce good forecast performance (Del Negro and Schorfheide, 2004), and, for the case of New Zealand forecast performance “competitive with” the Reserve Bank of New Zealand published forecasts (Lees et al., 2007). Explicitly estimated models also held the potential to address some of the issues raised in Durlauf and Vahey (2008). Estimation allows the production of density forecasts or fancharts (the so-called “rivers of blood”) for particular variables. The probability of explicit events, such as inflation lying outside 1

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[2] The description of how firms supply goods, was absent from the existing FPS model.

to 3 percent over the medium term, could be computed. These density forecasts could be combined with densities from alternative models to produce better density forecasts of key model variables (see Hall and Mitchell, 2007, for example).

Constructing a DSGE model in-house would also be useful for developing human capital. Developing this capital would tie in with a research programme that promotes the use of DSGE models to answer a rannge of specific policy questions and enhance interaction with researchers in academia and other central banks. The operation of a DSGE model in a forecasting environment held the potential to reinvigorate interaction between forecasters and researchers within the Reserve Bank — the model used to produce forecasts could be used to address research questions. Model redevelopment would also place the in-house model back near the frontier of macroeconomic modelling used in the central bank community.

The future at the Reserve Bank of New Zealand also holds the possibility of a tighter integration between macroeconomic policy and microeconomic data via DSGE models. A macroeconomic model grounded in microfoundations can be informed and challenged by microeconomic data, aiding understanding of the economy and the appropriate policy response (see Bils and Klenow, 2004; Angeloni et al., 2006; Gopinath et al., 2007, for example).

1.3 Model Design

Once it was decided that the Reserve Bank should build a DSGE model, attention turned to specifying more precisely what could be demanded from the model. Policymakers demanded that the model framework incorporate particular functionality with respect to the production of forecasts and that the model replicate key stylised facts about the New Zealand economy.

Indeed, a key driver for the model was to explain the key properties of the New Zealand business cycles and key stylized facts peculiar to New Zealand. For example, a large proportion of New Zealand's exports are agricultural commodities that are not particularly sensitive to the world price, at least in the short run. Exports are approximately a third of New Zealand Gross Domestic Production while Investment and Consumption components are respectively. Relative to the US, consumption data is volatile and not particularly persistent.

The Reserve Bank of New Zealand and local economists have a history of splitting the Consumer Price Index (CPI) into non-tradable and tradable components. One key stylised fact of the New Zealand economy is that the average rate of inflation in the tradable sector is considerably lower than in the non-tradable sector (averaging more than 3%).

One rationale for this persistent downward pressure on aggregate inflation is productivity advances in the production of consumable manufactured goods — the so called "China effect". Policymakers view this tradable/non-tradable split as important in understanding the inflation story and for making policy decisions.

More recently, New Zealand experienced a large boom in the residential property market. House prices doubled between the end of 2001 and the end of 2007. Construction costs form a non-trivial fraction of the CPI in New Zealand and drove a particularly large fraction of inflation over the most recent cycle Hargreaves et al. (2006).

Furthermore, the average New Zealand household's debt rose from around 100 percent of disposable income to around 170 percent (Bollard, 2006). Veirman and Dunstan (2008) also point to economically and statistically significant consumption effects from increases in housing wealth. Furthermore, fixed rate mortgage contracts are prevalent in New Zealand. The two-year fixed mortgage contract represents a competitive pricing point and has been the most popular contract in the market over the last five years. This implied a relatively sluggish transmission of of the policy rate to the effective mortgage rate, delaying the effectiveness of monetary policy. Relatively early in the development of the model, it became clear that a housing sector would prove useful in explaining New Zealand's most recent business cycle.

Furthermore, New Zealand is an oil importer. The small population base and geographically disperse population centres seem to imply a relatively high fraction of oil in the production of tradable goods (and even non-tradable goods). Petrol forms 5% percent of the consumer price index and is sensitive to oil price and exchange rate fluctuations, implying increases in the price of oil have relatively sizeable implications for the disposable incomes of New Zealand households.

Finally, a set of practical concerns centred around using KITT as the core forecasting and policy model also drove the model design. The model and model framework needed to be robust to the requirements of adding policymaker judgment to the production of forecasting for publication purposes.

To pursue this goal, we leveraged off the existing macroeconomic toolkit (that includes producing conditional forecasts, impulse responses, forecast decompositions etc.) that is extremely useful, required for understanding DSGE models.[3] In order to satisfy policymakers' demands to add judgment easily and effectively, we developed an algorithm based on Waggoner and Zha (1999) and detailed in Beneš et al. (2008) for this task. The algorithm computes forecasts conditioned on policymaker judgment that are the most likely set of forecasts from the perspective of the DSGE model, thereby maximising the influence of the model structure on the forecasts. Further, the algorithm uses the absolute magnitude of the structural shocks required to return the policymaker judgment as a metric for the size of judgment. We also used a technology that exploits the Kalman filter and linearity of the model to decompose forecasts into their constituents drivers. We found this extremely useful for understanding the model and as a starting point for conversations with policymakers about the appropriateness of specific forecasts.

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[3] These procedures were implemented with the IRIS toolbox (see Beneš, 2008), developed for use with Matlab software.

1.4 Compromises with KITT

Based on our set of observable data, we were unable to identify all the separate stochastic trends (required to provide the scale and richness of model regarded for a core forecasting and policy model) for the multi-sector framework. While some calibrations of the full trends model, that allows for permanent shocks to each productivity process and the terms of trade, matched some the along some dimensions they failed along other dimensions. We retain a version of the full trends model to examine the response of particular technology shocks but in order to produce a model for forecasting production, we detrend the data prior to estimating the model.

In the absence of a full trends model, we detrend by applying a multi-variate filter to the data to remove the trend. The multi-variate enables trends to be imposed consistently across the model.

1.5 Organisation of this book

The following section of this book provides a detailed description of the microfoundations that underpin the optimising behaviour of each agent in the economy. Section three details the estimation of the model and the treatment of the data. The properties of the model are documented in section four. Particular attention is paid to using impulse responses to a range of shocks to explain the dynamic properties of the model. Section five discusses the role of the model in the forecasting and policy environment and details some specific important techniques for using the model. Finally, section six provides some concluding comments and direct for future work.

Chapter 2

The Theoretical Model

2.1 Overview

KITT is a multi-sector DSGE model that describes the dynamics of aggregate macroeconomic variables by explaining the interaction between households, firms and the central bank. Within the multi-sector design, considerable emphasis is placed on explaining the key components of changes in the consumers price index. This multi-sector design facilitates the analysis of shocks that have implications for relative prices. Compared with the output gap philosophy that underpins FPS, where any shock that moves aggregate demand generates inflation, the origin of shocks in KITT can generate substantially different effects on the economy, depending on which components of GDP the shock impacts.

The SVAR literature has identified monetary policy as having a non-neutral impact on real macroeconomic variables in the short-run.[1] Woodford (2003) and Galí (2008) use this to motivate the inclusion of nominal rigidities into an otherwise standard business cycle model. Using the same reasoning we introduce nominal rigidities into the price setting problems of firms in the sectors that contribute directly to CPI inflation.[2]

Rather than being determined by the aggregate output gap, in KITT,

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[1] See Christiano et al. (1998) for a literature review

[2] We also introduce nominal adjustment costs into the households wage setting problem and the non-commodity exporters price setting problem.

inflation is determined by the pricing decisions of firms that are subject to nominal rigidities. Following Rotemberg (1982a) and Rotemberg (1982b) we introduce nominal rigidities in the form of quadratic adjustment costs. These nominal rigidities proxy for menu costs and customer's preferences for frequent and small price changes as opposed to infrequent but large price adjustments (Heijdra and van der Ploeg, 2002). In order to introduce nominal adjustment costs into a sector we must make the assumption that firms produce differentiated goods (that is the sector is monopolistically competitive). As a consequence firms set prices as a mark-up over marginal cost, and firms have enough market power to raise prices without losing all their market share (and lower prices without gaining the entire market share). The imposition of price adjustment costs prevent firms from moving prices instantly to their profit-maximising flexible level, causing real marginal cost pressures and inflation to persist.[3] Each sector contains a different production technology and competitive environment that determines sector specific inflation. The inflation components from each sector are aggregated according to their share of the consumer price index to produce headline inflation.

Figure 2.1 depicts the production structure of KITT including the factors of production and the ultimate destinations of final goods. The figure shows that three primary inputs (labour, capital and fuel) and a sector specific technology are combined to produce an intermediate production good. This sector is assumed to be perfectly competitive because the intermediate good is unobserved, not in the basket that makes up the consumer price index, and empirically, we don't require additional persistence in intermediate goods prices to match the data. The intermediate good is a factor of production across all four productive sectors.

More specifically, the model consists of a monopolistically competitive non-tradable goods producing sector, a monopolistically competitive trad-

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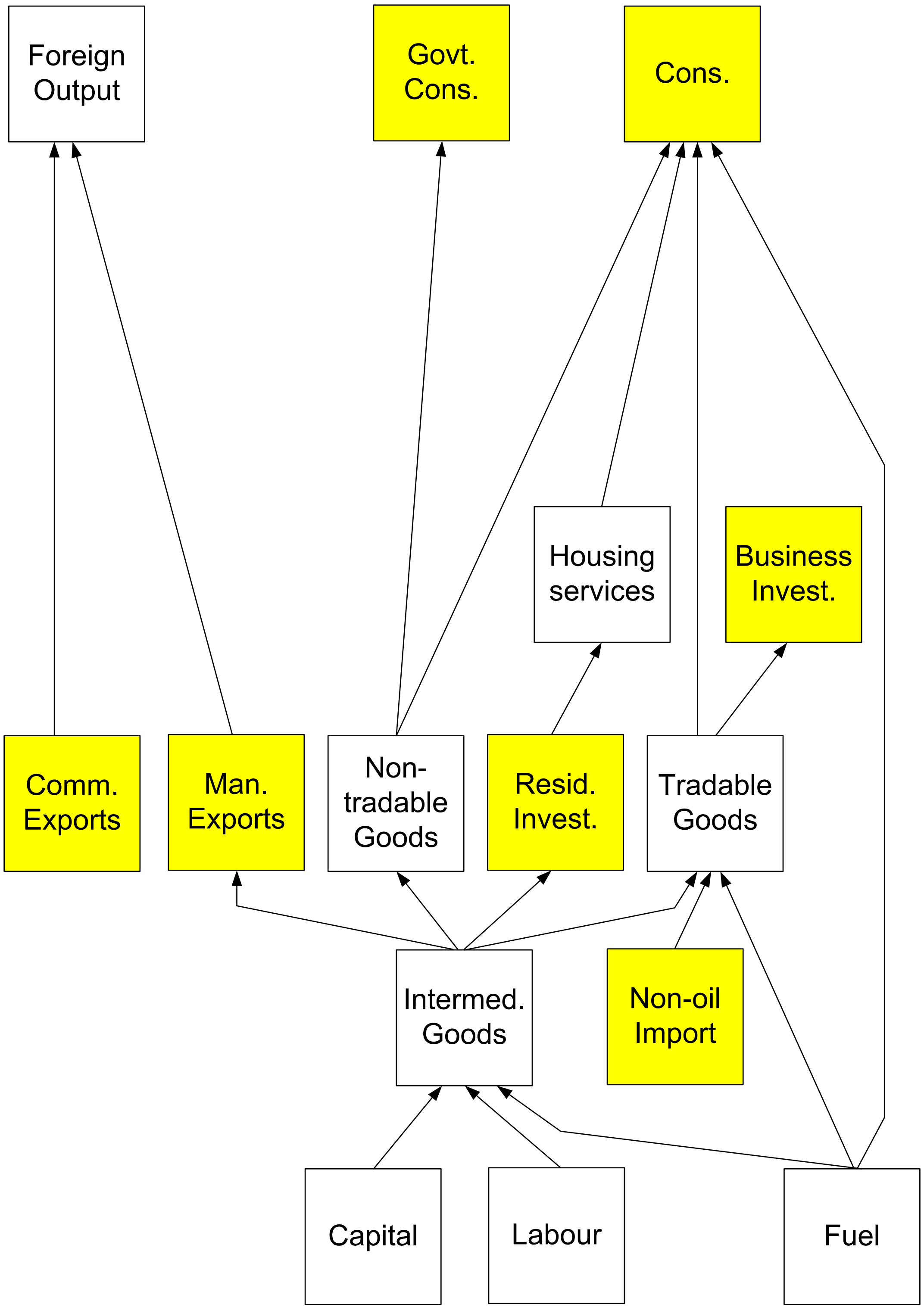
[3] The monopolistically competitive market structure means firms price as a markup over marginal cost. The imposition of adjustment costs means that this mark-up will be time varying. Without monopolistic competition the existence of sticky prices could mean that firms make negative profits and would have to exit the market.

Figure 2.1: The KITT production structure



able goods producing sector (i.e. local currency pricing), monopolistically competitive residential investment producers, a monopolistically competitive manufactured exports producing sector, and a perfectly competitive commodity exports sector. As mentioned, the tradable, non-tradable, manufactured exports, and residential investment sectors, are all subject to price adjustment costs that induce sticky prices and prevent firms from pricing at the flexible price optimum level following a disturbance. Each sector has its own price trend, degree of nominal rigidity, and pricing persistence. This allows for a detailed analysis of the inflation process at the sectoral level.

Recently, the housing sector in New Zealand has undergone a large boom, with concomitant increases in household debt. Typically core forecasting models do not contain housing sectors. However, we include a housing sector within KITT, reflecting the importance of the housing sector on the New Zealand business cycle.

More specifically, the tradable sector imports non-oil manufactured goods and uses the intermediate good, an additional fuel input (in order to transport the tradable good to distribution points) and a sector specific technology to produce the final tradable good. The tradable good is consumed domestically, but can also be used to build business capital. We use monopolistic competition to introduce local market pricing. This reduces exchange rate pass through in the short run.

The non-tradable sector uses the intermediate good and a sector specific technology to produce the non-tradable good. This good does not require either the imported good or the additional fuel input required for production. The non-tradable good can be either consumed by households or by the government.

The residential investment sector combines the intermediate good with a sector specific technology to produce the residential investment good. The residential investment good is supplied to the housing assembler who combines it with a fixed factor of production assumed to be land to build new houses. These new houses are added to the housing stock and used to generate housing services to be consumed by households. The housing assembler and the housing stock are owned by the household. House prices are assumed to be determined by an endogenous fundamental component and an exogenous non-fundamental component. The fundamental house price is equal to the expected sum of the discounted future stream of imputed rentals, while the non-fundamental component is assumed to be a shock.

Manufactured exports are produced using the intermediate good and a sector specific technology. These goods are exported to the foreign country. Manufactured exporters have some market power and set their prices in foreign currency (local currency pricing in the foreign country). This makes the supply of manufactured exports a function of the nominal exchange rate. The demand for manufactured exports will be a function of foreign output, the foreign price level and the price of the manufactured export in foreign currency.

Finally, demand for commodity exports (agricultural exports, mining etc.) is endogenous but determined by a demand function that is a function of foreign output which is assumed to be exogenous. We assume that commodity exporters are price takers and are too small to impact the world price for commodities. We also assume that the demand for commodity exports is perfectly inelastic, that is the foreign sector demands a fixed quantity of the commodity exports regardless of the market price.

By using an intermediate good in the production of tradables, non-tradables, residential investment and manufactured exports, we ensure that capital, labour and fuel are used in the production of each of these goods. Because the factor price of the intermediate good must be equated across sectors, we are ensuring that each of the sectors will have some sensitivity to the overall business cycle. The relative sensitivities of each of the sectors will be determined by the intermediate good's share of production in each sector. We allow these shares to be different across sectors.

In fact, the treatment of relative price trends in the model is unique for a core policy model. We use the constant nominal share property of the Cobb Douglas aggregation function to allow for different price trends in each of the sectors (Greenwood et al., 1997). Each sector has its own distinct price and technology trend. Sectors with a higher inflation rate have a lower growth rate in technology, and sectors with a lower inflation rate have a higher growth rate in technology. However, Cobb Douglas aggregation restricts the elasticity of substitution between factors to unity which is too restrictive — a lower elasticity of substitution between consumption goods in the short run would be more plausible. We accommodate this by allowing time varying elasticities through the use of deep habit formation as in Ravn et al. (2006). Deep habit formation breaks the short run demand into a price sensitive component, and a perfectly inelastic component, with the elasticity of substitution a weighted average of the two components.

Such a production structure provides a rich framework to decompose inflation into its different sectoral pressures. Headline inflation is determined by the behaviour and relative shares of each component: non-tradable, construction costs, tradable and fuel. Furthermore, because firms are monopolistic competitors, it is real marginal costs and demand in conjunction with quadratic adjustment costs that determine the profit maximising price in each sector.[4] Figure 2.2 shows the components that determine inflation.

Note that although it is current and expected future real marginal costs (along with indexation and cost push shocks) that determine inflation, sector specific demand is a key determinant of real marginal cost and inflation. This is because the production technologies in the non-tradables and construction sectors are Cobb-Douglas with decreasing returns to scale.[5] This implies that firms face an upward sloping marginal cost curve, and a given increase in demand requires the reproducible factors of production to be increased more than proportionately to the increase in demand. As a consequence, the firms profit maximising price must increase because of the reproducible input's diminishing marginal product.

Not all agents in the model solve an explicit optimisation problem based on specific assumptions about preferences and constraints. In particular,

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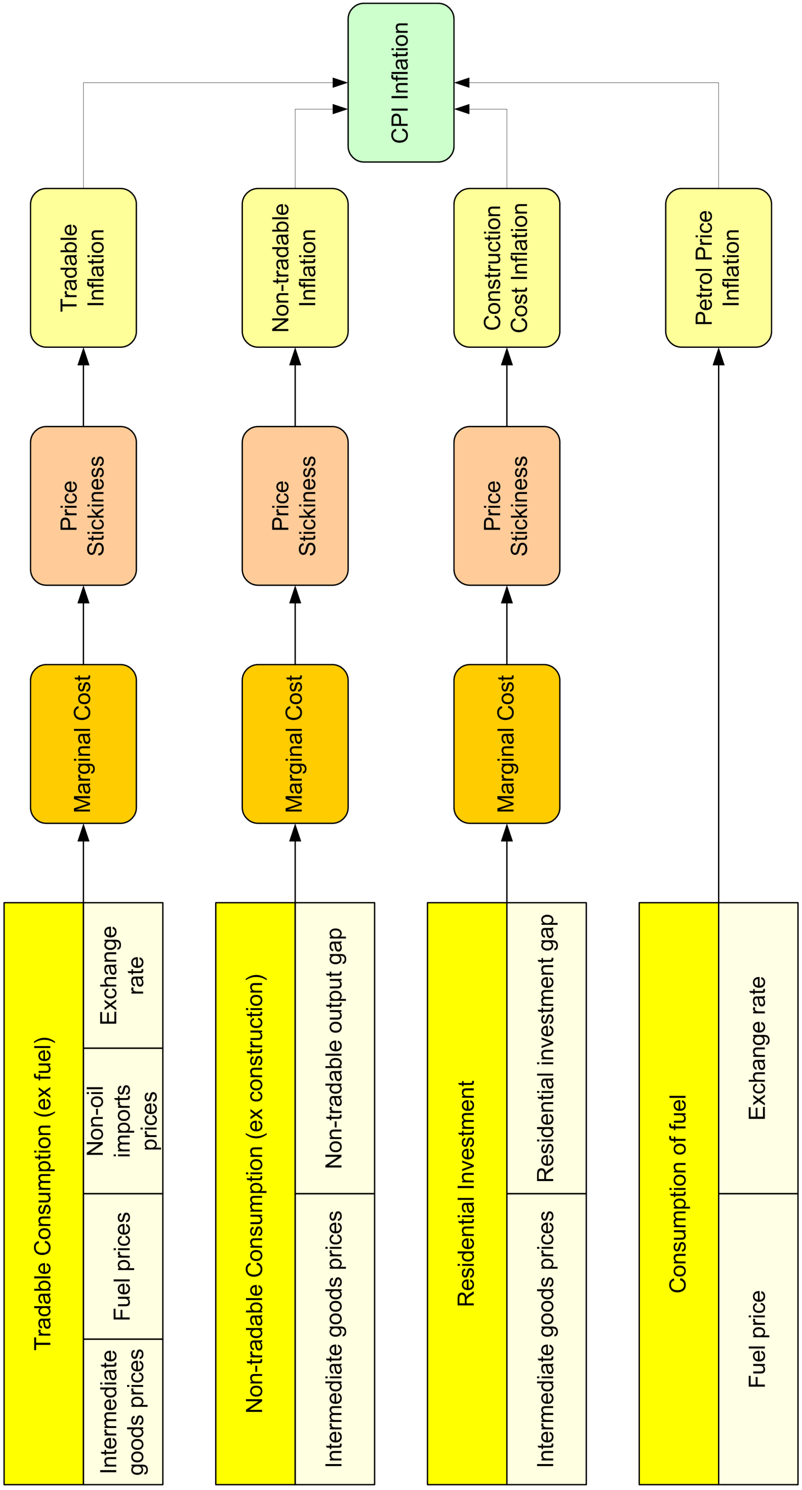
[4] Note that the nature of the demand for individual firms output plays a roll through the markup being a function of elasticity of demand. Sectoral demand can play a roll if firms use a technology with decreasing returns to scale.

[5] This implies a fixed factor of production, in addition to the intermediate good.

Figure 2.2: Determinants of inflation



monetary policy is set according to a simple policy rule that is not derived from first-best welfare considerations, or as a rule optimised with respect to the stabilisation provisions in the Policy Targets Agreement negotiated between the Minister of Finance and the Governor of the Reserve Bank of New Zealand. Similarly, the evolution of government spending is not derived from an optimal policy problem. Rather, in KITT, government follows a simple rule for determining their expenditure. Government spending is funded through lump sum taxes and government balances its budget in every period. Government spending increases the consumption of non-tradable goods, which in turn increases inflationary pressure in the non-tradables sector.

That said, the model is very precise about the assumptions regarding the microeconomic structure of both households and firms. Households gain utility from leisure and the consumption of specific goods, namely tradable goods, non-tradable goods, fuel and housing services. However, the presence of deep habit formation on the part of households implies utility is derived from consumption of a particular good with respect to the average consumption of that good in the previous period. More specifically, this lowers not only the intertemporal elasticity of substitution away from the balanced growth path, but also the intratemporal elasticity of substitution in utility function. Households choose consumption, wages, labour and bond holdings subject to their budget constraint in each period. Households receive income from wages, the rental on capital and profits from firms of which they are shareholders. Figure 2.3 summarizes the consumption decisions of households made at the intertemporal, intratemporal and inter-firm level.

The impact of the housing boom over the last business cycle has been significant. In particular we believe housing equity withdrawal has played a large role in explaining the observed consumption path. In KITT we assume that homeowners live in their own houses. As a result an increase in the value of the housing stock does not result in a wealth effect. If house prices go up, this is directly offset by the increased opportunity cost of foregoing higher rents assuming homeowners were able to rent out their properties.

To get around this problem we introduce a financial intermediary into the model that imposes a collateral constraint on the borrowing conditions of households.[6] This collateral constraint relates the interest rate charged on loans to the value of net foreign debt relative to the value of the housing stock and the policy rate through a reduced form relationship. Aoki et al. (2004) and Bernanke et al. (1999) provide micro-foundations motivating this relationship in a closed economy setting as representing the monitoring costs associated with a defaulting firm in a costly state verification problem. That is when the level of debt relative to the level of collateral increases it is assumed the probability of default also increases. In the case of default it is assumed that the financial intermediary has to incur some cost to monitor the borrower in order to recover some of their funds. The financial intermediary takes this increased probability of default into account when lending and charges a higher market rate on higher levels of debt relative to the collateral. We relate the premia on the market rate to the stock of net foreign debt allowing us to close the economy in a fashion consistent with the debt elastic interest rate example in Schmitt-Grohé and Uribe (2003).[7] This setup allows the supply of debt to increase in times of high house prices and to fall in times of low house prices. The increased supply of debt in times of high house prices results in increased consumption.

The next section in this chapter details the mathematical foundations of the model, explicitly detailing the assumptions about the behaviour of firms and households upon which the model is built. The final section of the paper lists the log-linear equations that summarize the dynamics of the model that can be derived from solving the households' and firms' problems. Note that we do not explicitly derive the underlying first-order conditions for choosing the optimal variety of differentiated goods on the demand side of such markets.[8]

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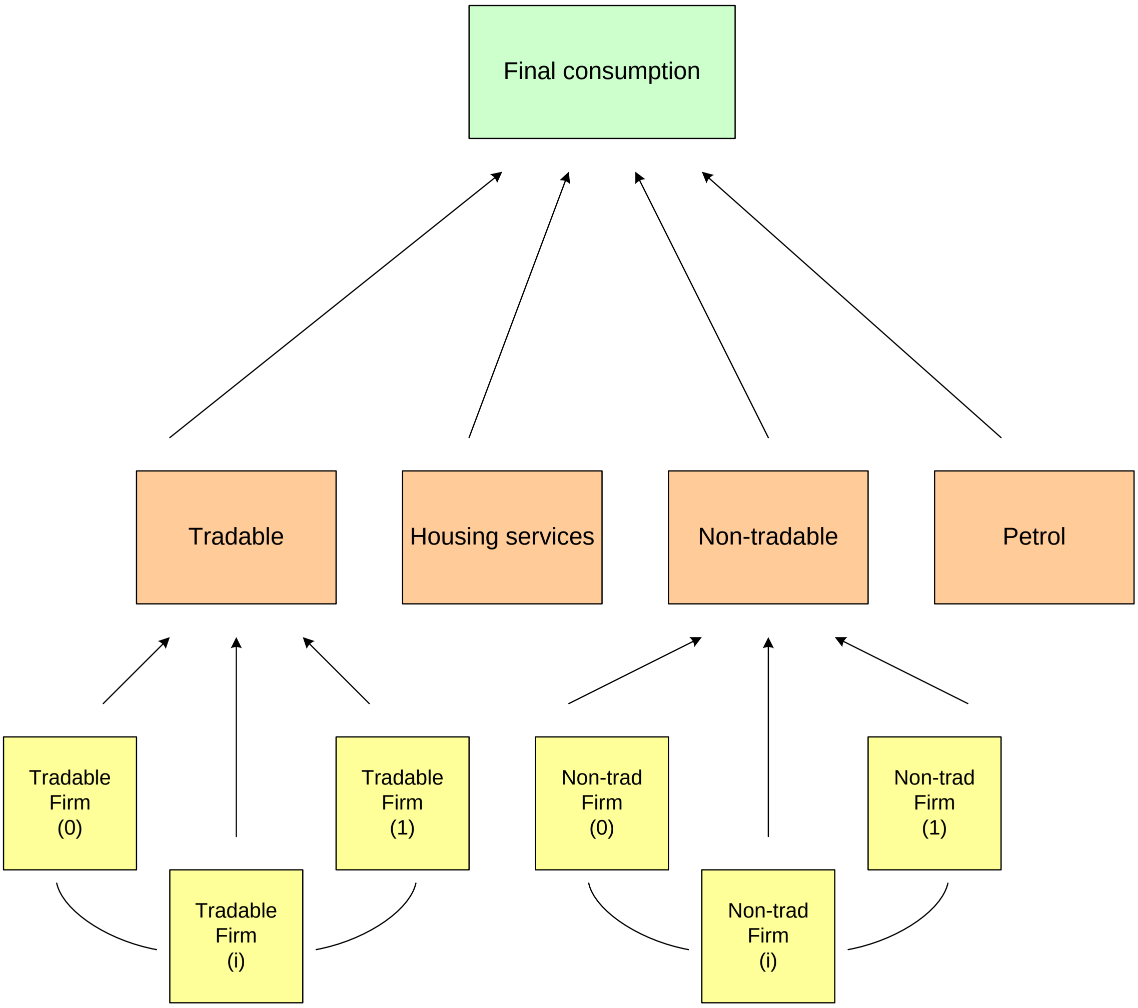
[6] We assume that the debt market is incomplete.

[7] This results in an upward sloping debt supply curve. When debt is above the long-run level it is costly to hold and households have an incentive to decrease their debt holdings.

[8] Galí (2008) provides a textbook treament.

Figure 2.3: The components of sectoral consumption



2.2 Some preliminary notation and naming conventions

Before describing in detail the mathematical relationships that underpin the DSGE model in detail, we outline our conventions for notation. Throughout the paper we:

1. use upper-case letters for competitive prices and quantities, and market-wide CES aggregates in monopolistically competitive markets;

2. use lower-case letters for prices and quantities related to individual differentiated agents in monopolistically competitive markets;

3. use lower-case Greek letters for parameters and upper-case Greek letters for Lagrange multipliers;

4. denote by bars the endogenous variables that are externalised from an agent's decision;[9]

5. use Dixit and Stiglitz's CES indices defined over continua of differentiated agents (firms or households) on intervals $[0,\ 1]$ in all monopolistically competitive markets with sticky prices/wages.

2.3 Households

The representative household consists of a continuum of members, with each of them supplying a differentiated labor service. The expected lifetime utility function is given by:

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(C_t^{\tau}, C_t^{f}, C_t^{n}, C_t^{h} \right) - V\left(\textstyle\int_0^1 \ell_{it}\, \mathrm{d}i \right) \right] \tag{2.1}$$

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[9] In equilibrium agents (and their decision rules) are symmetric so we drop the bars once we have solved each of their respective problems.

where E_0 is the expectations operator conditional on information available to the household at date 0, β is the household's time preference, C_t^{τ} is consumption of tradables, C_t^f is consumption of petrol, C_t^n is consumption of non-tradables, C_t^h is consumption of housing services, $U(\cdot)$ is the household's period utility function, ℓ_{it} is the ith household's supply of labour, and $V(\cdot)$ is the household's period disutility of labour function.

The utility and disutility functions have the following functional forms:

$$\begin{aligned} U(\cdots) \equiv\ & \omega_{\tau}(1-\chi)\log(C_t^{\tau} - \chi\bar{C}_{t-1}^{\tau}) + \omega_f(1-\chi)\log(C_t^f - \chi\bar{C}_{t-1}^f) \\ & + (1-\omega_{\tau}-\omega_f-\omega_h)(1-\chi)\log(C_t^n - \chi\bar{C}_{t-1}^n) + \omega_h \log C_t^h, \end{aligned}$$

and

$$V(\cdots) \equiv \tfrac{1}{1+\eta}\left[\int_0^1 \ell_{it}\,\mathrm{d}i\right]^{1+\eta}$$

where ω_{τ} is tradables share of consumption, ω_f is fuel's share of consumption, ω_h is housing service's share of consumption, χ is the deep habit parameter, and η is the inverse of the Frisch elasticity of labour supply. Lagged variables with a bar indicate aggregate variables the household takes as exogenous because the household is too small relative to the size of the economy to make a material impact on aggregate variables.

Tradable consumption, non-tradable consumption, petrol consumption and housing services consumption, are aggregated using a Cobb Douglas aggregator function to create the consumption index the households derive utility from.[10] Using a Cobb Douglas aggregation function allows for the incorporation of relative price trends. The unit elasticity of substitution between factors ensures the nominal factor shares remain constant on a balanced growth path. This means the income shares are not a function of the relative prices, and that regardless of the direction of relative prices, there will always be a positive demand for each input in the household's utility. However Cobb Douglas aggregation implies an elasticity of substi-

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[10] The aggregate consumption index is also created using a Cobb Douglas aggregator function.

tution that would ordinarily be too large. In practice, households find it difficult to substitute between goods in the short run (for example, it is difficult to immediately substitute away from consuming petrol). We model this by using deep habits (Ravn et al., 2006), that is, we allow for habits not only over intertemporal consumption decisions but also intratemporal consumption decisions. This means habit not only lowers the intertemporal elasticity of substitution when we are away from the balanced growth path, but it also lower the intratemporal elasticity of substitution. In the short run (off the balanced growth path) the different varieties of consumption are less substitutable.

Households maximise their expected lifetime utility by choosing allocations of tradables consumption C_t^τ, non-tradables consumption C_t^n, petrol consumption C_t^f, wages w_{it}, labour ℓ_{it}, debt holdings B_t, the business capital stock K_t, the housing capital stock H_t, business investment I_t^k, and residential investment I_t^h, subject to four constraints. First, the budget constraint, consisting of the following four basic parts:

(i) the change in household's debt and the debt servicing costs,

$$B_t \exp\left(\varepsilon_t^c\right) - B_{t-1}(1 + r_{t-1}^h) \cdots$$

where r_{t-1}^h is the effective interest rate and $\exp\left(\varepsilon_t^c\right)$ is a consumption preference shock;[11]

(ii) consumption and investment expenditures,

$$\cdots - P_t^\tau(C_t^\tau + I_t^k) - P_t^f C_t^f - P_t^n C_t^n - P_t^c I_t^h \cdots$$

where P_t^τ is the aggregate price of tradables, P_t^f is the aggregate price of petrol, P_t^n is the aggregate price of non-tradables and P_t^c is aggregate construction costs;

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[11] Although the shock term ε_t^c is appended to the debt term in the household's budget constraint, the will only appear in the household's debt Euler equation affecting the relative price of consumption today and tomorrow, hence its interpretation as a consumption preference shock.

(iii) factor payments (that is, labour income plus business capital rental), cash-flow claims on all domestic firms, and private costs paid by these firms to the household,

$$\cdots + \int_0^1 w_{it}\ell_{it}\,\mathrm{d}i + R_t K_{t-1} + \Pi_t \cdots$$

where R_t is the rental on business capital, and Π_t is profits and adjustment costs repatriated to households; and

(iv) adjusment costs of changing the consumption of fuel, business and housing investment, and re-optimising each individual wage rate,

$$\begin{aligned} &\cdots - \tfrac{1}{2}\phi_c P_t^f \bar{C}_t^f \left(\log C_t^f - \log C_{t-1}^f\right)^2 \\ &- \tfrac{1}{2}\iota_h P_c^\tau \bar{I}_t^h \left(\log I_t^h - \log \bar{I}_{t-1}^h - \varepsilon_t^{ih}\right)^2 \\ &- \tfrac{1}{2}\iota_k P_t^\tau \bar{I}_t^k \left(\log I_t^k - \log \bar{I}_{t-1}^k - \varepsilon_t^{ik}\right)^2 \\ &\qquad\qquad - \int_0^1 \left[\tfrac{1}{2}\xi_w \bar{W}_t \bar{L}_t \left(\Delta \log w_{it} - \Delta \log \bar{W}_{t-1} - \varepsilon_t^w\right)^2\right] \mathrm{d}i \end{aligned}$$

where ϕ_c is the adjustment cost parameter on petrol consumption, ι_h is the adjustment cost parameter on residential investment, ε_t^{ih} is a shock to the residential investment adjustment cost, ι_k is the adjustment cost parameter on business investment, ε_t^{ik} is a shock to the business investment adjustment cost, ξ_w is a wage flexibility parameter and W_t is the aggregate wage. (Recall that bars over variables indicate aggregate variables the household takes as exogenous because they are too small to influence them.)

Second, a business capital accumulation constraint,

$$K_t = (1 - \delta_k) K_{t-1} + I_t^k. \tag{2.2}$$

where δ_k is the depreciation rate on business capital.
Third, a housing capital accumulation constraint,

$$H_t \exp(-\varepsilon_t^{\Phi h}) = (1 - \delta_h) H_{t-1} + \left(I_t^h\right)^{\gamma_h}, \tag{2.3}$$

where δ_h is the depreciation rate on housing capital and γ_h is residential investment's share in the production of new housing. The production of new additions to the housing stock implicitly involves a fixed factor (say land), normalised to one. This is a shortcut that allows diminishing returns in the one reproducable factor in the production of new houses. Diminishing returns in residential investment ensures a downward sloping demand curve for residential investment by housing assemblers and an upward sloping supply curve for new houses.
Last, CES demand functions for individual labour services,

$$\ell_{it} = \left(w_{it}/W_t\right)^{-\epsilon} L_t. \tag{2.4}$$

where ϵ is the elasticity of substitution between differentiated labour, and $L_t \equiv \left(\int_0^1 \ell_t^{1-\frac{1}{\epsilon}}\,\mathrm{d}i\right)^{\frac{\epsilon}{\epsilon-1}}$ is aggregate labour. The CES demand function for labour is the standard result of the cost minimisation problem subject to the labour aggregation constraint.

We include an additional adjustment cost term for petrol consumption to lower the intratemporal elasticity of substitution further between petrol and other consumption goods. This allows us to maintain a plausible deep habit parameter in the utility function, but obtain a more realistic short run elasticity of substitution for petrol.

We also include quadratic adjustment costs for both business and residential investment; we interpret these as capital installation costs. These serve two purposes; first, they ensure that we get a hump shaped response in investment following a shock. Second, as described in Hayashi (1982) the introduction of convex adjustment costs for investment (or alternatively concavity in the production/installation function for new capital) allows the price of new capital to differ from the price of investment. The second feature means the price of new capital goods will have non-trivial dynamics and will be equal to the expected sum of the discounted future stream of marginal products of capital. As a consequence monetary policy is able to influence the demand for investment through the discounting.

When solving the household's problem, we use three types of current-value Lagrange multipliers: Λ_t on the budget constraint, $\Lambda_t \Phi_t^k$ on the business capital constraint, and $\Lambda_t \Phi_t^h$ on the housing capital constraint. Furthermore, we use labour demand functions, (2.4), to substitute individual labour services away in (2.1). We also assume that housing services are proportional to last period's stock of housing capital and a shock, $C_t^h = H_{t-1} \exp(\varepsilon_t^{ch})$.

The household's first order conditions with respect to consumption of non-tradables, tradables and fuel are given by:

$$C_t^n = \frac{(1-\omega_\tau-\omega_f-\omega_h)(1-\chi)}{\Lambda_t P_t^n} + \chi C_{t-1}^n \tag{2.5}$$

$$C_t^\tau = \frac{\omega_\tau (1-\chi)}{\Lambda_t P_t^\tau} + \chi C_{t-1}^\tau \tag{2.6}$$

$$C_t^f = \frac{\omega_f (1-\chi)}{\Lambda_t P_t^f \varphi_t^C} + \chi C_{t-1}^f \tag{2.7}$$

where $\varphi_t^C = 1 + \phi_c \left(\log C_t^f - \log C_{t-1}^f\right) - \mathrm{E}_t \, \beta \phi_c \left(\log C_{t+1}^f - \log C_t^f\right)$ is the derivative of the adjustment cost on fuel with respect to consumption of fuel.

Deep habit implies that the demand for each variety of consumption is a weighted average of a price sensitive component (with unit elasticity) and a price inelastic component, where the weights are determined by the deep habit parameter χ. As a consequence the intratemporal elasticity of substitution between consumption goods in the utility function will be lower than the unit elasticity in the standard Cobb Douglas aggregation function.

The first order conditions for today's debt holdings B_t, today's level of business capital K_t, and today's level of housing capital H_t give the bond Euler equation, the business capital Euler equation, and the housing capital Euler equation, respectively:

$$\Lambda_t = \mathrm{E}_t \, \beta \left[\Lambda_{t+1}\left(1+r_t^h\right)\right] \exp\left(-\varepsilon_t^c\right) \tag{2.8}$$

$$\Lambda_t \Phi_t^h = \mathrm{E}_t \, \beta \left[\frac{\omega_h \exp\left(\varepsilon_{t+1}^{ch}\right)}{H_t} + \Lambda_{t+1} \Phi_{t+1}^h (1-\delta_h)\right] \exp(\varepsilon_t^{\Phi h}) \tag{2.9}$$

$$\Lambda_t \Phi_t^k = \mathrm{E}_t \, \beta \Lambda_{t+1} \left[R_{t+1} + (1-\delta_k) \Phi_{t+1}^k\right] \tag{2.10}$$

Combining equations (2.9) and (2.8) and solving forward relates the price of housing Φ_t^h to the demand for housing:

$$\Phi_0^h = \mathrm{E}_0 \sum_{t=0}^{\infty} \left[\frac{\frac{\omega_h \exp\left(\varepsilon_{t+1}^{ch}\right)}{\Lambda_{t+1} H_t} \left(1 - \delta_h\right)^t \exp\left(\sum_{s=0}^{s=t} \varepsilon_s^c + \varepsilon_s^{\Phi h}\right)}{\prod_{s=0}^{s=t} \left(1 + r_s^h\right)} \right] \tag{2.11}$$

This shows that house prices are related to a fundamental component, the expected present value of the imputed rentals receive from housing services, and an exogenous component.

Combining equations (2.10) and (2.8) and solving forward relates the price of business capital Φ_t^k to the demand for business capital:

$$\Phi_0^k = \mathrm{E}_0 \sum_{t=0}^{\infty} \left[\frac{R_{t+1} \left(1 - \delta_k\right)^t}{\prod_{s=0}^{s=t} \left(1 + r_s^h\right) \exp\left(\sum_{s=0}^{s=t} -\varepsilon_s^c\right)} \right] \tag{2.12}$$

The fundamental price of business capital is equal to the expected present value of the rentals received from that unit of capital.

The household's first order conditions for business investment I_t^k, and residential investment I_t^h are given by:

$$\Phi_t^k / P_t^{\tau} = 1 + \iota_k \left(\log I_t^k - \log I_{t-1}^k - \varepsilon_t^{ik}\right) \tag{2.13}$$

$$\frac{\gamma_h \Phi_t^h \left(I_t^h\right)^{\gamma_h - 1}}{P_t^c} = 1 + \iota_h \left(\log I_t^h - \log I_{t-1}^h - \varepsilon_t^{ih}\right) \tag{2.14}$$

These equations form the capital assemblers demand and supply equations for investment and new capital goods respectively.

Combining equation (2.8) with equations (2.5), (2.6) and (2.7) gives the following consumption Euler equations for non-tradable, tradable and petrol respectively:

$$\frac{\mathrm{E}_t\, C^n_{t+1} - \chi C^n_t}{C^n_t - \chi C^n_{t-1}} = \beta\, \mathrm{E}_t \left(\frac{1+\pi^n_{t+1}}{1+r^h_t}\right) \exp(\varepsilon^c_t) \tag{2.15}$$

$$\frac{\mathrm{E}_t\, C^\tau_{t+1} - \chi C^\tau_t}{C^\tau_t - \chi C^\tau_{t-1}} = \beta\, \mathrm{E}_t \left(\frac{1+\pi^\tau_{t+1}}{1+r^h_t}\right) \exp(\varepsilon^c_t) \tag{2.16}$$

$$\frac{\mathrm{E}_t\, C^f_{t+1} - \chi C^f_t}{C^f_t - \chi C^f_{t-1}} = \beta\, \mathrm{E}_t \left(\frac{(1+\pi^f_{t+1})\frac{\varphi^C_{t+1}}{\varphi^C_t}}{1+r^h_t}\right) \exp(\varepsilon^c_t) \tag{2.17}$$

$$\tag{2.18}$$

The household's first order condition with respect to wages w_{it}, is given by:

$$\begin{aligned} &\tfrac{\epsilon}{\epsilon-1}\Phi^w_t/W_t - (w_{it}/W_t) = \\ &\tfrac{\xi_w}{\epsilon-1}\left[A_w\left(\Delta \log w_{it} - \Delta \log \bar{W}_{t-1} - \varepsilon^w_t\right) - \mathrm{E}_t\, B_w\left(\Delta \log w_{it+1} - \Delta \log \bar{W}_t\right)\right] \end{aligned} \tag{2.19}$$

where:

$$A_w \equiv (w_{it}/W_t)^\epsilon, \tag{2.20}$$

$$B_w \equiv \frac{\beta \Lambda_{t+1}}{\Lambda_t}\frac{W_{t+1}L_{t+1}}{W_t L_t}\left(\frac{w_{it}}{W_t}\right)^\epsilon \tag{2.21}$$

$$\Phi^w_t = L^\eta_t/\Lambda_t \tag{2.22}$$

and Φ^w_t is the marginal rate of substitution between the utility of consumption and the disutility of working.

2.4 Financial intermediary

Households are unable to access foreign debt markets directly. Instead they must operate via a financial intermediary. The financial intermediary borrows from abroad at the policy rate and then loans the money out at the effective rate. The effective rate is a function of the ratio of net foreign debt stock to the nominal housing stock (loan to value ratio). The reduced form relationship we assume follows the intuition of Aoki et al. (2004) and Bernanke et al. (1999).[12] Implicitly there is a collateral constraint in the model. Households must secure debt using collateral, in this case their homes. When household debt is high relative to the value of the housing stock, the market rate is high reflecting the extra risk of default.[13] When household debt is low relative to the value of the housing stock, the market rate is lower reflecting the reduced risk of default. We link the value of the housing stock to net foreign debt to close the model in a similar way to the debt elastic interest premia in Schmitt-Grohé and Uribe (2003).

We have taken this approach to closing the model in order to link house price movements more directly with consumption. As Aoki et al. (2004) point out assuming households own their own houses means that changes in the value of the housing stock does not have wealth effects. By including the value of the housing stock in the premia charged on debt links the supply of debt (and hence consumption) to house prices.

We assume the effective interest rate charged by the financial intermediary has the following reduced form:

$$r_t^h = r_t + \zeta \left(\frac{B_t}{\Phi_t^h H_t} - \lambda \right) \tag{2.23}$$

where r_t is the policy rate, $\frac{B_t}{\Phi_t^h H_t}$ is the loan to value ratio, ζ influences the semi-elasticity of the effective interest rate with respect to the loan to value ratio, and λ is the steady state loan to value ratio.

The effective interest rate is a function of the policy rate set by the central bank and a risk premia term that is a function of the loan to value

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[12] Although Aoki et al. (2004) and Bernanke et al. (1999) are both set in a closed economy environment.

[13] Bernanke et al. (1999) motivate this premia as the result of the financial intermediary solving a costly state verification problem. If borrowers default on their loans, the financial intermediary must incur a cost to observe the borrowers revenue and reclaim what they can. Increased debt relative to collateral is assumed to increase the probability of default. As a consequence the financial intermediary charges a higher rate on loans with less collateral.

ratio relative to its steady state level. An increase (decrease) in debt relative to the steady state level, will push up (down) the effective interest. The effective interest rate closes the model by introducing an upward sloping supply of debt relative to the housing stock, making it costly for households to increase their demand for debt.

2.5 Supply of factor services

We represent the competitive factor services market by a single firm. This is because firms are too small to influence the behaviour of other firms, and they are symmetric in equilibrium. The factor services sector is perfectly competitive because we do not want to generate any pricing persistence in this sector. We maximise the firm's present value profits including adjustment costs of changing the fuel to output ratio,

$$\begin{aligned}\max_{F_t^z, L_t, K_t', Z_t} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \Big\{ & P_t^z Z_t - P_t^f F_t^z - W_t L_t - R_t K_t' \\ & -\tfrac{1}{2}\, \phi_z P_t^f \bar{F}_t^z \left[\log(F_t^z/\bar{Z}_t) - \log(\bar{F}_{t-1}^z/\bar{Z}_{t-1})\right]^2 \Big\}, \quad (2.24)\end{aligned}$$

subject to a production function,

$$Z_t = (F_t^z)^{\gamma_{z1}} \left[A_t(L_t - L_0)\right]^{\gamma_{z2}} (K_t')^{1-\gamma_{z1}-\gamma_{z2}}. \quad (2.25)$$

where E_0 is the conditional expectations operator, Λ_t is the shadow price of wealth, β is the time preference, P_t^z is the price of the intermediate good, Z_t is the intermediate good, P_t^f is the price of petrol, F_t^z is the demand for petrol in the production of intermediate goods, W_t is the wage level, L_t is hours worked, R_t is the rental rate on capital, K_t' is the demand for capital, ϕ_z is a sector specific cost parameter, A_t is a sector specific cost parameter, L_0 is overhead labour, γ_{z1} is fuel's share of production in the intermediate good and γ_{z2} is labour's share of production in the intermediate good's producing sector.

We use a Cobb Douglas production function to allow for relative price trends between the different factor inputs. However the unit elasticity of subsitution implied by the Cobb Douglas production function is too high for the elasticity of substitution between oil and other inputs. We reduce this elasticity, in the short run, with the addition of an adjustment cost on oil. This reflects the difficulties, at least in the short run, that firms will face substituting away from oil given changes in the relative factor prices.

We follow Rotemberg and Woodford (1999) and use overhead labour to match both the observed elasticity of labour to intermediate output, and the observed labour's share. The use of a Cobb Douglas production function imposes an elasticity of intermediate output to labour equal to labour's share of output γ_{z1}. Including overhead labour changes this elasticity to $\frac{\gamma_{z1}L}{L-L_0}$, where L is the steady state level of labour. By choosing a suitable value of overhead labour L_0, relative to the steady state level of labour, we can alter this elasticity to match the data without changing labour's share of production.

Denoting by Φ_t^z the Lagrange multiplier on the production function (i.e. the nominal marginal cost), we obtain the following first-order conditions with respect to the three input factors, F_t^z, L_t, and K_t', respectively:

$$\gamma_{z1}\,\Phi_t^z Z_t = P_t^f F_t^z\left\{1+\phi_z\left[\log(F_t^z/Z_t)-\log(F_{t-1}^z/Z_{t-1}\right]\right\}, \tag{2.26}$$

$$\gamma_{z2}\Phi_t^z Z_t = W_t\left(L_t-L_0\right), \tag{2.27}$$

$$\left(1-\gamma_{z1}-\gamma_{z2}\right)\Phi_t^z Z_t = R_t K_t', \tag{2.28}$$

and with respect to output, Z_t:

$$P_t^z = \Phi_t^z. \tag{2.29}$$

That is, price P_t^z is equal to marginal cost Φ_t^z, the usual profit maximising condition under perfect competition.

2.6 Production of tradables

There is a continuum of monopolistically competitive firms in the tradables sector. We require monopolistic competition to incorporate sticky pricing into the tradables sector. This allows firms to set prices that are different to their competitors and not go out of business, this means firms do not have to price at the frictionless optimal price to stay in business. Sticky prices are required to generate non-nuetralities in the short-run. We maximise each firm's present value which includes two types of adjustment costs: one associated with changing the fuel to output ratio, this is to lower the elasticity of substitution between fuel and the other factors of production, and the other associated with re-optimising the final price, to introduce sticky prices,

$$\begin{aligned} \max_{z^{\tau}_{it}, f^{\tau}_{it}, m^{q}_{it}, p^{\tau}_{it}} \; & \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \Big\{ p^{\tau}_{it} y^{\tau}_{it} - P^f_t f^{\tau}_{it} - P^z_t z^{\tau}_{it} - P^q_t m^q_{it} \\ & - \tfrac{1}{2}\, \phi_\tau P^f_t F^{\tau}_t \left[\log(f^{\tau}_{it}/Y^{\tau}_t) - \log(F^{\tau}_{t-1}/Y^{\tau}_{t-1}) \right]^2 \\ & \qquad - \tfrac{1}{2}\, \xi_\tau P^{\tau}_t Y^{\tau}_t \left[\Delta \log p^{\tau}_{it} - \Delta \log P^{\tau}_{t-1} - \varepsilon^{p\tau}_t \right]^2 \Big\}, \end{aligned} \tag{2.30}$$

subject to a production function

$$y^{\tau}_{it} = (f^{\tau}_{it})^{\gamma_{\tau 1}} (z^{\tau}_{it})^{\gamma_{\tau 2}} (m^q_{it})^{1-\gamma_{\tau 1}-\gamma_{\tau 2}}, \tag{2.31}$$

and a CES demand curve

$$y^{\tau}_{it} = (p^{\tau}_{it}/P^{\tau}_t)^{-\epsilon} Y^{\tau}_t .$$

where p^{τ}_{it} is the price set by firm i, y^{τ}_{it} is the demand for the ith tradable firm, $Y^{\tau}_t = \left(\int_0^1 (y^{\tau}_{it})^{1-\frac{1}{\epsilon}} \, \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}}$ is aggregate tradables output, where $\epsilon > 1$ is the elasticity of substitution between differentiated tradable goods, f^{τ}_{it} is the demand for fuel by the ith tradable firm, z^{τ}_{it} is the ith tradable firm's demand for intermediate goods, P^q_t is the price of non-oil imports, m^q_{it} is the ith firm's demand for non-oil imports, ϕ_τ is a sector specific cost parameter associated with the demand for fuel, ξ_τ is a sector specific cost term associated with changing prices, $\gamma_{\tau 1}$ is fuel's share of production in tradables and $\gamma_{\tau 2}$ is the intermediate good's share of tradables production.

We introduce monopolistic competition into the model by using CES aggregation following Dixit and Stiglitz (1977). Convexity of the CES aggregator function implies goods are differentiated, that is consumer's have preferences for variety, and there are diminishing returns to each firm's output in the production of the aggregate tradable good. As a consequence each firm faces a downward sloping demand curve for their produce, a necessary condition for firms to be able to set their own prices. Imposing $\epsilon > 1$ ensures the different firms outputs will be sufficiently substitutable, and hence the demand curves are sufficiently flat that marginal revenue will be positive for all levels of output. In fact marginal revenue will always be a constant fraction, $\frac{\epsilon-1}{\epsilon}$ of the price level implied by the demand curve.

The firms producing tradable goods use a constant returns to scale Cobb Douglas production technology. This allows for relative price trends between the different factors of production, for the same reasons outlined above when describing the Cobb Douglas aggregation of consumption in the utility function. We motivate the inclusion of an additional adjustment cost on fuel prices for similar reasons outlined above when discussing consumption of fuel in the utility function and the budget constraint. The unitary elasticity of substitution between the factors of production that guarantee a balanced growth path with relative price trends, may be too high, at least in the short run. We assume that it may be quite difficult in the short run to substitute away from oil in the production of tradables given a shift in the relative prices. The inclusion of the adjustment cost term will lower the elasticity of substitution between oil and other inputs quite substantially in the short run (off the balanced growth path).

Denoting by Φ^τ_{it} the Lagrange multiplier on the firm's production function, and substituting for output from the demand equation, we obtain the following first-order conditions with respect to input factors, f^τ_{it}, z^τ_{it}, and

m^q_{it}, respectively:

$$\gamma_{\tau 1}\,\Phi^\tau_{it} y^\tau_{it} = P^f_t\, f^\tau_{it} + \phi_\tau P^f_t F^\tau_t \left[\log(f^\tau_{it}/Y^\tau_t) - \log(F^\tau_{t-1}/Y^\tau_{t-1})\right], \tag{2.32}$$

$$\gamma_{\tau 2}\,\Phi^\tau_{it} Y^\tau_t = P^z_t\, z^\tau_{it}, \tag{2.33}$$

$$(1-\gamma_{\tau 1}-\gamma_{\tau 2})\,\Phi^\tau_{it} Y^\tau_t = P^q_t\, m^q_{it}, \tag{2.34}$$

and with respect to the final price, p^τ_{it}:

$$\begin{aligned} &\tfrac{\epsilon}{\epsilon-1}\,\Phi^\tau_{it}/P^\tau_t - (p^\tau_{it}/P^\tau_t) = \\ &\tfrac{\xi_\tau}{\epsilon-1}\left[A_\tau\left(\Delta\log p^\tau_{it} - \Delta\log P^\tau_{t-1} - \varepsilon^{pn}_t\right) - \mathrm{E}_t B_\tau\left(\Delta\log p^\tau_{it+1} - \Delta\log P^\tau_t\right)\right] \end{aligned} \tag{2.35}$$

where

$$A_\tau \equiv (p^\tau_{it}/P^\tau_t)^\epsilon, \tag{2.36}$$

$$B_\tau \equiv \frac{\beta\Lambda_{t+1}}{\Lambda_t}\,\frac{P^\tau_{t+1}Y^\tau_{t+1}}{P^\tau_t\,Y^\tau_t}\left(\frac{p^\tau_{it}}{P^\tau_t}\right)^\epsilon. \tag{2.37}$$

$$\frac{\Phi^\tau_{it}}{P^\tau_t} = \left(\tfrac{1}{P^\tau_t}\right)\left(\tfrac{P^f_t\,\varphi^f_t}{\gamma_{\tau 1}}\right)^{\gamma_{\tau 1}}\left(\tfrac{P^z_t}{\gamma_{\tau 2}}\right)^{\gamma_{\tau 2}}\left(\tfrac{P^q_t}{1-\gamma_{\tau 1}-\gamma_{\tau 2}}\right)^{1-\gamma_{\tau 1}-\gamma_{\tau 2}} \tag{2.38}$$

where $\varphi^f_t = 1 + \phi_\tau\left[\log\left(F^\tau_t/Y^\tau_t\right) - \log\left(F^\tau_{t-1}/Y^\tau_{t-1}\right)\right]$ is the first derivative of the oil adjustment cost with respect to oil. $\frac{\Phi^\tau_{it}}{P^\tau_t}$ can be interpreted as the real marginal cost in the tradable sector.

2.7 Production of non-tradables

There is a continuum of monopolistically competitive firms in the non-tradables sector producing non-tradable goods. We require monopolistic competition to allow for sticky prices in the non-tradable sector. We require sticky prices to ensure monetary policy is non-neutral in the short run. Firm's maximise their present value which includes their production costs and the cost associated with changing prices:

$$\max_{z_{it}^n, p_{it}^n} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left\{ p_{it}^n y_{it}^n - P_t^z z_{it}^n - \tfrac{1}{2} \xi_n P_t^n Y_t^n \left[\Delta \log p_{it}^n - \Delta \log P_{t-1}^n - \varepsilon_t^{pn} \right]^2 \right\}, \tag{2.39}$$

subject to the following production function

$$y_{it}^n = A_t^n \left(z_{it}^n \right)^{\gamma_n}, \tag{2.40}$$

and the CES demand for their variety of product

$$y_{it}^n = \left(p_{it}^n / P_t^n \right)^{-\epsilon} Y_t^n \tag{2.41}$$

where p_{it}^n is the price set by the ith firm, y_{it}^n is the demand for the ith firms variety, z_{it}^n is the ith firm's demand for the intermediate good, P_t^n is the aggregate price of non-tradable goods, Y_t^n is the aggregate non-tradable output, A_t^n is the sector specific technology for non-tradables, γ_n is the intermediate good's share of non-tradables and ξ_n is a sector specific cost parameter.

Denoting Φ_{it}^n the Lagrange multiplier for the production constraint and substituting in the demand for the ith firm's output we get the following first order condition with respect to intermediate goods.

$$\gamma_n \Phi_{it}^n y_{it}^n = P_t^z z_{it}^n \tag{2.42}$$

and with respect to the ith firm's price p_{it}^n

$$\begin{aligned}&\tfrac{\epsilon}{\epsilon-1}\Phi^n_{it}/P^n_t-(p^n_{it}/P^n_t)=\\&\tfrac{\xi_\tau}{\epsilon-1}\left[A_n\left(\Delta\log p^n_{it}-\Delta\log P^n_{t-1}-\varepsilon^{pn}_t\right)-\mathrm{E}_t\,B_n\left(\Delta\log p^n_{it+1}-\Delta\log P^n_t\right)\right]\end{aligned} \tag{2.43}$$

where

$$A_n\equiv\left(p^n_{it}/P^n_t\right)^\epsilon, \tag{2.44}$$

$$B_n\equiv\frac{\beta\Lambda_{t+1}}{\Lambda_t}\frac{P^n_{t+1}Y^n_{t+1}}{P^n_tY^n_t}\left(\frac{p^n_{it}}{P^n_t}\right)^\epsilon \tag{2.45}$$

$$\frac{\Phi^n_{it}}{P^n_t}=\frac{P^z_t z^n_{it}}{\gamma_n P^n_t y^\tau_{it}}=(1/\gamma_n)\left(\frac{P^z_t}{P^n_tA^\tau_t}\right)\left(\frac{y^\tau_{it}}{A^\tau_t}\right)^{\frac{1}{\gamma_n}-1} \tag{2.46}$$

where $\frac{\Phi^n_{it}}{P^n_t}$ can be interpreted as the real marginal cost for firm i in the non-tradable sector.

2.8 Production of residential investment goods

There is a continuum of monopolistically competitive firms producing residential investment goods. We require monopolistic competition to introduce sticky prices to ensure monetary policy is non-neutral in the short run. These firms maximise their present which includes production costs and the cost associated with adjusting prices:

$$\max_{z^c_{it},p^c_{it}}\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\Lambda_t\left\{p^c_{it}y^c_{it}-P^z_tz^c_{it}-\tfrac{1}{2}\xi_cP^c_tY^c_t\left[\Delta\log p^c_{it}-\Delta\log P^c_{t-1}-\varepsilon^{pc}_t\right]^2\right\}$$

(2.47)

subject to a production function

$$y_{it}^{c} = A_t^{c} \left(z_{it}^{c}\right)^{\gamma_c} \tag{2.48}$$

and the CES production function

$$y_{it}^{c} = \left(p_{it}^{c}/P_t^{c}\right)^{-\epsilon} Y_t^{c} \tag{2.49}$$

where p_{it}^{c} is the price of the ith firms output, y_{it}^{c} is the demand for the ith firm's output, z_{it}^{c} is the ith construction firms demand for intermediate goods, P_t^{c} is the aggregate price for residential investment, Y_t^{c} is aggregate residential investment, ξ_c is a sector specific adjustment cost, A_t^{c} is sector specific technology in the construction sector and γ_c is intermediate's share of production in residential investment.

Solving for the lagrange multiplier associated with the production function as Φ_c and the first order condition with respect to the intermediate good,

$$\gamma_c \Phi_{it}^{c} y_{it}^{c} = P_t^{z} z_{it}^{c} \tag{2.50}$$

and the first order condition with respect to the price as,

$$\begin{aligned} &\tfrac{\epsilon}{\epsilon-1} \Phi_{it}^{c}/P_t^{c} - \left(p_{it}^{c}/P_t^{c}\right) = \\ &\tfrac{\xi_c}{\epsilon-1} \left[A_c \left(\Delta \log p_{it}^{c} - \Delta \log P_{t-1}^{c} - \varepsilon_t^{pc}\right) - \mathrm{E}_t\, B_c \left(\Delta \log p_{it+1}^{c} - \Delta \log P_t^{c}\right)\right] \end{aligned} \tag{2.51}$$

where

$$A_c \equiv \left(p^c_{it}/P^c_t\right)^{\epsilon}, \tag{2.52}$$

$$B_c \equiv \frac{\beta\Lambda_{t+1}}{\Lambda_t}\frac{P^c_{t+1}Y^c_{t+1}}{P^c_tY^c_t}\left(\frac{p^c_{it}}{P^c_t}\right)^{\epsilon}, \tag{2.53}$$

$$\frac{\Phi^c_{it}}{P^c_t} = \frac{P^z_t z^c_{it}}{\gamma_c P^c_t y^c_{it}} = (1/\gamma_c)\left(\frac{P^z_t}{P^c_t A^c_t}\right)\left(\frac{y^c_{it}}{A^c_t}\right)^{\frac{1}{\gamma_c}-1} \tag{2.54}$$

where $\frac{\Phi^c_{it}}{P^c_t}$ can be interpreted as the real marginal cost faced by the ith firm in the construction sector.

2.9 Production of manufactured exports

There is a continuum of monopolistically competitive manufactured exports producers. These firms set prices in the foreign currency. From the foreign countries perspective, this will reduce exchange rate pass-through. From the domestic countries perspective, their reduced form Phillips curve will be in terms of the export price in foreign currency, that is the supply of the non-commodity export will a function of the nominal exchange rate. They maximise their present value which includes production costs and a cost to changing prices:

$$\max_{z^v_{it}, p^{v*}_{it}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left\{ p^{v*}_{it} x^v_{it} - S_t P^z_t z^v_{it} - \tfrac{1}{2}\xi_v P^{v*}_t X^v_t \left[\Delta \log p^{v*}_{it} - \Delta \log P^{v*}_{t-1} - \varepsilon^{pv*}_t\right]^2 \right\} \tag{2.55}$$

subject to a production function

$$x_{it}^{v} = A_{t}^{v}\left(z_{it}^{v}\right)^{\gamma_{v}} \tag{2.56}$$

and a CES demand function

$$x_{it}^{v} = \left(p_{it}^{v*}/P_{t}^{v*}\right)^{-\epsilon} X_{t}^{v} \tag{2.57}$$

where p_{it}^{v*} is the price set by the ith manufacturing export firm, note that this price is in foreign currency that is the exporting firm prices in the currency of the export market. The domestic currency price of the export is $p_{it}^{v} = p_{it}^{v*}/S_{t}$ where S_{t} is the nominal exchange rate. x_{it}^{v} is the demand for the ith firm's variety of export good, z_{it}^{v} is the ith exporting firm's demand for the intermediate good, ξ_{v} is a sector specific cost parameter, A_{t}^{v} is sector specific technology in the manufactured export sector and γ_{v} is the intermediate's share of manufactured exports.

Letting Φ_{it}^{v} denote the lagrange multiplier (in domestic currency) associated with the production constraint, we get the first order condition associated with the intermediate good

$$\gamma_{v}\Phi_{it}^{v}x_{it}^{v} = P_{t}^{z}z_{it}^{v} \tag{2.58}$$

and the first order condition associated with prices

$$\begin{gathered}\tfrac{\epsilon}{\epsilon-1}\Phi_{it}^{v}/P_{t}^{v} - \left(p_{it}^{v*}/P_{t}^{v*}\right) = \\ \tfrac{\xi_{v}}{\epsilon-1}\left[A_{v}\left(\Delta\log p_{it}^{v*} - \Delta\log P_{t-1}^{v*} - \varepsilon_{t}^{pv*}\right) - \mathrm{E}_{t}\,B_{v}\left(\Delta\log p_{it+1}^{v*} - \Delta\log P_{t}^{v*}\right)\right]\end{gathered} \tag{2.59}$$

where

$$A_v \equiv \left(p_{it}^{v*}/P_t^{v*}\right)^{\epsilon}, \tag{2.60}$$

$$B_v \equiv \frac{\beta\Lambda_{t+1}}{\Lambda_t}\frac{P_{t+1}^{v*}X_{t+1}^{v}}{P_t^{v*}X_t^{v}}\left(\frac{p_{it}^{v*}}{P_t^{v*}}\right)^{\epsilon}. \tag{2.61}$$

$$\frac{\Phi_{it}^{v}}{P_t^{v}} = \frac{P_t^{z}z_{it}^{v}}{\gamma_v P_t^{v}x_{it}^{v}} = (1/\gamma_v)\left(\frac{P_t^{z}}{P_t^{v}A_t^{v}}\right)\left(\frac{x_{it}^{v}}{A_t^{v}}\right)^{\frac{1}{\gamma_v}-1} \tag{2.62}$$

where $\frac{\Phi_{it}^{v}}{P_t^{v}}$ can be interpreted as the real marginal cost (in domestic currency) of producing manufactured exports by firm i.

2.10 International flows

Debt in New Zealand is mainly denominated in domestic currency. As a result we model the balance of payments equation as follows:

$$B_t = (1+r_{t-1})\,B_{t-1}\exp\left(\varepsilon_t^{b}\right) - \left(P_t^{v}X_t^{v} + P_t^{d}X_t^{d} - P_t^{q}M_t^{q} - P_t^{o}M_t^{o}\right) \tag{2.63}$$

where $\exp(\varepsilon_t^{b})$ is a shock, P_t^{v} is the domestic currency price of manufactured exports, P_t^{d} is the domestic currency price of commodity exports, P_t^{q} is the domestic currency price of non-oil imports, P_t^{o} is the domestic currency price of oil and M_t^{o} is the oil imports.

We have the following modified uncovered interest parity equation.

$$r_t - r_t^{*} + \Delta\log S_{t+1} = \theta\left(r_{t-1} - r_{t-1}^{*} + \Delta\log S_t\right) + \varepsilon_t^{s} \tag{2.64}$$

where r^{*} is the foreign interest rate, S_t is the exchange rate, θ is the weight on backward looking agents (chartists), and ε_t^s is a portfolio shock. We can interpret $\theta\left(r_{t-1} - r_{t-1}^* + \Delta \log S_t\right)$ as an endogenous risk premia.

Standard uncovered interest rate parity is extremely forward looking, and tends to be too 'jumpy' relative to the data. To see why this is, we need to solve the standard UIP equation forward for an infinite number of periods.[14] This reveals that today's exchange rate is equal to the sum of all future interest rate differentials, and the undiscounted terminal exchange rate (the exchange rate in period infinity). Because the nominal exchange rate does not have a steady state, it is relative price movements that determine the value of the terminal exchange rate. In the first period following a shock, the exchange rate jumps because agents in the model have full knowledge of the terminal exchange rate. To avoid this excess volatility we use a modified version of UIP that has some backward looking behaviour.[15] This reduces the initial impact of the terminal condition on today's exchange rate.

2.11 Policy

The monetary and fiscal authorities do not set policy according to optimising rules, but instead follow simple rules for setting policy. The monetary authority sets policy according to the following rule:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)\left[r + \kappa\left(\mathrm{E}_t\, \pi_{t+1} - \mathrm{E}_t\, \bar{\pi}_{t+1}\right)\right] + \varepsilon_t^r \tag{2.65}$$

---

[14] We can write the standard UIP condition as:

$$\log S_t = r_t - r_t^* + \mathrm{E}_t \log S_{t+1}$$

which we can solve forward to get:

$$S_t = \mathrm{E}_t \sum_{T=0}^{\infty} \left(r_t - r_t^*\right) + \mathrm{E}_t \log S_{t+\infty}$$

where the terminal exchange rate $\mathrm{E}_t \log S_{t+\infty}$ is driven by movements in relative prices.

[15] Adolfson et al. (2007b) and Murchison and Rennison (2006) both take a similar approach to modelling UIP.

where r_t is the policy rate, ρ_r is a smoothing parameter, r is the steady state policy rate, $E_t\bar{\pi}_{t+1}$ is the expected date $t+1$ inflation target, κ is the gap between expected inflation and the inflation target, and ε_t^r is a monetary policy shock, representing deviations from the monetary policy rule.

CPI inflation, π_t is defined as:

$$\pi_t = (1 - \nu_c - \nu_\tau - \nu_f)\pi_t^n + \nu_\tau\pi_t^\tau + \nu_c\pi_t^c + \nu_f\pi_t^f \tag{2.66}$$

where π_t^n is non-tradable inflation, π_t^τ is tradable inflation, π_t^c is construction cost inflation and π_t^f is petrol price inflation.

The fiscal authority sets government spending according to the following rule:

$$\log G_t = \rho_g \log G_{t-1} + (1 - \rho_g)\log\left(\frac{\sigma NGDP_t}{P_t^n}\right) + \varepsilon_t^g \tag{2.67}$$

where G_t is government consumption, ρ_g is a smoothing parameter, $\sigma \in [0, 1]$ is the weight put on the business cycle, $NGDP_t$ is nominal GDP and ε_t^g is a government spending shock. Because the government always runs balanced budgets, net transfers are zero and hence, do not appear in the household's budget constraint. The second term in the government spending rule means government spending is procyclical to generate extra inflationary pressure.

2.12 Foreign sector

Foreign GDP is constructed using commodity and non-commodity exports. They choose allocations of commodity and non-commodity exports to maximise profits, subject to some adjustment costs.

$$\begin{aligned} P_t^{w*}Y_t^* - P_t^{d*}X_t^d - P_t^{v*}X_t^v - \tfrac{1}{2}\eta_d P_t^{d*}\overline{X}_t^d\left[\log X_t^d - \log \overline{X}_{t-1}^d\right]^2 \\ - \tfrac{1}{2}\eta_v P_t^{v*}\overline{X}_t^v\left[\log X_t^v - \log \overline{X}_{t-1}^v\right]^2 \end{aligned} \tag{2.68}$$

subject to the production function

$$Y_t^* = \left(X_t^d\right)^{\omega} \left(X_t^v\right)^{1-\omega} \tag{2.69}$$

The adjustment costs represent the difficulties in adjusting output to changes in foreign demand or foreign prices. We get the following first order conditions for the demand for commodity and non-commodity exports.

$$\begin{align}
\omega \frac{P_t^{w*} Y_t^*}{P_t^{d*} X_t^d} &= 1 + \eta_d \left[\log X_t^d - \log X_{t-1}^d\right], \tag{2.70}\\
(1-\omega) \frac{P_t^{w*} Y_t^*}{P_t^{v*} X_t^v} &= 1 + \eta_v \left[\log X_t^v - \log X_{t-1}^v\right] \tag{2.71}
\end{align}$$

We assume the following exogenous processes for foreign variables:

$$\begin{align}
\log P_t^{w*} &= \rho_{pw*} \log P_{t-1}^{w*} + (1-\rho_{pw*}) \log P^{w*} + \varepsilon_t^{pw*} \tag{2.72}\\
\log P_t^{o*} &= \rho_{po*} \log P_{t-1}^{o*} + (1-\rho_{po*}) \log P^{o*} + \varepsilon_t^{po*} \tag{2.73}\\
\log P_t^{d*} &= \rho_{pd*} \log P_{t-1}^{d*} + (1-\rho_{pd*}) \log P^{d*} + \varepsilon_t^{pd*} \tag{2.74}\\
\pi_t^f &= EE1 \left(\pi_t^{o*} - \log S_t\right) + (1-EE1)\bar{\pi}A \notag\\
&+ EE2 \left(\log \left(P_{t-1}^o \left(1+\tau^f\right)\right) - \log P_{t-1}^f\right) + \varepsilon_t^{pf} \tag{2.75}\\
P_t^q S_t &= P^{w*} \exp \varepsilon_t^{pq} \tag{2.76}\\
r_t^* &= \rho_{r*} r_{t-1}^* + (1-\rho_{r*}) r^* + \varepsilon_t^{r*} \tag{2.77}\\
\log Y_t^* &= \rho_{y*} \log Y_{t-1}^* + \varepsilon_t^{y*} \tag{2.78}
\end{align}$$

where P_t^{w*} is the foreign price level, P_t^{o*} is the foreign oil price level, P^{d*} is the foreign commodity export price level, P_t^q is the price of non-oil imports, π_t^f is domestic petrol price inflation, π_t^{o*} is foreign oil price inflation, P_t^f is the domestic price of petrol, P_t^o is the domestic price of oil. $\rho_\star$ for $\star = pw*, po*, pd*, r*, y*$ are the autoregressive parameters for foreign prices, foreign oil prices, foreign commodity prices, foreign interest rates and foreign output. $EE1$ is the weight on the level term in fuel price inflation, $EE2$ is the weight on the change term in the fuel price equation. $\varepsilon_t^\star$ for $\star = pw*, po*, pd*, pf, pq, r*, y*$ are the shock terms for the foreign price level, the foreign price level of oil, the foreign price level of commodities, petrol price inflation, non-oil import prices, the foreign interest rate and foreign output.

We have the following definitions for foreign variables:

$$P_t^v S_t = P_t^{v*} \tag{2.79}$$

$$P_t^o S_t = P_t^{o*} \tag{2.80}$$

$$P_t^d S_t = P_t^{d*} \tag{2.81}$$

2.13 Technology

The technology processes in each of the productive sectors all follow the same AR(1) process;

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_t^A \tag{2.82}$$

$$\log A_t^n = \rho_{An} \log A_{t-1}^n + \varepsilon_t^{An} \tag{2.83}$$

$$\log A_t^c = \rho_{Ac} \log A_{t-1}^c + \varepsilon_t^{Ac} \tag{2.84}$$

$$\log A_t^\tau = \rho_{A\tau} \log A_{t-1}^\tau + \varepsilon_t^{A\tau} \tag{2.85}$$

$$\log A_t^v = \rho_{Av} \log A_{t-1}^v + \varepsilon_t^{Av} \tag{2.86}$$

Where $\rho_\star$ for $\star = A, An, Ac, A\tau, Av$ is the autoregressive parameter for the intermediate goods producing, non-tradables, construction, tradables and the manufactured sectors respectively. $\varepsilon_t^\star$ for $\star = A, An, Ac, A\tau, Av$ is the technology shock for the intermediate goods producing, non-tradables, construction, tradables and the manufactured sectors respectively.

2.14 Aggregation

Without idiosyncratic uncertainty, the model exhibits symmetric equilibria at all times. Consequently, the individual prices and quantities are identical for all agents in monopolistically competitive markets, and coincide with both the respective integrals over all agents and the market-wide CES indices (if defined):

$$w_{it} = W_t, \quad \ell_{it} = \int_0^1 \ell_{it} \,\mathrm{d}i = L_t, \tag{2.87}$$

$$p_{it}^{\tau} = P_t^{\tau}, \quad y_{it}^{\tau} = \int_0^1 y_{it}^{\tau} \,\mathrm{d}i = Y_t^{\tau}, \tag{2.88}$$

$$p_{it}^{n} = P_t^{n}, \quad y_{it}^{n} = \int_0^1 y_{it}^{n} \,\mathrm{d}i = Y_t^{n}, \tag{2.89}$$

$$p_{it}^{c} = P_t^{c}, \quad y_{it}^{c} = \int_0^1 y_{it}^{c} \,\mathrm{d}i = Y_t^{c} = I_t^{h}, \tag{2.90}$$

$$p_{it}^{v*} = P_t^{v*}, \quad x_{it}^{v} = \int_0^1 x_{it}^{v} \,\mathrm{d}i = X_t^{v}, \tag{2.91}$$

$$f_{it}^{\tau} = \int_0^1 f_{it}^{\tau} \,\mathrm{d}i = F_t^{\tau}, \quad z_{it}^{\tau} = \int_0^1 z_{it}^{\tau} \,\mathrm{d}i = Z_t^{\tau}, \quad m_{it}^{q} = \int_0^1 m_{it}^{q} \,\mathrm{d}i = M_t^{q}, \tag{2.92}$$

$$z_{it}^{n} = \int_0^1 z_{it}^{n} \,\mathrm{d}i = Z_t^{n}, \tag{2.93}$$

$$z_{it}^{c} = \int_0^1 z_{it}^{c} \,\mathrm{d}i = Z_t^{c}, \tag{2.94}$$

$$z_{it}^{v} = \int_0^1 z_{it}^{v} \,\mathrm{d}i = Z_t^{v}, \tag{2.95}$$

Under such symmetric equilibria, we can simplify some of the first-order conditions. Using the fact that in (2.36)

$$A_{\tau} = 1$$

at all times, and that

$$B_{\tau} = \beta$$

not only in steady state but up to first order also at all times (Taylor's first-order expansion yields zero coefficients for all terms included in B because $\Delta \log p_{it}^{\tau} - \Delta \log P_{t-1}^{\tau}$ is zero in steady state), we can write

$$\begin{aligned} & \tfrac{\epsilon}{\epsilon-1}\,\Phi_{it}^{\tau}/P_t^{\tau} - 1 = \\ & \quad \tfrac{\xi_{\tau}}{\epsilon-1}\left[\left(\Delta \log P_t^{\tau} - \Delta \log P_{t-1}^{\tau}\right) - \beta \mathrm{E}_t\left(\Delta \log P_{t+1}^{\tau} - \Delta \log P_t^{\tau}\right)\right], \end{aligned} \tag{2.96}$$

where the left-hand side is the deviation of the sector-specific real marginal cost from its flexible-price level, and the right-hand side is the marginal cost associated with re-optimising final prices. Which can be rewritten as:

$$\pi_t^{\tau} - \pi_{t-1}^{\tau} = \xi^{\tau}\left[\gamma_{\tau 1}\hat{p}_t^{f/\tau} + \gamma_{\tau 2}\hat{p}_t^{z/\tau} + \left(1-\gamma_{\tau 1}-\gamma_{\tau 2}\right)\hat{p}_t^{q/\tau}\right] \tag{2.97}$$

$$+\,\mathrm{E}_t\,\beta\left(\pi_{t+1}^{\tau} - \pi_t^{\tau}\right) + \varepsilon_t^{p\tau} \tag{2.98}$$

where $\hat{p}_t^{f/\tau} \equiv \log\left(\frac{P_t^f \varphi_t^f}{P^f}\right) - \log\left(\frac{P_t^{\tau}}{P^{\tau}}\right)$, $\hat{p}_t^{z/\tau} \equiv \log\left(\frac{P_t^z}{P^z}\right) - \log\left(\frac{P_t^{\tau}}{P^{\tau}}\right)$, $\hat{p}_t^{q/\tau} \equiv \log\left(\frac{P_t^q}{P^q}\right) - \log\left(\frac{P_t^{\tau}}{P^{\tau}}\right)$.

Similarly for non-tradables, the construction sector and the manufactured export sector, we obtain:

$$\pi_t^{n} - \pi_{t-1}^{n} = \left(\tfrac{\epsilon-1}{\xi^{n}}\right)\left[\hat{p}_t^{z/n} + \left(\tfrac{1}{\gamma_n} - 1\right)\hat{y}_t^{n}\right] + \mathrm{E}_t\,\beta\left(\pi_{t+1}^{n} - \pi_t^{n}\right) + \varepsilon_t^{pn} \tag{2.99}$$

$$\pi_t^{c} - \pi_{t-1}^{c} = \left(\tfrac{\epsilon-1}{\xi^{c}}\right)\left[\hat{p}_t^{z/c} + \left(\tfrac{1}{\gamma_c} - 1\right)\hat{y}_t^{c}\right] + \mathrm{E}_t\,\beta\left(\pi_{t+1}^{c} - \pi_t^{c}\right) + \varepsilon_t^{pc} \tag{2.100}$$

$$\pi_t^{v*} - \pi_{t-1}^{v*} = \left(\tfrac{\epsilon-1}{\xi^{v}}\right)\left[\hat{p}_t^{z/v} + \left(\tfrac{1}{\gamma_v} - 1\right)\hat{x}_t^{v}\right] + \mathrm{E}_t\,\beta\left(\pi_{t+1}^{v*} - \pi_t^{v*}\right) + \varepsilon_t^{pv} \tag{2.101}$$

$$\pi_t^{w} - \pi_{t-1}^{w} = \left(\tfrac{\epsilon-1}{\xi^{\tau}}\right)\left[\eta\hat{l}_t - \hat{\lambda}_t - \hat{w}_t\right] + \mathrm{E}_t\,\beta\left(\pi_{t+1}^{w} - \pi_t^{w}\right) + \varepsilon_t^{w} \tag{2.102}$$

where $\hat{p}_t^{z/n} \equiv \log\left(\frac{P_t^z}{P^z}\right) - \log\left(\frac{P_t^n}{P^n}\right)$, $\hat{y}_t^{n} \equiv \log\left(\frac{Y_t^n}{Y^n}\right)$, $\hat{p}_t^{z/c} \equiv \log\left(\frac{P_t^z}{P^z}\right) - \log\left(\frac{P_t^c}{P^c}\right)$, $\hat{y}_t^{c} \equiv \log\left(\frac{Y_t^c}{Y^c}\right)$, $\hat{p}_t^{z/v} \equiv \log\left(\frac{P_t^z}{P^z}\right) - \log\left(\frac{P_t^v}{P^v}\right)$, $\hat{x}_t^{v} \equiv \log\left(\frac{X_t^c}{X^c}\right)$, $\hat{\lambda} \equiv$

$\log\left(\frac{\Lambda_t}{\Lambda}\right)$, $\hat{l}_t \equiv \log\left(\frac{L_t}{L}\right)$ and $\hat{w}_t \equiv \log\left(\frac{W_t}{W}\right)$.

2.15 Market clearing

Using the aggregation results from the previous section, we can write the market clearing conditions as follows:

- Market for factor services:

$$Z_t = Z_t^\tau + Z_t^n + Z_t^c + Z_t^v \tag{2.103}$$

- Market for tradables:

$$Y_t^\tau = C_t^\tau + I_t^k \tag{2.104}$$

- Market for non-tradables:

$$Y_t^n = C_t^n + G_t \tag{2.105}$$

- Market for oil (fuel) imports:

$$M_t^o = F_t^z + F_t^\tau + C_t^f \tag{2.106}$$

- Market for business capital:

$$K_t' = K_{t-1} \tag{2.107}$$

- Market for housing services:

$$C_t^h = H_{t-1} \exp(\varepsilon_t^{ch}) \tag{2.108}$$

- Consumption:

$$C_t = (C_t^\tau)^{\omega_\tau} (C_t^f)^{\omega_f} \left(C_t^h\right)^{\omega_h} (C_t^n)^{1-\omega_\tau-\omega_f-\omega_h} \tag{2.109}$$

2.16 Log-linearisation

This section contains the equations that described the log linearised model.

Households: consumers

Log linearisng equations (2.8), (2.5), (2.6) and (2.7) around the steady state gives:

$$
\begin{aligned}
\hat{\lambda}_t &= \tilde{r}_t^h + \mathrm{E}_t\,\hat{\lambda}_{t+1} - \varepsilon_t^c && (2.110)\\
\hat{p}_t^n + \frac{\hat{c}_t^n}{1-\chi} + \hat{\lambda}_t &= \left(\frac{\chi}{1-\chi}\right)\hat{c}_{t-1}^n && (2.111)\\
\hat{p}_t^\tau + \frac{\hat{c}_t^\tau}{1-\chi} + \hat{\lambda}_t &= \left(\frac{\chi}{1-\chi}\right)\hat{c}_{t-1}^\tau && (2.112)\\
\hat{p}_t^f + \frac{\hat{c}_t^f}{1-\chi} + \hat{\lambda}_t + \phi_c\left(\hat{c}_t^f - \hat{c}_{t-1}^f\right) &= \left(\frac{\chi}{1-\chi}\right)\hat{c}_{t-1}^f + \\
&\quad \beta\phi_c\left(\mathrm{E}_t\,\hat{c}_{t+1}^f - \hat{c}_t^f\right) && (2.113)
\end{aligned}
$$

where $\hat{\lambda}_t \equiv \log\left(\frac{\Lambda_t}{\Lambda}\right)$, $\tilde{r}_t^h \equiv r_t^h - r^h$, $\hat{p}_t^n \equiv \log\left(\frac{P_t^n}{P^n}\right)$, $\hat{c}_t^n \equiv \log\left(\frac{C_t^n}{C^n}\right)$, $\hat{p}_t^\tau \equiv \log\left(\frac{P_t^\tau}{P^\tau}\right)$, $\hat{c}_t^\tau \equiv \log\left(\frac{C_t^n}{C^n}\right)$, $\hat{p}_t^f \equiv \log\left(\frac{P_t^f}{P^f}\right)$ and $\hat{c}_t^f \equiv \log\left(\frac{C_t^f}{C^f}\right)$.

Households: workers

Log-linearising equations (2.22) and (2.19) around the steady state gives:

$$
\begin{aligned}
\eta\hat{l}_t &= \hat{\phi}_t^w + \hat{\lambda}_t && (2.114)\\
\tilde{\pi}_t^w - \tilde{\pi}_{t-1}^w &= \xi_w\left(\eta\hat{l}_t - \hat{\lambda}_t - \hat{w}_t\right) + \beta\left(\mathrm{E}_t\,\tilde{\pi}_{t+1}^w - \tilde{\pi}_t^w\right) + \varepsilon_t^w && (2.115)
\end{aligned}
$$

where $\hat{l}_t \equiv \log\left(\frac{L_t}{L}\right)$, $\hat{\phi}_t^w \equiv \log\left(\frac{\Phi_t^w}{\Phi^w}\right)$ and $\tilde{\pi}_t^w \equiv \pi_t^w - \pi^w$.

Households: housing

Log-linearising equations (2.108),(2.9),(2.3) and (2.14) around the steady state gives:

$$\begin{aligned}
\hat{c}_t^h &= \hat{h}_{t-1} + \varepsilon_t^{ch} && (2.116)\\
\hat{\lambda}_t + \hat{\phi}_t^h + \delta_h \operatorname{E}_t \hat{c}_{t+1}^h &= (1-\delta_h)\left(\operatorname{E}_t \hat{\lambda}_{t+1} + \operatorname{E}_t \hat{\phi}_{t+1}^h + \varepsilon_t^{\phi h}\right) && (2.117)\\
\hat{h}_t &= \delta_h \gamma_h \hat{i}_t^h + (1-\delta_h)\,\hat{h}_{t-1} && (2.118)\\
\hat{\phi}_t^h + (\gamma_h - 1)\,\hat{i}_t^h - \hat{p}_t^c &= \iota_h\left(\hat{i}_t^h - \hat{i}_{t-1}^h - \varepsilon_t^{ih}\right) && (2.119)
\end{aligned}$$

where $\hat{c}_t^h \equiv \log\left(\frac{C_t^h}{C^h}\right)$, $h_t \equiv \log\left(\frac{H_t}{H}\right)$,$\hat{\phi}_t^h \equiv \log\left(\frac{\Phi_t^h}{\Phi^h}\right)$, $\hat{i}_t^h \equiv \log\left(\frac{I_t^h}{I^h}\right)$ and $\hat{p}_t^c \equiv \log\left(\frac{P_t^c}{P^c}\right)$.

Households: investors

Log-linearising equations (2.10), (2.2) and (2.13) around the steady state gives:

$$\begin{aligned}
\hat{\lambda}_t + \hat{\phi}_t^k &= \hat{\lambda}_{t+1} + \delta_k \operatorname{E}_t \hat{r}_{t+1} + (1-\delta_k) \operatorname{E}_t \hat{\phi}_{t+1}^k && (2.120)\\
\hat{k}_t &= \delta \hat{i}_t^k + (1-\delta_k)\,\hat{k}_{t-1} && (2.121)\\
\hat{\phi}_t^k - \hat{p}_t^\tau &= \iota_k\left(\hat{i}_t^k - \hat{i}_{t-1}^k - \varepsilon_t^{ik}\right) && (2.122)
\end{aligned}$$

where $\hat{\phi}_t^k \equiv \log\left(\frac{\Phi_t^k}{\Phi^k}\right)$, $\operatorname{E}_t\{\hat{r}_{t+1}\} \equiv \log\left(\frac{\operatorname{E}_t\{R_{t+1}\}}{R}\right)$, $\hat{k}_t \equiv \log\left(\frac{K_t}{K}\right)$, $\hat{i}_t^k \equiv \log\left(\frac{I_t^k}{I^k}\right)$ and $\hat{p}_t^\tau \equiv \log\left(\frac{P_t^\tau}{P^\tau}\right)$.

Supply of domestic factor services

Log-linearising equations (2.25), (2.27), (2.28) and (2.26) around the steady state gives:

$$
\begin{aligned}
\hat{z}_t &= \hat{a}_t + \gamma_{z1}\left(\tfrac{L}{L-L_0}\right)\hat{l}_t + \gamma_{z2}\hat{k}_{t-1} + \\
&\quad (1-\gamma_{z1}-\gamma_{z2})\,\hat{f}^z_t \qquad (2.123)\\
\hat{p}^z_t + \hat{z}_t &= \tfrac{WL}{\gamma_{z1}P^z Z}\left(\hat{w}_t + \hat{l}_t\right) - \tfrac{WL_0}{\gamma_{z1}P^z Z}\hat{w}_t \qquad (2.124)\\
\hat{p}^z_t - \hat{z}_t &= \hat{r}_t - \hat{k}_{t-1} \qquad (2.125)\\
\hat{p}^z_t + \hat{z}_t - \hat{p}^f_t - \hat{f}^z_t &= \phi_z\left(\hat{f}^z_t - \hat{f}_t - \hat{z}_t + \hat{z}_{t-1}\right) \qquad (2.126)
\end{aligned}
$$

where $\hat{z}_t \equiv \log\left(\frac{Z_t}{Z}\right)$, $\hat{a}_t \equiv \log\left(\frac{A_t}{A}\right)$, $\hat{f}^z_t \equiv \log\left(\frac{F^z_t}{F^z}\right)$ and $\hat{p}^z_t \equiv \log\left(\frac{P^z_t}{P^z}\right)$.

Non-tradables firms

Log-linearising equations (2.40), (2.42) and (2.43) around the steady state gives:

$$
\begin{aligned}
\hat{y}^n_t &= \hat{a}^n_t + \gamma_n \hat{z}^n_t \qquad (2.127)\\
\hat{\phi}^n_t + \hat{y}^n_t &= \hat{p}^z_t + \hat{z}^n_t \qquad (2.128)\\
\tilde{\pi}^n_t - \tilde{\pi}^n_{t-1} &= \xi_n\left[\hat{p}^{z/n}_t + \left(\tfrac{1}{\gamma_n} - 1\right)\hat{y}^n_t\right] + \beta\left(\mathrm{E}_t\,\tilde{\pi}^n_{t+1} - \tilde{\pi}^n_t\right) + \varepsilon^{pn}_t \qquad (2.129)
\end{aligned}
$$

where $\hat{y}^n_t \equiv \log\left(\frac{Y^n_t}{Y^n}\right)$, $\hat{a}^n_t \equiv \log\left(\frac{A^n_t}{A^n}\right)$, $\hat{z}^n_t \equiv \log\left(\frac{Z^n_t}{Z^n}\right)$, $\hat{\phi}^n_t \equiv \log\left(\frac{\Phi^n_t}{\Phi^n}\right)$, $\tilde{\pi}^n_t \equiv \pi^n_t - \pi^n$, and $\hat{p}^{z/n}_t \equiv \hat{p}^z_t - \hat{p}^n_t$.

Construction firms

Log-linearising equations (2.48), (2.50) and (2.51) around the steady state gives:

$$
\begin{aligned}
\hat{i}^h_t &= \hat{a}^c_t + \gamma_c \hat{z}^c_t \qquad (2.130)\\
\hat{\phi}^c_t + \hat{i}^h_t &= \hat{p}^z_t + \hat{z}^c_t \qquad (2.131)\\
\tilde{\pi}^c_t - \tilde{\pi}^c_{t-1} &= \xi_n\left[\hat{p}^{z/c}_t + \left(\tfrac{1}{\gamma_c} - 1\right)\hat{y}^c_t\right] + \beta\left(\mathrm{E}_t\,\tilde{\pi}^c_{t+1} - \tilde{\pi}^c_t\right) + \varepsilon^{pn}_t \qquad (2.132)
\end{aligned}
$$

where $\hat{a}_t^c \equiv \log\left(\frac{A_t^c}{A^c}\right)$, $\hat{z}_t^c \equiv \log\left(\frac{Z_t^c}{Z^c}\right)$, $\hat{\phi}_t^c \equiv \log\left(\frac{\Phi_t^c}{\Phi^c}\right)$, $\tilde{\pi}_t^c \equiv \pi_t^c - \pi^c$ and $\hat{p}_t^{z/c} \equiv \hat{p}_t^z - \hat{p}_t^c$.

Tradables firms

Log-linearising equations (2.31), (2.33), (2.34), (2.32) and (2.35) around the steady state gives:

$$\begin{aligned}
\hat{y}_t^\tau &= \hat{a}_t^\tau + \gamma_{\tau 1}\hat{z}_t^\tau + \gamma_{\tau 2}\hat{m}_t^q + (1-\gamma_{\tau 1}-\gamma_{\tau 2})\,\hat{f}_t^\tau && (2.133)\\
\hat{\phi}_t^\tau + \hat{y}_t^\tau &= \hat{p}_t^z + \hat{z}_t^\tau && (2.134)\\
\hat{\phi}_t^\tau + \hat{y}_t^\tau &= \hat{p}_t^q + \hat{m}_t^q && (2.135)\\
\hat{\phi}_t^\tau + \hat{y}_t^\tau - \hat{p}_t^f - \hat{f}_t^\tau &= \phi_\tau\left(\hat{f}_t^z - \hat{f}_{t-1}^z - \hat{y}_t^\tau + \hat{y}_{t-1}^\tau\right) && (2.136)\\
\tilde{\pi}_t^\tau - \tilde{\pi}_{t-1}^\tau &= \xi_\tau\left[\gamma_{\tau 1}\hat{p}_t^{z/\tau} + \gamma_{\tau 2}\hat{p}_t^{q/\tau} + (1-\gamma_{\tau 1}-\gamma_{\tau 2})\,\hat{p}_t^{f/\tau}\right] + \\
&\quad \beta\left(\mathrm{E}_t\,\tilde{\pi}_{t+1}^\tau - \tilde{\pi}_t^\tau\right) + \varepsilon_t^{p\tau} && (2.137)
\end{aligned}$$

where $\hat{y}_t^\tau \equiv \log\left(\frac{Y_t^\tau}{Y^\tau}\right)$, $\hat{a}_t^\tau \equiv \log\left(\frac{A_t^\tau}{A^\tau}\right)$, $\hat{z}_t^\tau \equiv \log\left(\frac{Z_t^\tau}{Z^\tau}\right)$, $\hat{m}_t^q \equiv \log\left(\frac{M_t^q}{M^q}\right)$, $\hat{f}_t^\tau \equiv \log\left(\frac{F_t^\tau}{F^\tau}\right)$, $\hat{\pi}_t^\tau \equiv \log\left(\frac{\Phi_t^\tau}{\Phi^\tau}\right)$, $\tilde{\pi}_t^\tau \equiv \pi_t^\tau - \pi^\tau$, $\hat{p}_t^{z/\tau} \equiv \hat{p}_t^z - \hat{p}_t^\tau$, $\hat{p}_t^{q/\tau} \equiv \hat{p}_t^q - \hat{p}_t^\tau$ and $\hat{p}_t^{f/\tau} \equiv \hat{p}_t^f - \hat{p}_t^\tau$.

Manufactured exports

Log-linearising equations (2.56), (2.58) and (2.71) around the steady state gives:

$$\begin{aligned}
\hat{x}_t^v &= \hat{a}_t^v + \gamma_v\hat{z}_t^v && (2.138)\\
\hat{\phi}_t^v + \hat{x}_t^v &= \hat{p}_t^v + \hat{z}_t^v && (2.139)\\
\tilde{\pi}_t^{v*} - \tilde{\pi}_{t-1}^{v*} &= \xi_v\left[\hat{p}_t^{z/v*} + \left(\frac{1}{\gamma_{v*}} - 1\right)\hat{x}_t^{v*}\right] + \\
&\quad \beta\left(\mathrm{E}_t\,\tilde{\pi}_{t+1}^v - \tilde{\pi}_t^v\right) + \varepsilon_t^{v*} && (2.140)\\
\hat{p}_t^{w*} + \hat{y}_t^* - \hat{p}_t^v - \hat{s}_t - \hat{x}_t^v &= \eta_v\left(\hat{x}_t^v - \hat{x}_{t-1}^v - \varepsilon_t^{xv}\right) && (2.141)
\end{aligned}$$

where $\hat{x}_t^v \equiv \log\left(\frac{X_t^v}{X^v}\right)$, $\hat{a}_t^v \equiv \log\left(\frac{A_t^v}{A^v}\right)$, $\hat{\phi}_t^v \equiv \log\left(\frac{\Phi_t^v}{\Phi^v}\right)$, $\tilde{\pi}_t^{v*} \equiv \pi_t^{v*} - \pi^{v*}$, $\hat{p}^{z/v*} = \hat{p}_t^z - \hat{p}_t^{v*}$, $\hat{p}_t^{w*} \equiv \log\left(\frac{P_t^{w*}}{P^{w*}}\right)$ and $\hat{y}_t^* \equiv \log\left(\frac{Y_t^*}{Y^*}\right)$.

Commodity exports

Log-linearising equation (2.70) around the steady state gives:

$$\hat{y}_t^* - \hat{x}_t^d = \eta_d \left(\hat{x}_t^d - \hat{x}_{t-1}^d - \varepsilon_t^{xd}\right) \tag{2.142}$$

where $\hat{x}_t^d \equiv \log\left(\frac{X_t^d}{X^d}\right)$.

International flows

Log-linearising equations (2.63), (2.23) and (2.64) around the steady state gives:

$$\begin{aligned}\hat{b}_t = r\left(\tilde{r}_{t-1} + \hat{b}_{t-1} + \varepsilon_t^b\right) - \Big[\left(\tfrac{P^v X^v}{B}\right)\left(\hat{p}_t^v + \hat{x}_t^v\right) + \left(\tfrac{P^d X^d}{B}\right)\left(\hat{p}_t^d + \hat{x}_t^d\right) \\ - \left(\tfrac{P^q M^q}{B}\right)\left(\hat{p}_t^q + \hat{m}_t^q\right) - \left(\tfrac{P^o M^o}{B}\right)\left(\hat{p}_t^o + \hat{m}_t^o\right)\Big]\end{aligned} \tag{2.143}$$

$$\tilde{r}_t^h = \tilde{r}_t + \zeta\lambda\left(\hat{b}_t - \hat{\phi}_t^h - \hat{h}_t\right) \tag{2.144}$$

$$\tilde{r}_t - \tilde{r}_t^* + \mathrm{E}_t\,\hat{s}_{t+1} - \hat{s}_t = \theta\left(\tilde{r}_{t-1} - \tilde{r}_{t-1}^*\right) + \varepsilon_t^s \tag{2.145}$$

where $\hat{b}_t \equiv \log\left(\frac{B_t}{B}\right)$, $\hat{p}_t^v \equiv \log\left(\frac{P_t^v}{P^v}\right)$, $\hat{p}_t^d \equiv \log\left(\frac{P_t^d}{P^d}\right)$, $\hat{p}_t^o \equiv \log\left(\frac{P_t^o}{P^o}\right)$, $\hat{m}_t^o \equiv \log\left(\frac{M_t^o}{M^o}\right)$, $\hat{s}_t \equiv \log\left(\frac{S_t}{S}\right)$ and $\tilde{r}_t^* \equiv r_t^* - r^*$.

Central bank

Log-linearising equations (2.65) and (2.66) around the steady state gives:

$$\tilde{r}_t = \rho_r \tilde{r}_{t-1} + (1-\rho_r)\left(\mathrm{E}_t\,\tilde{\bar{\pi}}_{t+1} + \kappa\left(\mathrm{E}_t\,\tilde{\pi}_{t+1} - \mathrm{E}_t\,\tilde{\bar{\pi}}_{t+1}\right)\right) \tag{2.146}$$

$$\hat{p}_t = (1-\nu_c-\nu_\tau-\nu_f)\,\hat{p}_t^n + \nu_c\hat{p}_t^c + \nu_\tau\hat{p}_t^\tau + \nu_f\hat{p}_t^f \tag{2.147}$$

where $\mathrm{E}_t\,\tilde{pi}_{t+1} \equiv \mathrm{E}_t\,\pi_{t+1} - \pi$, $\mathrm{E}_t\,\tilde{\bar{\pi}}_{t+1} \equiv \mathrm{E}_t\,\bar{\pi}_{t+1} - \bar{\pi}$, $\hat{p}_t \equiv \log\left(\frac{P_t}{P}\right)$, $\hat{p}_t^n \equiv \log\left(\frac{P_t^n}{P^n}\right)$, $\hat{p}_t^\tau \equiv \log\left(\frac{P_t^\tau}{P^\tau}\right)$, $\hat{p}_t^c \equiv \log\left(\frac{P_t^c}{P^c}\right)$ and $\hat{p}_t^f \equiv \log\left(\frac{P_t^f}{P^f}\right)$.

Government

Log-linearising equation (2.67) around the steady state give:

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + (1-\rho_g)\left(n\hat{g}dp_t - \hat{p}_t^n\right) \tag{2.148}$$

where $\hat{g}_t \equiv \log\left(\frac{G_t}{G}\right)$ and $n\hat{g}dp_t \equiv \log\left(\frac{NGDP_t}{NGDP}\right)$.

Market clearing

Log-linearising equations (2.103), (2.105), (2.104) and (2.106) around the steady state gives:

$$\begin{aligned}
\hat{z}_t &= \left(\tfrac{Z^n}{Z}\right)\hat{z}_t^n + \left(\tfrac{Z^c}{Z}\right)\hat{z}_t^c + \left(\tfrac{Z^\tau}{Z}\right)\hat{z}_t^\tau + \left(\tfrac{Z^v}{Z}\right)\hat{z}_t^v && (2.149)\\
\hat{y}_t^n &= \left(\tfrac{C^n}{Y^n}\right)\hat{c}_t^n + \left(\tfrac{G}{Y^n}\right)\hat{g}_t && (2.150)\\
\hat{y}^\tau &= \left(\tfrac{C^\tau}{Y^\tau}\right)\hat{c}_t^\tau + \left(\frac{I^k}{Y^\tau}\right)\hat{i}_t^k && (2.151)\\
\hat{m}_t^o &= \left(\tfrac{F^\tau}{M^o}\right)\hat{f}_t^\tau + \left(\tfrac{F^z}{M^o}\right)\hat{f}_t^z + \left(\frac{C^f}{M^o}\right)\hat{c}_t^f && (2.152)
\end{aligned}$$

Foreign Processes

Log-linearising equations (2.72), (2.73), (2.74), (2.76), (2.77) and (2.78) around the steady state gives:

$$\hat{p}_t^{w*} = \rho_{p^{w*}} \hat{p}_{t-1}^{w*} + \varepsilon_t^{p^{w*}} \tag{2.153}$$

$$\hat{p}_t^{o*} = \rho_{p^{o*}} \hat{p}_{t-1}^{o*} + \varepsilon_t^{p^{o*}} \tag{2.154}$$

$$\hat{p}_t^{d*} = \rho_{p^{d*}} \hat{p}_{t-1}^{d*} + \varepsilon_t^{p^{d*}} \tag{2.155}$$

$$\begin{aligned} \pi_t^f &= EE1 * (\pi_t^{o*} - \log S_t) + (1 - EE1) * \bar{\pi} \\ &+ EE2 * \left(\log\left(P_{t-1}^o\left(1 + \tau^f\right)\right) - \log P_{t-1}^f\right) + \varepsilon_t^{pf} \end{aligned} \tag{2.156}$$

$$\tilde{r}_t^* = \rho_{r*} \tilde{r}_{t-1}^* + \varepsilon_t^{r*} \tag{2.157}$$

$$\hat{y}_t^* = \rho_{y*} \hat{y}_{t-1}^* + \varepsilon_t^{y*} \tag{2.158}$$

Technology Processes

Log-linearising equations (2.82), (2.83), (2.84), (2.85) and (2.86) around the steady state gives:

$$\hat{a}_t = \rho_A \hat{a}_{t-1} + \varepsilon_t^A \tag{2.159}$$

$$\hat{a}_t^n = \rho_{An} \hat{a}_{t-1}^n + \varepsilon_t^{An} \tag{2.160}$$

$$\hat{a}_t^c = \rho_{Ac} \hat{a}_{t-1}^c + \varepsilon_t^{Ac} \tag{2.161}$$

$$\hat{a}_t^\tau = \rho_{A\tau} \hat{a}_{t-1}^\tau + \varepsilon_t^{A\tau} \tag{2.162}$$

$$\hat{a}_t^v = \rho_{Av} \hat{a}_{t-1}^v + \varepsilon_t^{Av} \tag{2.163}$$

Chapter 3

Data

3.1 Overview

KITT is a model of the dynamics of the economy around its steady state, or trend. Prior to estimation and forecasting, therefore, we remove trends from the data. Table 3.1 lists the data we use to estimate the model. It is important to note that the official tradable and non-tradable price indices are inclusive of petrol and construction costs respectively. To match the official data, we aggregate the model constructs of non-tradable and construction cost inflation to form the official non-tradable price series. We also aggregate the model constructs for tradable and petrol prices to form the official tradable series.

In terms of sample period selected, New Zealand undertook a number of large scale policy reforms in the late 1980s (including the beginning of inflation targeting) and the macroeconomic data generally suffers from a structural break in inflation, interest rates and the headline components of production GDP at this time and we estimate the model on macroeconomic data from 1992Q1 to 2008Q4.

Alternatively, we could build a structural model of the trends. However, because we assume different price and technology trends for each of the sectors that make up CPI inflation, this implies four separate trends. Further, there are other areas of the model that require additional trends.

Table 3.1: Observable variables

	description
$\widehat{r}$	Domestic interest rate
$\widehat{\pi}$	Headline inflation
$\widehat{\triangle s}$	Exchange rate growth
$\widehat{P^t/P}$	Relative price of tradables
$\widehat{P^n/P}$	Relative price of non-tradables
$\widehat{P^c/P}$	Real construction costs
$\widehat{P^o/P}$	Real world oil price
$\widehat{P^v/P}$	Real price of non-commodity exports
$\widehat{P^q/P}$	Relative price of non-oil imports
$\widehat{P^d/P}$	Real price of commodity exports
$\widehat{\Phi^h/P}$	Relative price of houses
$\widehat{W/P}$	Real wages
$\widehat{P^w/P}$	Real world price
$\widehat{C}$	Real total consumption
$\widehat{C^s}$	Real consumption of housing services
$\widehat{C^n}$	Real consumption of non-tradables
$\widehat{I^k}$	Real business investment
$\widehat{I^h}$	Real housing investment
$\widehat{G}$	Real government spending
$\widehat{X^v}$	Real non-commodity exports
$\widehat{X^d}$	Real commodity exports
$\widehat{M^q}$	Real non-oil imports
$\widehat{M^o}$	Real oil imports
$\widehat{b}$	Debt to nominal GDP
$\widehat{L}$	Labour (hours paid)
$\widehat{r^*}$	Foreign interest rate
$\widehat{Y^*}$	Foreign real output

For example, the terms of trade is exogenous in the model and follows a random walk process. The model assumes that tradable goods can either be consumed or invested, where both have the same underlying price and technology trend. This proliferation of separate trends suggested that we would not have enough observables to pin down the trends in the model.

In addition, we use the X12 procedure to both deseasonalise the series and remove the high frequency or noise component in the data. The raw New Zealand data are relatively noisy and we take the view that the DSGE model is designed to explain movements in the data at the business cycle frequency only. For example, a regressing the growth of seasonally adjusted consumption data on its lag (and a constant) returns a negative coefficient. DSGE models will struggle to explain this volatility in consumption.

3.2 Detrending the data

There is a multitude of different ways of extracting a trend from a macroeconomic time series, each with a different set of underlying assumptions. In essence, our trend model (TM) is an empirical device to remove the trends from the data while remaining broadly consisent with the assumptions underpinning KITT. TM allows for trends that converge to a well-defined steady state. In addition, TM also imposes multivariate consistency constraints on the trends, such as the national accounting identities and an equation for the evolution of debt.

To further explain the dynamics of TM, consider a time series y_t expressed as the sum of a trend component τ_t and a cyclical component c_t:

$$y_t = \tau_t + c_t \tag{3.1}$$

where c_t is a white noise process with zero mean and a variance of σ_c^2

Now let the following expressions describe the evolution of the trend:

$$\begin{aligned} \tau_t &= \phi\tau_{t-1} + ss + \beta_{t-1} \\ \beta_t &= \rho\beta_{t-1} + \varepsilon_t \end{aligned} \tag{3.2}$$

where ss is the steady state, β is a time-varying parameter, ϕ is a parameter governing the time series properties of the trend (discussed below), ρ is a parameter determining the speed of adjustment to steady state, and ε_t is a white noise process with zero mean and variance of σ_ε^2. Note: by setting $\phi = 1$ and $\rho = 1$ we have the popular Hodrick-Prescott (HP) filter, which assumes that the trend is an $I(2)$ process.

Consistent with the assumptions underlying KITT, TM assumes that all variables are classified as either $I(0)$ or $I(1)$. The stationary $I(0)$ trends are modelled by setting $\phi = 0$ and $0 < \rho < 1$ in 3.2:

$$\tau_t = \rho\tau_{t-1} + (1-\rho)ss + \varepsilon_t \tag{3.3}$$

where ss is a steady state $level$. The non-stationary $I(1)$ trends, on the other hand, are modelled by setting $\phi = 1$ and $0 < \rho < 1$ in 3.2:

$$\Delta\tau_t = \rho\Delta\tau_{t-1} + (1-\rho)ss + \varepsilon_t \tag{3.4}$$

where ss is a steady state rate of $growth$.

TM is essentially a collection of equations describing the trends and cycles of the data required to produce the observable data for KITT, where the trend and cycle of each series are modelled using equation 3.1 and equation 3.3 or 3.4. As mentioned above, there are also some consistency restrictions imposed on the short-run trends. More technically, the trend equations and consistency constraints are written in state space form, and the (unobservable) trends are estimated using the Kalman filter.

Below, we describe the assumed time series behaviour of each of our time series, the consistency restrictions that are applied, the steady state of the model, the parameterisation of the model, and how the model will be used in practice.

Debt and the national accounts

KITT has relative price trends across sectors and Cobb-Douglas aggregation, implying constant nominal shares to GDP in steady state. These assumptions underpin the steady state in TM.

Leting $i = \{C^n, C^s, C^t, C^f, I^k, I^h, G, X^v, X^d, M^q, M^o\}$ be the expenditure components of GDP, we define Pi, i, and $Pi \times i$ to be the implicit price deflator, real value-added, and nominal expenditure of component i, respectively. Our assumptions for the time series properties of the national accounts data are then:

$$\begin{aligned} Pi/PC &\sim I(1) \\ i &\sim I(1) \\ Pi \times i/NY &\sim I(0) \end{aligned} \tag{3.5}$$

where NY is aggregate nominal GDP and PC is the implicit price deflator for consumption.

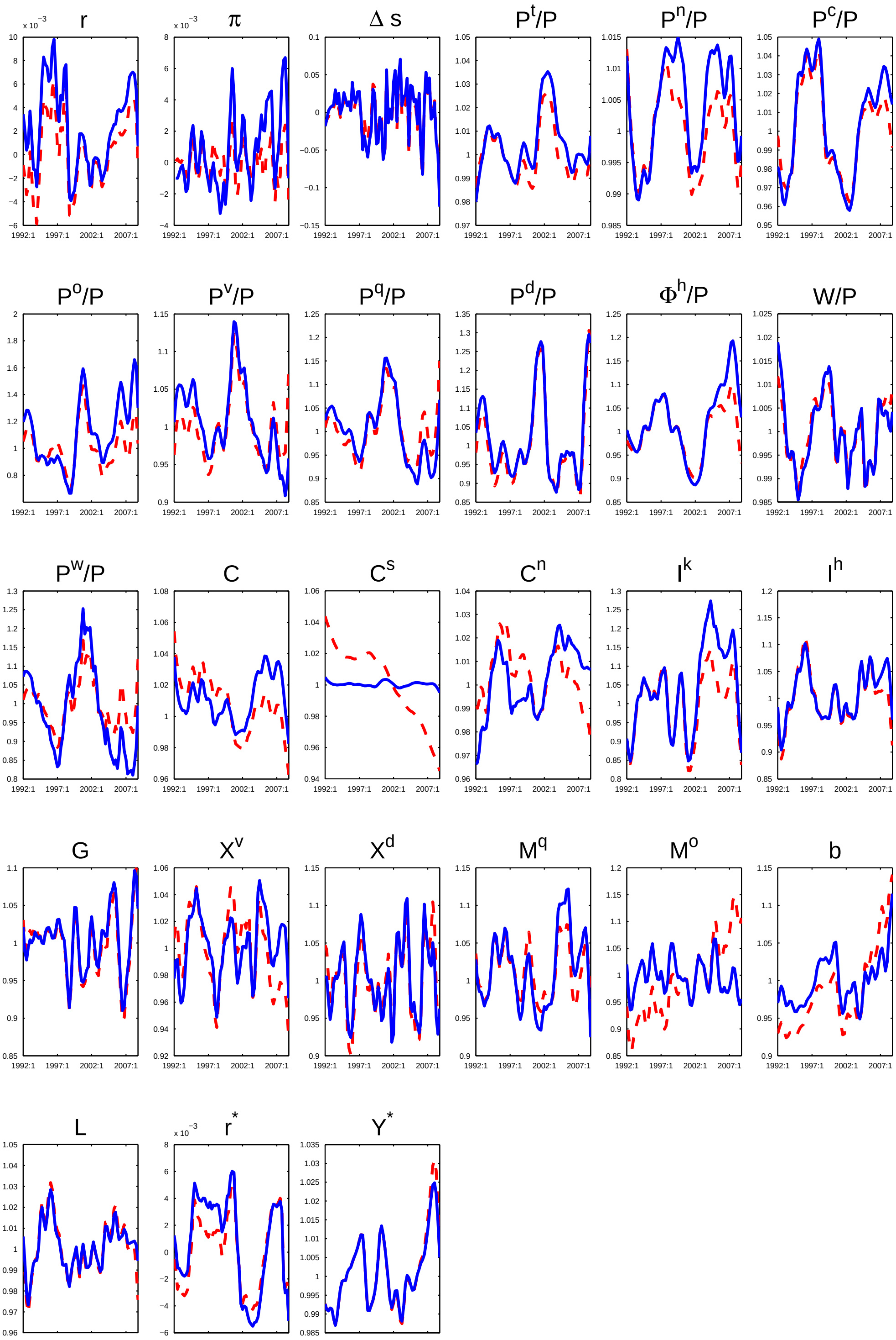
We define the law of motion of the debt trend as:

$$b_t = b_{t-1} \exp^{(r_t^b - \Delta \log(NY_t))} - (PX_t^v \times X_t^v + PX_t^d \times X_t^d - (PM_t^q \times M_t^q + PM_t^o \times M_t^o))/NY_t \tag{3.6}$$

where b is debt as a share of nominal GDP and r^b is effective interest rate on debt, with $b \sim I(0)$ and $r^b \sim I(0)$.

In addition to equation 3.6, we impose the following identities on the

Figure 3.1: Observable data in gap terms



NB. Note that the data are in deviations from trend with the red dashed lines representing the baseline case and the blue solid line the judgementally adjusted case.

short-run trends and the steady state:

$$\begin{aligned} \sum_i Pi \times i/NY = 1 \qquad (3.7) \\ \sum_i i = Y \\ (Pi \times i/NY)/(PC \times C/NY) = i/C \times Pi/PC \\ PC \times C = PC^n \times C^n + PC^s \times C^s + PC^t \times C^t + PC^f \times C^f \\ C = C^n + C^s + C^t + C^f \end{aligned}$$

where Y is real GDP, $Y \sim I(1)$. These constraints simply ensure that the components of nominal and real GDP add up to their corresponding aggregates, and that the trends for relative prices and the nominal and real GDP components are consistent with each other.

We determine the steady state by exogenising the steady state debt level, the interest rate on debt, the growth rate of nominal GDP, relative prices, aggregate real GDP, and all nominal shares except for consumption, tradable consumption, and manufactured imports. We then solve for the steady state nominal shares of consumption, tradable consumption, and manufactured imports, along with all real GDP components.

The steady state level for debt is a key parameter in determining the steady state levels for the nominal consumption and manufactured import shares. A change to the steady state level of debt leads to a change in the steady state share of tradable consumption to GDP and an equivalent change in steady state manufactured imports share. An increase in the steady state debt position, for example, implies that the steady state tradable consumption share (and the manufactured imports share) is lower. Intuitively, in order to fund a higher long run debt position, the economy requires a rise in the trade balance. This occurs via a fall in manufactured imports that are used for tradable consumption.

3.2.1 Other variables

The link between the national accounts and the CPI in TM comes from the consumption deflator relative to the CPI, $PC/P \sim I(0)$. This trend is used to convert the relative price trends above into trends for the national accounts deflators relative to the CPI, such as the relative price of manufactured imports. As with the national accounts, we assume that the remaining relative prices are $I(1)$:

$$P^t/P, P^n/P, P^c/P, P^f/P, P^o/P, \Phi^h/P, W/P, P^w/P \sim I(1) \tag{3.8}$$

The short run trends and the steady states of the CPI components must also satisfy the CPI add-up constraint:

$$(1-\nu_n-\nu_c-\nu_f)\log(P^t/P)+\nu_n\log(P^n/P)+\nu_s\log(P^c/P)+\nu_f\log(P^f/P)=0$$

We assume that domestic and foreign interest rates, the growth in the exchange rate, and inflation are $I(0)$ variables, and labour hours paid and foreign output are assumed to be $I(1)$.

$$\begin{aligned} r, r^*, \pi, \Delta s &\sim I(0) \\ Y^*, L &\sim I(1) \end{aligned} \tag{3.9}$$

Baseline parameterisation

The parameters required to decribe the steady state of TM are displayed in table 3.2. Consisent with KITT, we allow for five different steady state inflation rates across the national accounts and the CPI. Notice that, given our assumption for constant expenditure shares to GDP and a steady state growth rate of nominal GDP, this implies five different rates of growth for the components of real GDP.

As can be seen in equation 3.1, 3.3 and 3.4, aside from the steady state parameters ss, there are two parameters that need to be calibrated for each series in TM. These parameters are the speed with which the trend converges back to steady state ρ and the signal to noise ratio $\lambda = \sigma_c^2/\sigma_\varepsilon^2$.[1]

As a baseline, we set the speed of adjustment parameter to be the same across all series in TM, $\rho = 0.95$. Likewise, we set two different signal to noise ratios, one for the $I(0)$ variables and one for the $I(1)$ variables, $\lambda = 100$ and $\lambda = 1600$, respectively.

The observable data derived from TM using the baseline specification are represented by the red dashed lines in figure 3.1. We find that the baseline specification produces reasonable data for most variables. There are, however, several variables for which the baseline specification does not do such a good job. For example, the trends for real consumption of housing services, C^s, and real manufactured imports, M^o, appear to be too inflexible.

Fine-tuning and adding judgement

It is clear that TM will be subject to judgement in practice. These judgements may be minor, such as adjustments to the dynamic parameters and signal to noise ratios, or more substantial, such as setting some of the trend shocks to zero (making the associated trend deterministic).

Indeed, the flexibility of the Kalman filter framework we use to estimate TM allows us to go as far as making the unobservable trends entirely observable, or even observable in some periods and not in others. This allows us to exogenously determine a subset of the trends and let the model 'back-out' the remaining trends in a consistent manner. Forecasters might wish to impose, for example, an aggregate real GDP trend exogenously, perhaps from a HP filter, and then determine the remaining trends endogenously.

For the purposes of this book, we make some adjustments to the baseline specification to reflect the RBNZ's understanding of economy over our sample period (the blue lines in figure 3.1). Some of the judgements we

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[1] Note, λ has the same interpretation as in the HP filter framework.

apply to the baseline specification are: a constant trend for interest rates (r and r^*), the change in the (log) exchange rate Δs, and inflation (π and π^*); lower signal to noise ratios for real consumption of housing services, C^s, and real manufactured imports, M^o; and the RBNZ's mulivariate filter estimate of trend real GDP is imposed exogenously.

Table 3.2: KITT steady state parameters

Variable		value	Variable		value
π	$=$	0.0050	b	$=$	3.0811
π^n	$=$	0.0075	$\Delta\log(C)$	$=$	0.0084
π^c	$=$	0.0075	$\Delta\log(NY)$	$=$	$\Delta\log(C)+\Delta\log(PC)$
π^t	$=$	0.0025			
π^f	$=$	0.0000	$PC^t \times C^t/NY$	$=$	0.2417
			$PC^n \times C^n/NY$	$=$	0.1977
r	$=$	0.0150	$PC^s \times C^s/NY$	$=$	0.1177
r^*	$=$	0.0100	$PC^f \times C^f/NY$	$=$	0.0136
r^b	$=$	0.0210	$PI^k \times I^k/NY$	$=$	0.1437
Δs	$=$	0	$PI^h \times I^h/NY$	$=$	0.0561
			$PG \times G/NY$	$=$	0.2060
$\Delta\log(PC)$	$=$	π	$PX^v \times X^v/NY$	$=$	0.2205
$\Delta\log(PC^n)$	$=$	π^n	$PX^d \times X^d/NY$	$=$	0.0827
$\Delta\log(PC^t)$	$=$	π^t	$PM^q \times M^q/NY$	$=$	0.2594
$\Delta\log(PC^s)$	$=$	π^n	$PM^o \times M^o/NY$	$=$	0.0202
$\Delta\log(PC^f)$	$=$	π^f			
$\Delta\log(PI^k)$	$=$	π^t	$\Delta\log(C^n)$	$=$	$\Delta\log(NY)-\Delta\log(PC^n)$
$\Delta\log(PI^h)$	$=$	π^c	$\Delta\log(C^t)$	$=$	$\Delta\log(NY)-\Delta\log(PC^t)$
$\Delta\log(PG)$	$=$	π^n	$\Delta\log(C^s)$	$=$	$\Delta\log(NY)-\Delta\log(PC^s)$
$\Delta\log(PX^v)$	$=$	π^v	$\Delta\log(C^f)$	$=$	$\Delta\log(NY)-\Delta\log(PC^f)$
$\Delta\log(PX^d)$	$=$	π	$\Delta\log(I^k)$	$=$	$\Delta\log(NY)-\Delta\log(PI^k)$
$\Delta\log(PM^q)$	$=$	π^t	$\Delta\log(I^h)$	$=$	$\Delta\log(NY)-\Delta\log(PI^h)$
$\Delta\log(PM^o)$	$=$	π^f	$\Delta\log(G)$	$=$	$\Delta\log(NY)-\Delta\log(PG)$
			$\Delta\log(X^v)$	$=$	$\Delta\log(NY)-\Delta\log(PX^v)$
$\Delta\log(P^o)$	$=$	π^f	$\Delta\log(X^d)$	$=$	$\Delta\log(NY)-\Delta\log(PX^d)$
$\Delta\log(P^w)$	$=$	π	$\Delta\log(M^q)$	$=$	$\Delta\log(NY)-\Delta\log(PM^q)$
$\Delta\log(W)$	$=$	$\pi+0.0024$	$\Delta\log(M^o)$	$=$	$\Delta\log(NY)-\Delta\log(PM^o)$
$\Delta\log(\Phi^h)$	$=$	π^c			
			$\Delta\log(Y^*)$	$=$	0.0077
			$\Delta\log(L)$	$=$	0.0043

Chapter 4

Model Evaluation

4.1 Estimation strategy

4.1.1 Overview

The key purpose of KITT is to act as a central forecasting and story telling device. It is used to help build central forecasts that will be communicated to policy makers during a policy making round, and published in the *Monetary Policy Statement*. This section of the book details the estimation of the model to be used to assist with the forecasting and policy process.

We use Bayesian methods to formally estimate the model and while we are precise about what what we treat as prior information and where we let the data speak, our estimation strategy could also be viewed as a more informed calibration exercise, where a selection (admittedly large) of parameters are estimated. Our strategy for estimating the model contains the following steps:

Table 4.1: Estimation strategy

1.	Obtain the steady-state
2.	Check identification (both the steady-state and dynamic model)
3.	Data treatment
4.	Formulate priors
5.	Estimate the dynamic model

Once we have pinned down the steady-state we can easily use Bayesian methods to estimate the dynamic model. Our experience with estimation of large-scale DSGE models suggests the likelihood function can be not particularly smooth. Incorporating prior information can help smooth the likelihood, however, this can obscure identification issues where the data only weakly informative (or not at all) with regard to particular parameter values. However, we conduct some checks for parameters where the data are not particularly informative.

In order to obtain priors for the estimation of the dynamic model we simulated the model and ruled out some sets of priors on structural parameters that produced impulse responses that were considered implausible. In part, this exercise included soliciting priors from senior forecasters and policymakers on their beliefs about the transmission mechanism.

Results from the estimation of the dynamics model are presented in section 4.3 and moments from the model are compared to the data in section 4.4. The following section describes how we treat the data.

4.1.2 Overview

The key purpose of KITT is to act as a central forecasting and story telling device. It is used to build central forecasts that will be communicated to policy makers during a policy making round, and published in the *Monetary Policy Statement*. This section of the book details the estimation of the model to be used to assist with the forecasting and policy process.

We use Bayesian methods to formally estimate the model and while we are precise about what what we treat as prior information and where we let the data speak, our estimation strategy can also be viewed as a more informed calibration exercises, where a selection (admittedly large) of parameters are estimated. Our strategy for estimating the model contained the following steps:

Log-linearisation of the model must be done around the model's steady-state and thus each dynamic model is only valid for the steady-state around which it is linearised. We separate estimation of the steady-state of the

Table 4.2: Estimation strategy

1.	Estimate the steady-state - Choose nominal ratios to match - Choose parameters to estimate
2.	Check identification (both the steady-state and dynamic model)
3.	Data treatment
4.	Formulate priors
5.	Estimate the dynamic model

model from the dynamic model for two reasons. Firstly, we choose to estimate the steady-state of the model by matching particular nominal ratios rather than matching the entire set of data properties. Secondly, and more pragmatically, estimating the steady-state, log-linearising, and checking the consistency of the dynamics and the steady-state takes enough computational time (at least thirty seconds) to prohibit estimating the steady-state and dynamic model simultaneously using Bayesian methods.

However, once we have pinned down the steady-state we can easily use Bayesian methods to estimate the dynamic model. Estimation of large models can suffer from a likelihood function that is not particularly smooth. Incorporating prior information can help smooth the likelihood, however, these techniques can obscure identification issues where the data are not at all, or only weakly informative, with regard to particular parameter values. We use techniques based on the Fisher information matrix to check for parameters where the data are not particularly informative.

In order to obtain priors for the estimation of the dynamic model we simulated the model and ruled out some sets of structural priors that produced impulse responses that were considered implausible. In part, this exercise included soliciting priors from a senior management group on their beliefs about the transmission mechanism.

Results from the estimation of the dynamics model are presented in section 4.3 and moments from the model are compared to the data in section 4.4. The following section describes how we treat the data.

4.1.3 Data

The model used for forecasting is detrended so that variables are in ‘gap’ terms, deviations of the data from steady-state. The decision to use data in a gaps format has been made for several reasons. We do not have enough observables to pin down the trends in the model. The full trends model assumes different price and technology trends for each of the sectors that make up CPI inflation. In addition, the terms of trade is exogenous in the model and follows a random walk process. The model assumes that tradable goods can either be consumed or invested, where both have the same underlying price and technology trend. This assumption does not match the data and we would require an additional trend in prices and technology to match the data. There are other examples in the model where further trends would have to be added. It turns out that this is not a practical solution to the problem.

The gap logic is also reasonably consistent with the current modelling framework and is well understood by forecasters, modellers and policymakers. So beginning with a gap logic presents less problems at both a technical level and at the presentation and communications level.

The gap logic requires some method for separating the lower frequencies (or trend) from the business cycle frequencies of the data. We use a simple univariate Hodrick-Prescott filter to separate the trend terms from the cycle. Furthermore, we use the X12 procedure to both deseasonalise the series and remove the high frequency or noise component in the data. The raw New Zealand data are relatively noisy and we take the view that the DSGE model is designed to explain movements in the data at the business cycle frequency only. For example, a regressing the growth of seasonally adjusted consumption data on its lag (and a constant) returns a negative coefficient. DSGE models will struggle to explain this volatility in consumption. Figure 4.1 provides a stylized representation of how we treat the data in KITT.[1]

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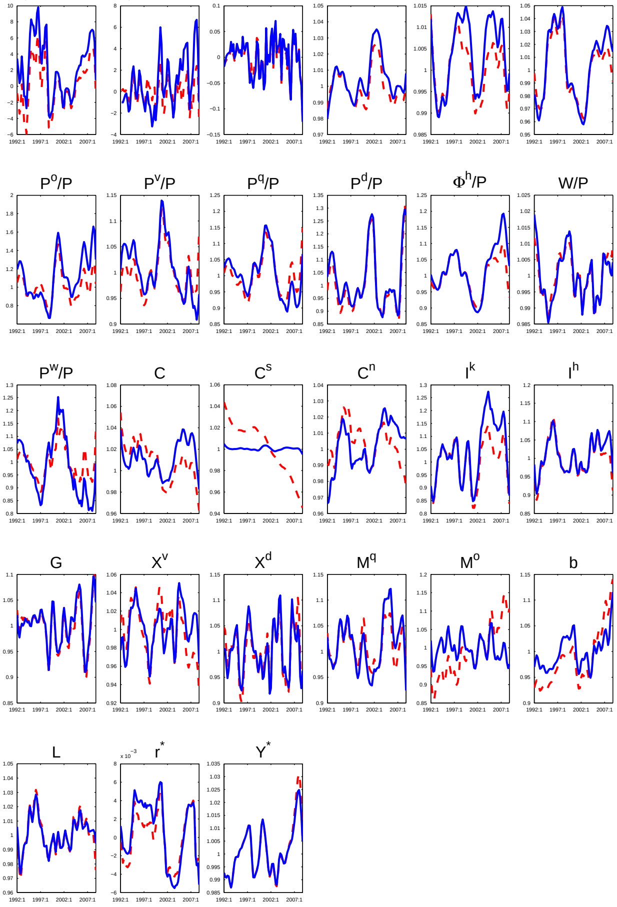
[1] The development of FPS faced similar issues. Indeed, **?** note: “The focus during the development of the core FPS model was on its medium-term dynamic properties. Consequently, the model does not attempt to reproduce the short-run idiosyncratic properties of

Figure 4.1: Data pretreatment in KITT



In terms of the raw data, New Zealand undertook a number of large scale policy reforms in the late 1980s (including the beginning of inflation targeting) and the macroeconomic data generally suffers from a structural break in inflation, interest rates and the headline components of production GDP at this time. Similar to recent research, we estimate the model on macroeconomic data from 1992Q1 to 2008Q1.

Table 4.3 lists the observable variables and their identifiers in the Reserve Bank of New Zealand's Aremos database.[2] It is important to note that the official tradable and non-tradable price indices are inclusive of petrol and construction costs respectively. To match the official data, we aggregate the model constructs of non-tradable and construction costs to form the official non-tradable price series. We also aggregate the model constructs for tradable and petrol prices to form the official tradable series.

4.1.4 Parametrising steady-state

To parametrise the model steady state, we use a numerical algorithm.[3] We look for the values of steady state parameters such that they minimise the distance between model implied nominal ratios and desired nominal ratios (e.g., great ratios), which we supply.

To help a smooth and fast convergence of the algorithm, we (i) do not require perfect match of the desired nominal ratios, but penalise deviations from them; (ii) fix a subset of steady-state parameters that we believe we have the most information about, (iii) assign prior weights on the remaining parameters we are backing out. The initial values of the parameters come from prior intervention analysis of the model, and the role of the weights is to penalise deviations from desired prior model dynamics.

The set of model steady-state parameters are listed in table 4.4. Of the 24 parameters, XX are fixed, and we numerically look for YY parameters. The choice fixed and free parameters is arbitrary, and reflects the ability

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the data."

[2] This data is available on request from the authors.

[3] We use the Newton method to minimise the objective function.

Table 4.3: Observable variables

	description	Aremos identifier
$\widehat{r}$	Domestic interest rate	R90D
$\widehat{\pi}$	Headline inflation	PCPIS
$\widehat{\triangle s}$	Exchange rate growth	RUSD, RAUD, RDEM, REUR, RGBP, RJPY
$\widehat{P^t/P}$	Relative price of tradables	PTR, PSPET, PSOVFL, PCPIS
$\widehat{P^n/P}$	Relative price of non-tradables	PNT, PCPIS
$\widehat{P^q/P}$	Relative price of non-oil imports	NM, NMo, M, Mo, PCPIS
$\widehat{P^w/P}$	Real world price	IAUPPOM, IUSPPOM, IJAPPOM, IUKPPOM, IGEPPOM
$\widehat{P^o/P}$	Real world oil price	TIPPP, TIPPP_
$\widehat{P^v/P}$	Real price of non-commodity exports	NXM_, NXDP_, NXCCP_, NX, NX_, PCPIS
$\widehat{P^c/P}$	Real construction costs	PZCON, PCPIS
$\widehat{P^d/P}$	Real price of commodity exports	NXM, NXDP, NXCCP, NXM_, NXDP_, NXCCP_, PCPIS
$\widehat{\Phi^h/P}$	Relative price of houses	PQHPI, PCPIS
$\widehat{W/P}$	Real wages	LQHOPR, PCPIS
$\widehat{C}$	Real total consumption	NCP_
$\widehat{C^s}$	Real consumption of housing services	NCPHS_
$\widehat{I^k}$	Real business investment	NI_, NIPD_
$\widehat{I^h}$	Real housing investment	NIPD_
$\widehat{G}$	Real government spending	NGDP_
$\widehat{X^v}$	Real non-commodity exports	NX_, NXM_, NXDP_, NXCCP_
$\widehat{X^d}$	Real commodity exports	NXM_, NXDP_, NXCCP_
$\widehat{M^q}$	Real non-oil imports	NM_, TIPPP_
$\widehat{M^o}$	Real oil imports	TIPPP_
$\widehat{b}$	Balance of payments	TIIN
$\widehat{L}$	Labour (hours paid)	LQTHTI
$\widehat{r^*}$	Foreign interest rate	RUS90D, RAU90D, RJP90D, RGB90D, REU90D
$\widehat{Y^*}$	Foreign real output	IAUQ_, IUSQ_Z, IUKQ_, IGEQ_, IJAQ_

of the numerical search algorithm to converge, and produce economically sensible parameter values.

We stress that after their parametrisation the steady-state parameters remain fixed and are not subject to any (data-driven) estimation at later stages.

4.1.5 Identification

When confronting KITT with the data, we want to know the information content of our data and whether the DSGE story produces a unique explanation for the data. Identifying both the structural parameters and the structural shocks is critical for this process. We apply two methodologies. First, the singular value decomposition of the Fisher information matrix (FIM), which locally checks the identifiability of structural (transitory and steady-state) parameters. And second, the adjusted SVAR identification methodology, which checks identifiability of structural shocks.

We pay particular attention to identification issues. The simplest way to think of the problem is to decompose the Fisher information matrix implied by the model structure into the information about the deep structural parameters ϑ, and the information about reduced form parameters Φ. The deep structural parameters are important for optimal policy and welfare analysis, but for forecasting the reduced form parameters are key. That is why we are analysing these two layers of FIM. The FIM carries the information about the curvature of likelihood function, $L(.)$ around those parameters, and thus on their identifiability.

The Fisher information matrix is the variance of scores. In principle, its elements are the first order derivatives of the likelihood function $L(\vartheta)$ with respect to deep structural parameters ϑ, $FIM = E\left[\frac{\partial L(\vartheta)}{\partial \vartheta}\right]^2$. If ϑ is not identified, the likelihood function is flat in that dimension, which leads to singularity of FIM, $\partial L(\vartheta)/\partial\vartheta = 0$. But decomposing the FIM as

$$E\left[\frac{\partial L(\vartheta)}{\partial \vartheta}\right]^2 = E\left[\frac{\partial L(\vartheta)}{\partial \Phi}\frac{\partial \Phi}{\partial \vartheta}\right]^2,$$

Table 4.4: Steady state parameters

Parameter	Description	Value
	Calibrated parameters	
β	Time preferences	0.9975
ω_t	Consumption share of tradables	0.4500
δ_h	Depreciation of housing stock	0.0260
μ	Price markup	1.5000
ω_d	Export share of diary products	0.4000
ϵ	Elasticity of substitution in monopol.comp. markets	1.0000
τ_f	Import share of oil	0.0000
v_c	Weight of construction costs in CPI	0.1000
v_t	Weight of tradables in CPI	0.4500
v_f	Weight of petrol in CPI	0.0500
	Calibrated steady-state ratios	
$NIh/NGDP$	Housing investment / GDP	0.0601
$B/NGDP$	Balance of payments / GDP	0.5000
$NX/NGDP$	Exports / GDP	0.3090
$NIk/NGDP$	Capital investment / GDP	0.1196
NMo/NM	Oil imports / total imports	0.0278
$NG/NGDP$	Government expenditures / GDP	0.2297
	Estimated parameters	
ω_h	Consumption share of housing services	0.1680
γ_h	Share of investment in new housing	0.1684
ψ_q	Tradables steady-state adjustment	-0.3200
γ_v	Share of factor services in manufactured exports	0.6252
γ_t	Share of factor services in tradables production	0.3010
λ	Loan-to-value ratio	0.1192
γ	Labour's share in no-fuel production inputs	0.7000
δ_k	Depreciation of capital	0.0300
γ_n	Share of factor services in non-tradables production	0.7645
γ_c	Share of factor services share in housing production	0.7083
γ_{f^t}	Fuel's share in tradable production	0.0050
γ_{f^z}	Fuel's share in production services	0.0116
ω_f	Consumption share of fuel	0.0300
σ	Output share of government expenditures	0.2297

where $\Phi = (\Phi_1(\vartheta), \Phi_\varepsilon(\vartheta))$, we can immediately see that identification of the model depends jointly on indentifiability of both the deep structural parameters ϑs, and reduced form parameters Φs. Even if ϑ is not identified, $\frac{\partial \Phi}{\partial \vartheta} = 0$, the reduced form parameters can, $\frac{\partial L(\vartheta)}{\partial \Phi} \neq 0$, and the model/forecast can be uniquely influenced by the data.

The model has 24 steady-state parameters and 24 transitory parameters (parameters that drive model dynamics, but do not influence the model steady state). There are 73 transitory variables (38 are state variables and 45 variables are identities), 26 observable variables, and 26 exogenous shocks. In estimation, we focus on the transitory parameters. We also re-scale the price stickiness parameters. The definition of the price stickiness in the model implies very high empirical values that causes the likelihood function to be ill-behaved. We invert the following parameters to correct for this effect, that is we scale the parameters according to the following: $\bar{\xi}_w = \frac{1}{\xi_w}$, $\bar{\xi}_n = \frac{1}{\xi_n}$, $\bar{\xi}_c = \frac{1}{\xi_c}$, $\bar{\xi}_t = \frac{1}{\xi_t}$, and $\bar{\xi}_v = \frac{1}{\xi_v}$.

The first method evaluates the Fisher information matrix around the initial model parameterisation as described in Iskrev (2008) This is a local check only.[4] We find that all parameters are (locally) identified, that is, the Fisher information matrix is not singular along any of the dimensions we consider. Further, we find that the parameters $\{\phi_c, \phi_t, \phi_z\}$ are the only parameters that suffer from weak identification. These parameters yield low singular values of the Fisher Information Matrix that indicates the likelihood function is particularly flat along the dimension given by these parameters. The remaining parameter estimates are significantly influenced by data information.

We use the methodology outlined in Fukač (2007) to check the identifiability of impulse responses. We represent KITT as a SVAR, and then check the identifiability of structural shocks. This methodology ignores identifiability of the deep structural parameters and focuses on the reduced form (SVAR) parameters required to generate a unique shock decomposition.

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[4] For the sake of robustness, we also evaluate the FIM around estimated parameters. The results are unchanged, and we conclude that the region where the parameters are identified is sufficiently large.

This is critical for determining and explaining to policymakers the nature of the shocks that are driving the economy within in a forecast.[5] The Fisher Information Matrix decomposition indicates that the standard errors of all shocks are locally identified. We find that shocks $\{\varepsilon_t^{Ik}, \varepsilon_t^{Pt}, \varepsilon_t^{Pv}, \varepsilon_t^{Xv}, \varepsilon_t^{Xd}, \varepsilon_t^{r}, \varepsilon_t^{g}, \varepsilon_t^{b}, \varepsilon_t^{s}, \varepsilon_t^{y*}, \varepsilon_t^{Pw^*}, \varepsilon_t^{Po^*}, \varepsilon_t^{Pd^*}, \varepsilon_t^{r^*}, \varepsilon_t^{Pq}, \varepsilon_t^{Pf}, \varepsilon_t^{Pn}, \varepsilon_t^{Pc}, \varepsilon_t^{Mo}, \varepsilon_t^{Mg}\}$ are identified almost everywhere, but we cannot identify $\{\varepsilon_t^{c}, \varepsilon_t^{w}, \varepsilon_t^{cs}, \varepsilon_t^{\Phi h}, \varepsilon_t^{Ih}, \varepsilon_t^{L}\}$.

4.1.6 Dynamic parameters

To estimate KITT, we use Bayesian methods. In our experience, central bankers hold strong priors about the transmission mechanism of the model and our comfortable with incorporating these prior beliefs into the estimation process. We also need to impose prior beliefs on the weakly identified parameters in the model. Bayesian methods held the appeal of combine prior information with the data in a consistent manner.[6]

4.2 Estimation

The log-linearised structure of KITT can be written in the canonical form as:

$$A_0\mathbf{x}_t = A_1 E_t \mathbf{x}_{t+1} + A_2 \mathbf{x}_{t-1} + \varepsilon_t. \tag{4.1}$$

where $\mathbf{x}_t$ is an $n \times 1$ vector of endogenous model variables. ε_t is an $n \times 1$ vector of structural shocks, and A_0, A_1, and A_2 are $n \times n$ structural matrices collecting structural parameters (see tables 4.4, 4.5, and 4.6), which we will denote as ϑ, $\vartheta \in R$.

We solve (4.1) for rational expectations, $E_t\mathbf{x}_{t+1}$, using the algorithm in

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[5] We used the methodology to explore alternative model structures, using the methodology to help place the shocks within the model so that we maximize the number of globally identified shocks.

[6] During the model development phase, we often used the regularised maximum likelihood method (RMLE), which can be considered a compromise between classical methods and full Bayesian methods. For details about the method we refer to Ljung (1999).

Klein (2000) to obtain the reduced model form that captures equilibrium dynamics:

$$\mathbf{x}_t = \Phi_1(\vartheta)\mathbf{x}_{t-1} + \Phi_\varepsilon(\vartheta)\varepsilon_t. \tag{4.2}$$

Matrices $\Phi_1(\vartheta)$ and $\Phi_\varepsilon(\vartheta)$ are functions of A_0, A_1, A_2, and their elements are again functions of the deep structural parameters ϑ.

The likelihood function is estimated using the Kalman filter by combining the state-space representation of the model solution (4.2) with a measurement equation (4.3), linking the state vector to the observed data:

$$\mathbf{y}_t = \mathbf{A}(\vartheta) + \mathbf{B}\mathbf{x}_t. \tag{4.3}$$

Here, **B** maps the elements of $\mathbf{x}_t$ into the $m \times 1$ (with $m \leq n$) vector of observable variables $\mathbf{y}_t$. $A(\vartheta)$ is related to the model parameters and captures the means of the variables contained in $\mathbf{x}_t$. We do not assume any measurement errors in the data.

The model (4.2)-(4.3) and its associated parameters ϑ are estimated using the methods outlined in An and Schorfheide (2007). Specifically, given a prior $p(\vartheta)$ and a sample of data $\mathbf{y}_T$, the posterior density of the model parameters ϑ is proportional to the likelihood of the data multiplied by the prior $p(\vartheta)$:

$$p(\vartheta|\mathbf{y}_T) \propto L(\vartheta|\mathbf{y}_T)p(\vartheta) \tag{4.4}$$

The list of 26 observable variables contained in our $\mathbf{y}_T$ is displayed in table 4.3. The estimates of posterior distributions are obtained using Markov Chain Monte Carlo methods. We make 500,000 draws from the posterior distribution using the random walk Metropolis-Hastings algorithm, discarding the first half of the draws to ensure convergence.[7]

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[7] The start-values for our Metropolis Hastings algorithm are found using Chris Sims's optimisation routine 'csminwel', available from his website.

4.3 Results

Consistent with the identification results above, we find that the data appear quite informative about most of the transitory parameters, with marked differences between the prior and posterior distributions for most parameters.[8] The estimated standard deviations of the shocks are displayed in table 4.6. The priors and posterior estimates for the dynamic parameters are displayed in table 4.5 and figure 4.2.

The deep habit in consumption parameter χ is estimated to be quite high, with the 90 percent probability interval ranging from 0.806 to 0.852. This parameter directly affects the intertemporal and intratemporal elasticity of substitution of consumers, where higher values make consumption less sensitive to price changes and increase persistence (see section 2).

Recall from section 2, that the elasticity of substitution is inversely related to adjustment costs for housing and business investment ι_h and ι_k, fuel consumption ι_c, demand for tradable goods ϕ_τ, and demand for intermediate goods ϕ_z. Our posterior estimates thus show that fuel consumption and demand for intermediate goods are less sensitive to price fluctuations (more inelastic) than housing and business investment and demand for tradable goods.

Our priors for the degree of price flexibility differ across each sector. In a given sector, the degree of price flexibility is likely to be strongly related to the degree of competition faced by firms. For this reason, our prior beliefs are that tradable prices are more flexible than non-commodity export prices and construction costs, and these prices are in turn more flexible than non-tradable prices and wages. Broadly speaking, we find that our posterior estimates match our prior ranking for the degree of price flexibility across sectors, with tradable prices estimated to be most flexible, followed by non-commodity export prices and construction costs. Non-tradable prices and wages are estimated to be the least flexible prices in the model.

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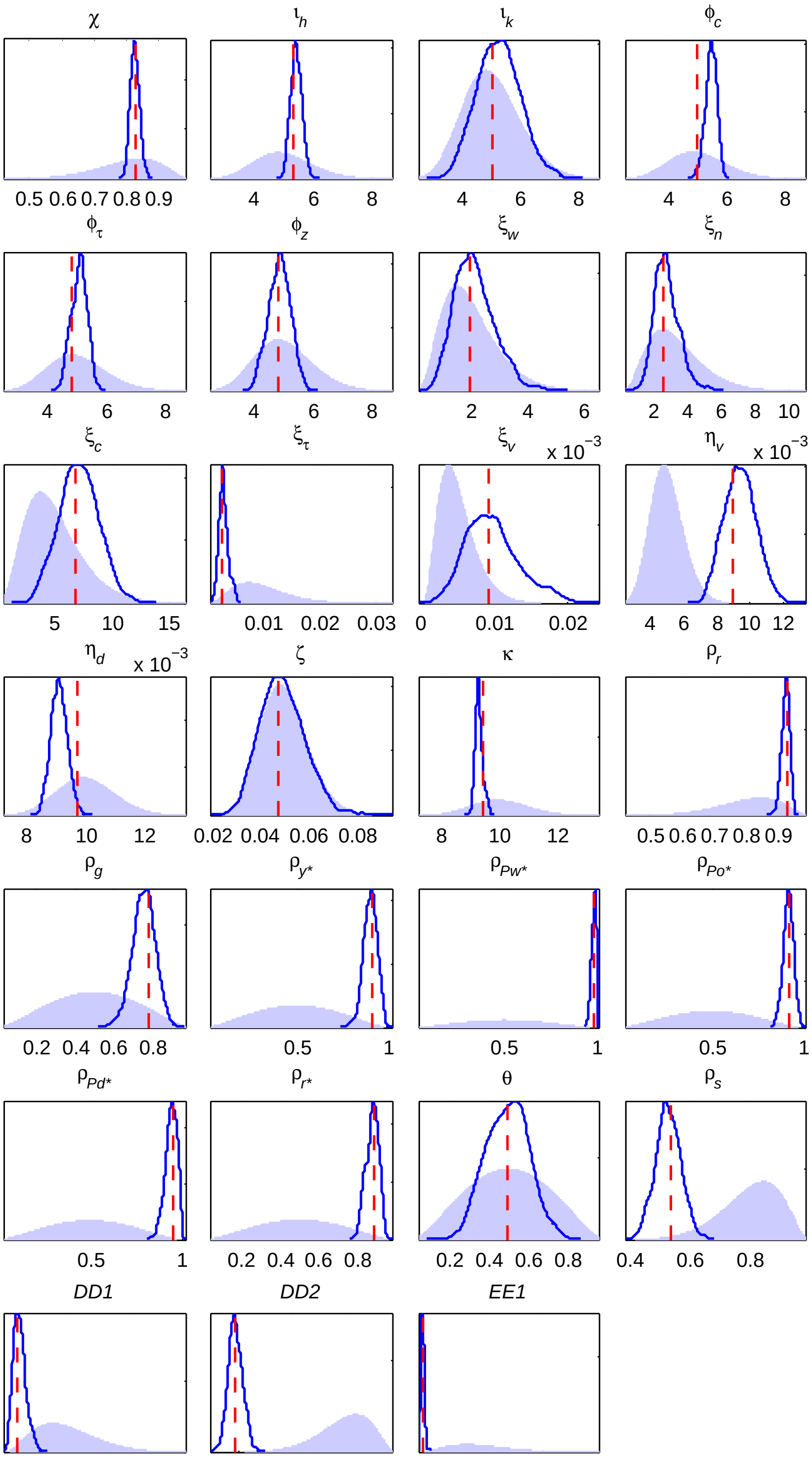
[8] Note in Figure 4.2 that the values of the multivariate posterior mode (dash line) and the mean of marginal posterior distribution (solid line) differ for ϕ_c and ϕ_t. This can be interpreted as a result of weak identification detected with the Fisher Information Matrix decomposition.

Figure 4.2: Parameter priors and posterior estimates



Note that grey shadow denotes the prior distribution with the posterior given by the solid line. The dashed line indicates the starting value for the posterior mode from the 'csminwel' routine.

Adjustment costs for commodity and non-commodity exports (η_d and η_v) can be shown to help determine the sensitivity of export production to foreign demand Y^*. Indeed, adjustment costs can be considered as approximately the inverse elasticity of export demand with respect to world demand. As such, a higher adjustment cost parameter for non-commodity exports (9.390 versus 9.092 for commodity exports) suggests that non-commodity exports are somewhat less sensitive to a given change in foreign demand than commodity exports.

The posteriors for the monetary policy rule ρ_r and κ sharpen markedly relative to our prior. Further, overall, the estimated policy rule is more reactive to inflation deviations from target. *Ceteris paribus*, a 1 percent inflation deviation from target, for example, prompts a 2 percentage point increase in the policy rate with our prior expectation ($= (1 - 0.8) \times 10$), but only a 0.70 percentage point increase in the policy rate with our posterior expectation ($(1 - 0.924) \times 9.279$). The degree of policy smoothing in the posterior is quite pronounced relative to our prior (0.8 versus 0.924 respectively), such that the estimated monetary policy response is more prolonged than the expected prior response.

The estimated smoothing parameters on the foreign variables tend to be slightly higher than our priors on average. In addition, the smoothing parameters on government expenditure is found to be slightly lower in the data. We also find that the persistence of the risk premium shock in the exchange rate equation ρ_ε^s is estimated to be relatively high. The estimated posterior for the exchange rate smoothing parameter θ is very similar to our prior expectation.

Our estimated empirical equations for non-oil import prices and fuel prices show that the short- and long-run adjustment coefficients are larger for non-oil import prices than for fuel prices. The short run adjustment coefficients (DD_1 and EE_1) reflect the contemporaneous impact foreign price fluctuations (expressed in domestic currency) have on the final prices paid by domestic agents. The long run adjustment coefficients (DD_2 and EE_2), on the other hand, reflect the speed at which past deviations from the law of one price return back to steady state. Our parameter estimates suggest that, for a given change in foreign prices (or the exchange rate), fuel price deviations from the law of one price take much longer to return to steady state than non-commodity export price deviations.

4.4 Evaluating empirical fit

This section describes our model's ability to match some key features of the New Zealand data.[9] Specifically, we assess the empirical fit of our model using two groups of variables: prices and GDP components. The price group contains non-tradable, tradable, and headline inflation, and the exchange rate and the policy rate. The GDP components we examine are consumption, investment, exports, imports, and total GDP.[10]

The simulated empirical data are generated using a bootstrapped VAR(2), estimated using the particular group of variables under consideration. Simulated data from the model, on the other hand, are generated by drawing from the estimated posterior distributions of the parameters and the shocks from section 4.3. Our experiment involves simulating 1000 data sets from the VAR and the model, with each of these data sets containing the same number of observations as the historical data described in section 4.1.3. We then compile summary statistics from these simulated samples. Recall that the official tradable and non-tradable price indices are inclusive of petrol and construction costs, respectively. To match the official data, we aggregate the model constructs of non-tradable and construction costs to form the official non-tradable price series. We also aggregate the model constructs for tradable and petrol prices to form the official tradable series.

We chose to compare autocorrelation functions, standard deviations, and a selection of contemporaneous cross-correlations. These simulated moments are displayed in figures 4.3 and 4.4: the grey shaded areas represent the empirical distributions and the lines represent the distributions generated by the model, where the dotted lines in the first column rep-

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[9] The simulation methods used here are described in further detail in Canova (2007).

[10] The investment, exports, imports, and GDP aggregates are constructed using the appropriate simulated model variables.

Table 4.5: Transitory parameters: priors and posteriors

	Description	Prior Distribution	Post mean	Probability interval
χ	Consumption habit persistence	$\beta(0.8, 0.1)$	0.829	[0.806, 0.852]
ι_h	Adjustment costs: housing investment	$\Gamma(5, 1)$	5.412	[5.097, 5.726]
ι_k	Adjustment costs: business investment	$\Gamma(5, 1)$	5.232	[3.974, 6.390]
ϕ_c	Adjustment costs: fuel consumption	$\Gamma(5, 1)$	5.437	[5.117, 5.750]
ϕ_τ	Adjustment costs: tradable output	$\Gamma(5, 1)$	5.055	[4.617, 5.494]
ϕ_τ	Adjustment costs: imports	$\Gamma(5, 1)$	3.094	[2.372, 3.834]
ϕ_z	Adjustment costs: intermediate goods	$\Gamma(5, 1)$	4.845	[4.225, 5.456]
$\hat{\xi}_w$	Price flexibility: wages	$\Gamma(\frac{1}{500}, \frac{1}{1000})$	0.002	[0.001, 0.003]
$\hat{\xi}_n$	Price flexibility: non-tradable	$\Gamma(\frac{1}{300}, \frac{1}{600})$	0.003	[0.002, 0.004]
$\hat{\xi}_c$	Price flexibility: construction	$\Gamma(\frac{1}{200}, \frac{1}{400})$	0.007	[0.004, 0.010]
$\hat{\xi}_\tau$	Price flexibility: tradable	$\Gamma(\frac{1}{100}, \frac{1}{200})$	0.003	[0.002, 0.005]
$\hat{\xi}_v$	Price flexibility: non-commodity exports	$\Gamma(\frac{1}{200}, \frac{1}{400})$	0.010	[0.004, 0.015]
η_v	Adjustment costs: non-comm. exports	$\Gamma(5, 1)$	9.390	[7.791,10.898]
η_d	Adjustment costs: commodity exports	$\Gamma(10, 1)$	9.092	[8.644, 9.565]
ζ	Lending interest premium	$\beta(0.05, 0.01)$	0.049	[0.034, 0.064]
κ	Inflation reaction: monetary policy	$\Gamma(10, 1)$	9.279	[9.055, 9.491]
ρ_r	Smoothing: monetary policy	$\beta(0.8, 0.1)$	0.924	[0.904, 0.945]
ρ_g	Smoothing: government	$\beta(0.5, 0.2)$	0.753	[0.660, 0.850]
ρ_{y^*}	Smoothing: foreign output	$\beta(0.5, 0.2)$	0.895	[0.841, 0.957]
$\rho_{p^{w*}}$	Smoothing: foreign prices	$\beta(0.5, 0.2)$	0.980	[0.962, 0.998]
$\rho_{p^{o*}}$	Smoothing: foreign oil prices	$\beta(0.5, 0.2)$	0.914	[0.868, 0.958]
$\rho_{p^{d*}}$	Smoothing: foreign commodity prices	$\beta(0.5, 0.2)$	0.937	[0.892, 0.990]
ρ_{r^*}	Smoothing: foreign monetary policy	$\beta(0.5, 0.2)$	0.879	[0.825, 0.931]
θ	Smoothing: exchange rate	$\beta(0.5, 0.2)$	0.496	[0.309, 0.663]
ρ_{ϵ^s}	Smoothing: risk premium shock	$\beta(0.5, 0.2)$	0.870	[0.814, 0.925]
DD_1	Short run adjustment: non-oil imports	$\beta(0.8, 0.1)$	0.527	[0.452, 0.594]
DD_2	Error correction: non-oil import prices	$\beta(0.2, 0.1)$	0.048	[0.014, 0.078]
EE_1	Short run adjustment: fuel prices	$\beta(0.8, 0.1)$	0.384	[0.341, 0.433]
EE_2	Error correction: fuel prices	$\beta(0.2, 0.1)$	0.012	[0.004, 0.020]

$\Gamma(., .)$ and $\beta(., .)$ are gamma and beta distributions, respectively: the first element is the mean and the second element is the standard deviation. The probability interval is a 90% interval.

Table 4.6: Shock standard deviations: priors and posteriors

Para	Description	Prior distribution	Post. mean	Probability interval
σ_{ε_c}	Consumption	$\Gamma^{-1}(0.01, \infty)$	0.053	[0.040,0.066]
σ_{ε_w}	Wage (MRS)	$\Gamma^{-1}(0.01, \infty)$	0.003	[0.002,0.003]
$\sigma_{\varepsilon_{c^{ch}}}$	Housing services	$\Gamma^{-1}(0.01, \infty)$	0.004	[0.004,0.005]
$\sigma_{\varepsilon_{c^{cn}}}$	Housing services	$\Gamma^{-1}(0.01, \infty)$	0.012	[0.010,0.014]
$\sigma_{\varepsilon_{\phi^h}}$	Housing services	$\Gamma^{-1}(0.01, \infty)$	0.086	[0.069,0.103]
$\sigma_{\varepsilon_{i^h}}$	Housing investment	$\Gamma^{-1}(0.01, \infty)$	0.040	[0.033,0.048]
$\sigma_{\varepsilon_{i^k}}$	Business investment	$\Gamma^{-1}(0.01, \infty)$	0.017	[0.013,0.020]
σ_{ε_l}	Labour demand	$\Gamma^{-1}(0.01, \infty)$	0.002	[0.001,0.002]
$\sigma_{\varepsilon_{p^n}}$	Cost push: Non-tradable	$\Gamma^{-1}(0.01, \infty)$	0.004	[0.003,0.005]
$\sigma_{\varepsilon_{p^c}}$	Cost push: Construction	$\Gamma^{-1}(0.01, \infty)$	0.002	[0.002,0.003]
$\sigma_{\varepsilon_{p^\tau}}$	Cost push: Tradable	$\Gamma^{-1}(0.01, \infty)$	0.106	[0.086,0.126]
$\sigma_{\varepsilon_{m^o}}$	Oil import demand	$\Gamma^{-1}(0.01, \infty)$	0.026	[0.021,0.031]
$\sigma_{\varepsilon_{m^q}}$	Non-oil import demand	$\Gamma^{-1}(0.01, \infty)$	0.039	[0.030,0.047]
$\sigma_{\varepsilon_{p^v}}$	Cost push: Non-commodity exports	$\Gamma^{-1}(0.01, \infty)$	0.016	[0.013,0.019]
$\sigma_{\varepsilon_{x^v}}$	Non-commodity export demand	$\Gamma^{-1}(0.01, \infty)$	0.034	[0.026,0.039]
$\sigma_{\varepsilon_{x^d}}$	Commodity exports demand	$\Gamma^{-1}(0.01, \infty)$	0.002	[0.002,0.002]
σ_{ε_r}	Monetary policy	$\Gamma^{-1}(0.01, \infty)$	0.020	[0.016,0.024]
σ_{ε_g}	Government spending	$\Gamma^{-1}(0.01, \infty)$	0.011	[0.009,0.013]
σ_{ε_b}	Current account	$\Gamma^{-1}(0.01, \infty)$	0.005	[0.003,0.007]
σ_{ε_s}	UIP	$\Gamma^{-1}(0.01, \infty)$	0.003	[0.003,0.004]
$\sigma_{\varepsilon_{y*}}$	Foreign demand	$\Gamma^{-1}(0.01, \infty)$	0.002	[0.002,0.002]
$\sigma_{\varepsilon_{p^{w*}}}$	Foreign prices	$\Gamma^{-1}(0.01, \infty)$	0.079	[0.064,0.094]
$\sigma_{\varepsilon_{p^{o*}}}$	Foreign oil prices	$\Gamma^{-1}(0.01, \infty)$	0.042	[0.034,0.050]
$\sigma_{\varepsilon_{p^{d*}}}$	Foreign commodity prices	$\Gamma^{-1}(0.01, \infty)$	0.001	[0.001,0.001]
$\sigma_{\varepsilon_{r*}}$	Foreign monetary policy	$\Gamma^{-1}(0.01, \infty)$	0.014	[0.011,0.017]
$\sigma_{\varepsilon_{p^{q*}}}$	Non-oil import price	$\Gamma^{-1}(0.01, \infty)$	0.019	[0.015,0.023]
$\sigma_{\varepsilon_{p^{f*}}}$	Oil import price	$\Gamma^{-1}(0.01, \infty)$	0.015	[0.012,0.017]

$\Gamma^{-1}(.,.)$ is an inverse gamma distribution: the first element is the mean and the second element is the standard deviation. The probability interval is a 90% interval. †denotes expressed in domestic prices. Note that both the no-oil import price and the oil-import prices are denominated in domestic currency terms.

resent a 95 percent probability interval. The first column of each figure contains the simulated autocorrelations and the second column contains simulated standard deviations. The third column contains simulated cross-correlations: these cross-correlations are with headline inflation for the GDP components and with the policy rate for prices.

Looking first at the moments associated with prices in the model, displayed in figure 4.3, the model generally has a good match to the empirical data. Certainly, the model appears to fit most of the standard deviations and cross-correlations well. However, the model suggests slightly less persistence than the data in tradables inflation and slightly more volatility in interest rates than seen in the empirical data. Nevertheless, the policy rule is particularly simple and one of the least structural equations in our model. Future work examining alternative policy rules might easily lead to an improvement in the match to the volatility of interest rates.

Turning to the GDP components (figure 4.4), we find that the model does a particularly good job at matching the autocorrelation, of consumption and investment. However, the autocorrelations do not match for exports and imports, contributing to relatively poor fit to the empirical autocorrelation function for GDP. While the simulated model matches the volatility of consumption and investment, the volatility of exports is over-predicted by the model and this also contributes to higher volatility for GDP than that implied by the data. This suggests that future model development should focus on reducing the variance of exports. The model generally does a good job at matching the empirical properties of the GDP components but tends to understate the contemporaneous correlation with headline inflation (displayed in the last column). The simulated model moments for government spending also closely match the simulated empirical moments, although these moments are not displayed in figure 4.4.

4.5 Business cycle properties

We also look at the business cycle properties of the model. We run 1,000 Monte Carlo experiments to simulate the time series of consumption (C), exports (X), imports (M), investment (I), government expenditures (G), and compile them in to GDP series. Each series in each experiment has 1,000 observations. We use the classical definition of business cycle. We identify peaks and troughs of a log series, and count the average duration and amplitude of contraction and expansion. In Table 4.5, we provide a summary of the experiment results, and actual data characteristics. The data span from 1992:Q1 to 2008:Q2.[11] On average, we can see that the model under-predicts the duration and size of expansion, and slightly over-predicts the duration and size of contractions.

Table 4.7: Actual and simulated business cycle characteristics

	Data (1992Q1-2008Q2)				Model			
	Expansion		Contraction		Expansion		Contraction	
	Dur	Amp	Dur	Amp	Dur	Amp	Dur	Amp
*Y	12.2	0.91	4	-0.8	9	0.63	5	-0.61
C	27	0.29	1	-0.01	13	0.17	2	-0.01
X	9	0.17	3	-0.04	10	0.19	3	-0.06
M	10	0.19	3	-0.02	11	0.53	4	-0.30
I	23	0.64	6	-0.12	10	0.26	3	-0.07
G	27	0.29	1	-0.01	13	0.17	2	-0.01

Note: *Hall and McDermott (2007) estimates for 1977:Q2 to 2002:Q1.

Having examined both the match of a selection of the moments from the model to the data, and the business cycle properties of the model, the following section uses the impulse responses from the model as a tool to help understand the model.

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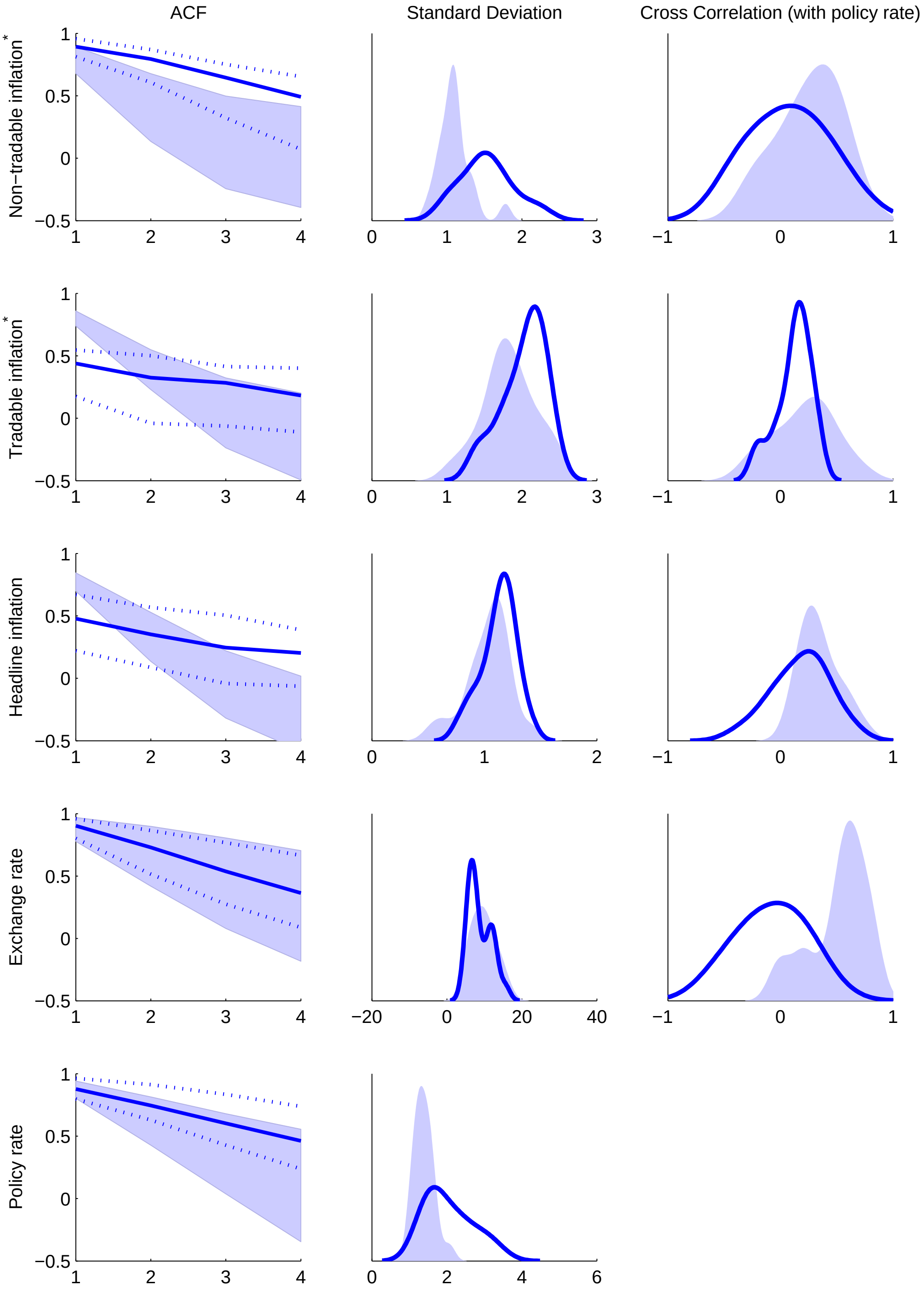
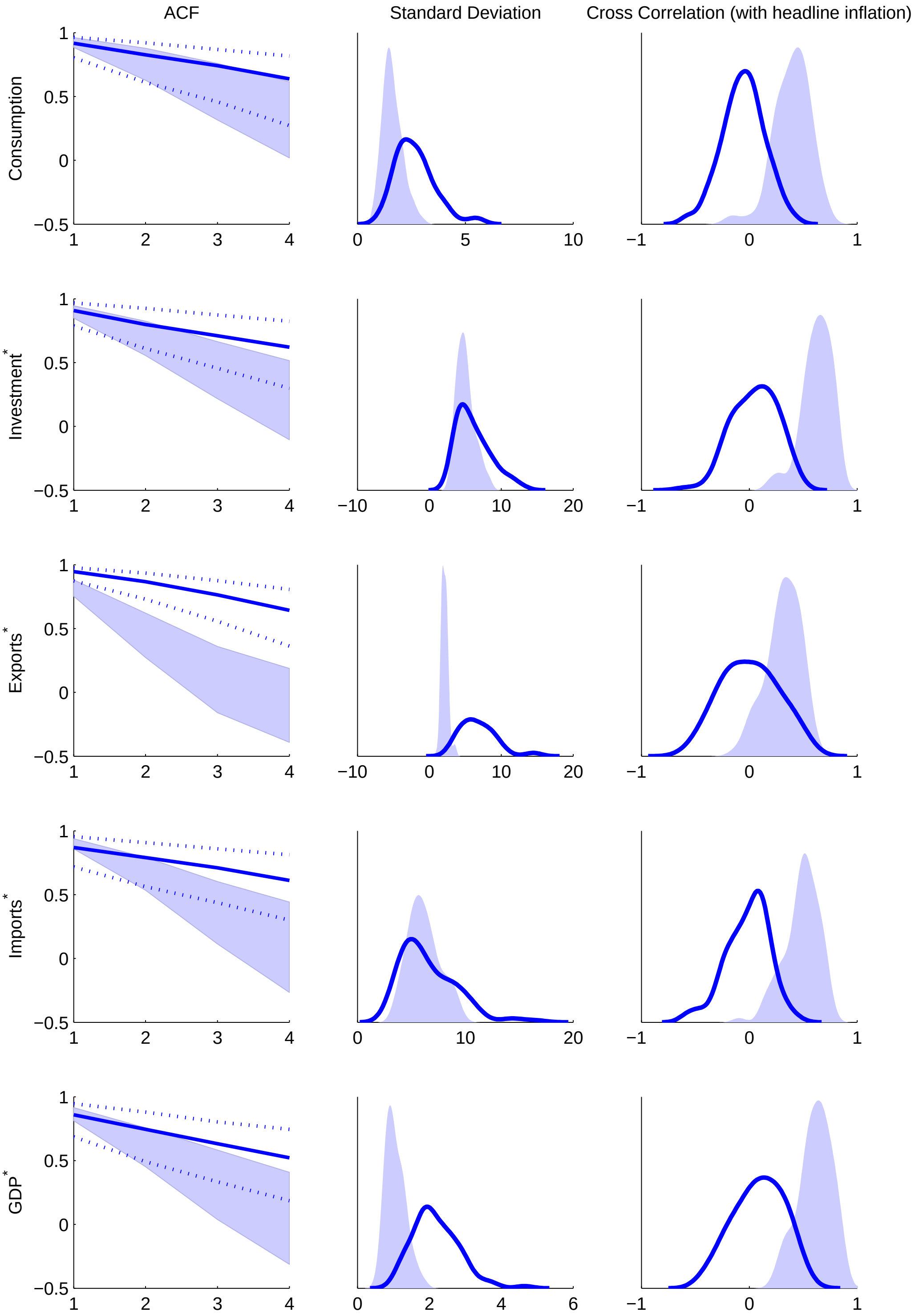
[11] We must stress that the data characteristics are reported only as an approximative benchmark, and they cannot be taken as representative numbers. The New Zealand has experienced a period of prolonged business cycle over the sample period, and thus the numbers might suffer significant biases.

Figure 4.3: Model moments: Prices



Note that the grey shadow denotes ninety percent probability intervals implies by the VAR model; the solid line indicates the moment from the posterior model with the dotted lines indicating 90 % probability intervals from the model.

Figure 4.4: Model moments: GDP components



Note that the grey shadow denotes ninety percent probability intervals implies by the VAR model; the solid line indicates the moment from the posterior model with the dotted lines indicating 90 % probability intervals from the model. * Simulated model variables aggregated to match official data.

Chapter 5

Model Properties

5.1 Model dynamics

This chapter presents the model's response to a selection of shocks and is designed to illustrate of the key mechanisms at work in the model. Each shock begins with the model at steady-state and the responses to the shock are presented as percentage deviations from steady-state. We focus on impulse responses from the posterior-mode but also display 90 % probability intervals around the impulse responses. We also focus attention on a selection of variables with the first nine panels of each figure depicting the following key model variables: the ninety day interest rate, the effective interest rate, the nominal exchange rate, headline consumer price inflation, tradable inflation, non-tradable inflation, fuel inflation, construction cost inflation, the price-level of the intermediate good, consumption, business investment and the manufactured exports. The final three panels are preserved for variables specific to understanding each shock.

5.1.1 Domestic shocks

Consumption preference shock

The consumption preference shock affects the utility households garner from aggregate consumption today relative to future consumption. Impulse responses to this shock are displayed in figure 5.1. The shock is constructed to produce an initial 1 percent increase in consumption.[1] This can be seen in the fourth row of the figure.

Because the consumption shock acts directly on the value households place on consuming goods today relative to the the future, this increases the shadow price of tomorrow's consumption as households seek to borrow to fund more consumption today. This new borrowing increases the loan-to-value ratio of consumers' homes, opening a small wedge between the policy rate and the effective interest rate faced by consumers, which, as discussed in section 2.4, is the relevant interest rate for the household's consumption decision.

The impact of the consumption shock on the loan-to-value ratio is displayed in the effective interest rate, in the top-middle panel of figure 5.1. The humped-shaped increase in the loan-to-value ratio drives a wedge of about six basis points between the effective interest rate (a direct function of the loan-to-value ratio) and the policy rate. The policy rate increases in the short term in response to the higher inflation profile over the short to medium term.

Note that while the shock impacts on the relative price of aggregate consumption across periods, the shock does not appear in the marginal rate of substitution. However, this shock will have indirect intratemporal effects. This is because the relative price of traded and non-traded consumption goods and consumers' willingness to substitute consumption between these goods determines their relative proportion of expenditure. The final two panels of figure 5.1 show that the profiles of tradable and non-tradable consumption are broadly similar, reflecting relative inelastic demand in the short run, due to deep habit. The small differences in profiles can be attributed to relative price differences. For example, the profile of tradable goods is (stronger/weaker) reflecting lower/higher prices for tradable goods due to the appreciation/depreciation of the exchange rate.

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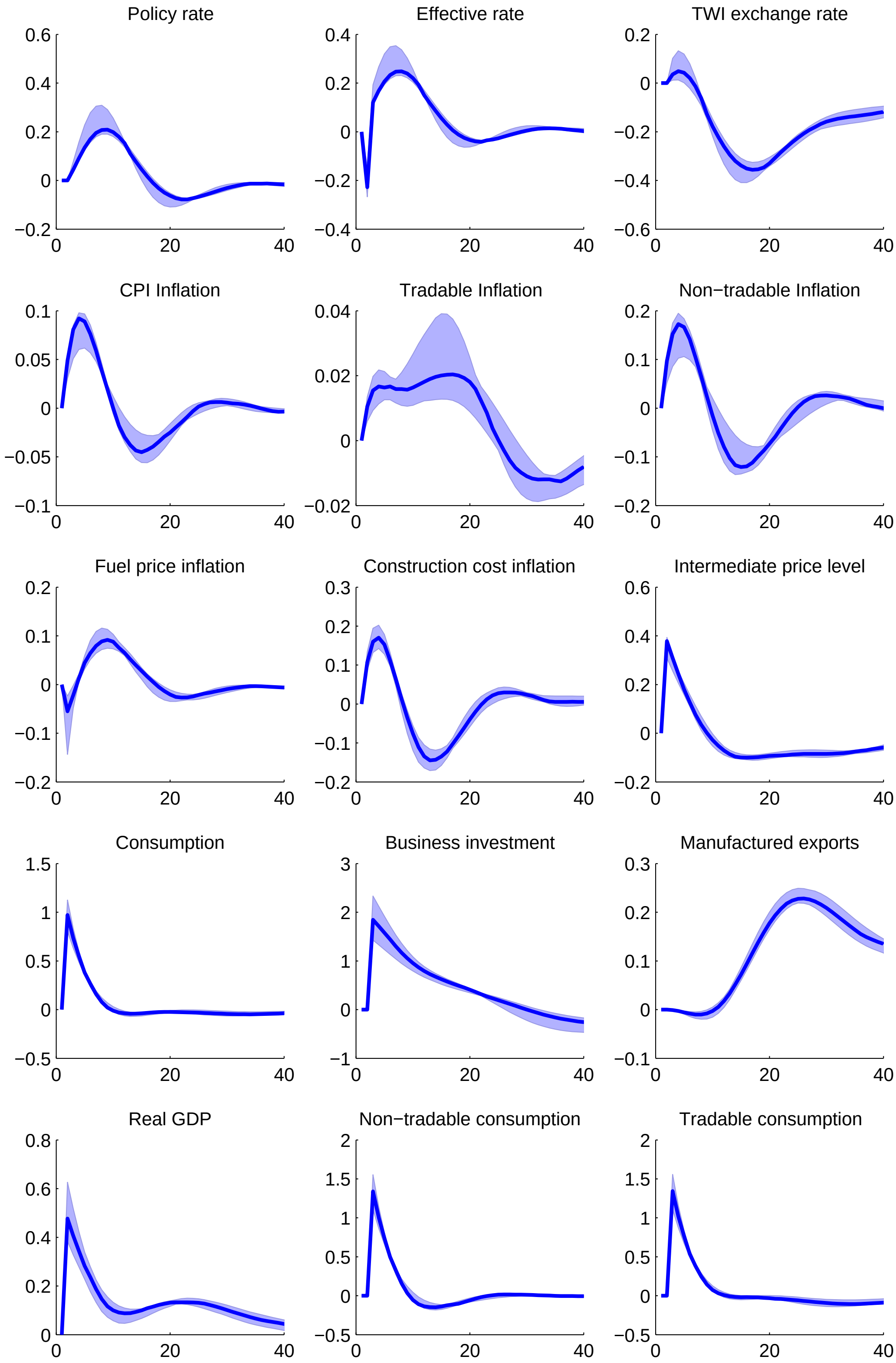
[1] This requires the magnitude of the initial shock to be 0.076.

Figure 5.1: Consumption preference shock



Note that the impulses are constructed from the posterior mode. The shaded area indicates a 90 % probability interval.

Similarly, the consumption preference shock increases consumption of fuel in the short run and also housing consumption. The impact on housing consumption is relatively small but persistence, reflecting the costs to adjusting consumption of housing services.

From the firms' perspective, the increase in consumption generates additional demand for their goods. As the firms begin to increase production to meet the additional demand, marginal costs increase due to the convexity of marginal costs in output in the non-tradable and construction sectors. The figure shows that this is also true of marginal costs of the producers of tradable goods. Supplying additional goods is relatively more costly and these additional costs are passed on to consumers in the form of price increases across all goods. However, the model assumes that firms face quadratic adjustment costs to changing prices and prefer to pass on a series of small price increases in preference to a single one-off increase in the price-level. This helps generate the hump-shaped inflation profile in the near term. It is also less costly to increase output today than it would be in the future, so both business and residential investment increase because they are relatively cheaper today than they would be in the future due to the fall in the shadow value of wealth.

The nominal exchange rate appreciates initially due to the monetary policy response. However, but in the medium- to long-run, the exchange rate depreciates because the inflationary impact of the shock raises the domestic price level and the exchange rate adjusts to maintain the relative price of export and imported goods. Overall, the inflation and relative price effects are larger than the relative interest rate differentials and the exchange rate depreciates. Finally, the depreciation in the exchange rate results in cheaper imports.

Non-tradables cost-push shock

Figure 5.2 depicts a non-tradable cost-push shock. The magnitude of the is shock constructed to produce a one percent increase in non-tradables inflation and is depicted in the rightmost panel of the second row of figure

5.2.

The non-tradable cost push shock effectively shifts the entire non-tradable Phillips curve up. We can think of the non-tradable Phillips curve as a dynamic aggregate supply schedule that relates the level of non-tradables output with non-tradables prices (inflation). For a given level of marginal costs and expectations of future non-tradables inflation, today's non-tradables inflation will be higher. This cost push shock and firm's indexation to lagged inflation ensures expectations of future non-tradable inflation remain above the starting point for some time, further contributing to the non-tradable Phillips curves deviation from its long-run position. At these higher prices, the demand for non-tradables consumption and government consumption is lower. Because non-tradable output has fallen, non-tradables marginal costs also fall. The profile for non-tradable marginal costs is shown in the bottom-left panel of the figure.

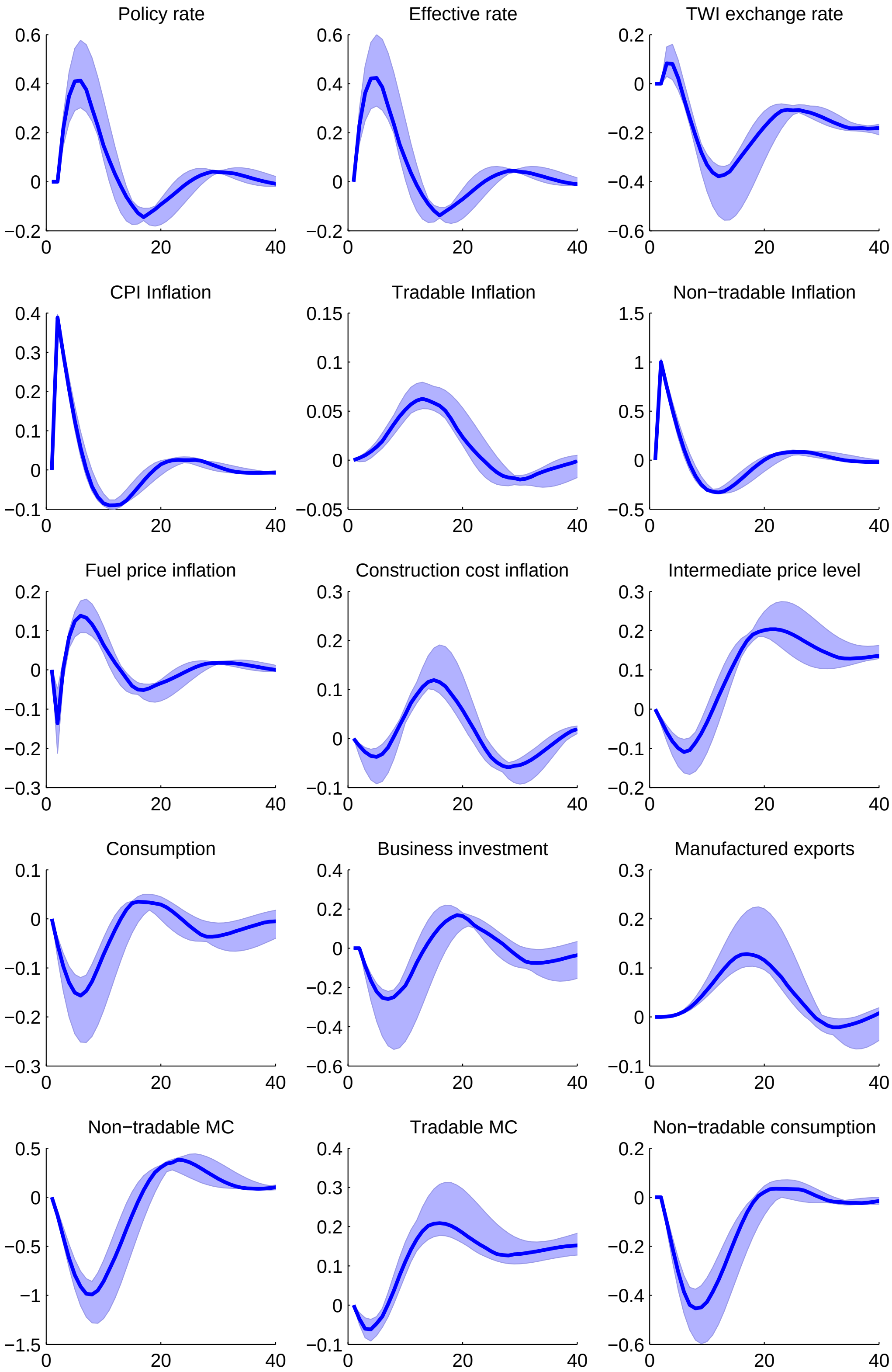
Higher non-tradables prices cause consumers to substitute away from non-tradable consumption into tradable consumption. This increased demand for tradable goods relative to non-tradable goods pushes up tradable inflation. This results in a net increase in aggregate inflation that can be seen in the second row of the figure. Higher headline inflation prompts a rise in the policy rate which in turn causes an appreciation of the nominal exchange rate (see the top-rightmost panel of figure 5.2).

Higher interest rates lead to a fall in consumption and investment demand, reducing production in the tradable and non-tradable sectors of the economy. The policy rate gradually returns back to steady state following the initial impact of the shock, and the exchange rate begins to depreciate. Because the relative price of domestic goods and services has risen relative to foreign prices, the exchange rate settles below its initial level.

Tradables cost-push shock

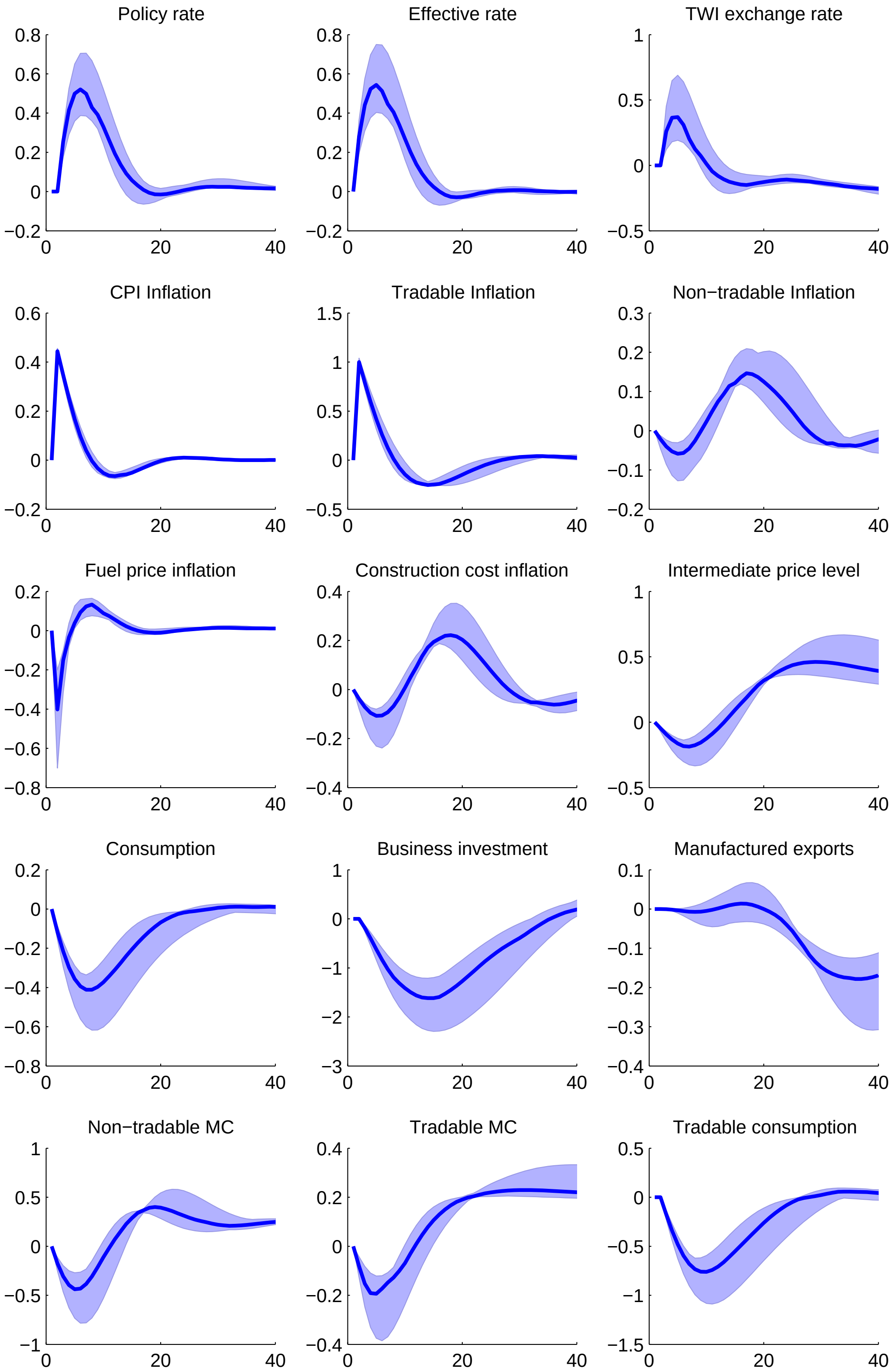
Figure 5.3 shows the tradables cost-push shock. The magnitude of the shock is constructed to produce a one percent increase in tradables inflation and is depicted in the middle panel of the second row of the figure.

Figure 5.2: Non-tradable cost-push shock



Note that the impulses are constructed from the posterior mode. The shaded area indicates a 90 % probability interval.

Figure 5.3: Tradable cost-push shock



Note that the impulses are constructed from the posterior mode. The shaded area indicates a 90 % probability interval.

In the same manner as its non-tradable counterpart, the tradable cost push shock lifts the tradable Phillips curve. The tradable Phillips curve can be thought of as a dynamic aggregate supply curve for tradable goods. The shock shifts the aggregate supply curve for tradables up, so that for a given level of marginal costs and expected future tradable inflation, tradable inflation is higher today. Expectations of future tradable inflation also remain above their long run level for awhile further contributing to tradable inflations positive deviation from trend. At these higher tradable prices, demand for tradable consumption and business investment is lower.

Households substitute away from tradable consumption towards non-tradable consumption because non-tradables are now relatively cheap. The increased demand for non-tradable consumption relative to tradable consumption pushes up non-tradable prices and inflation. This results in higher aggregate inflation. The monetary authority responds by raising interest rates, which results in a fall in aggregate consumption and investment.

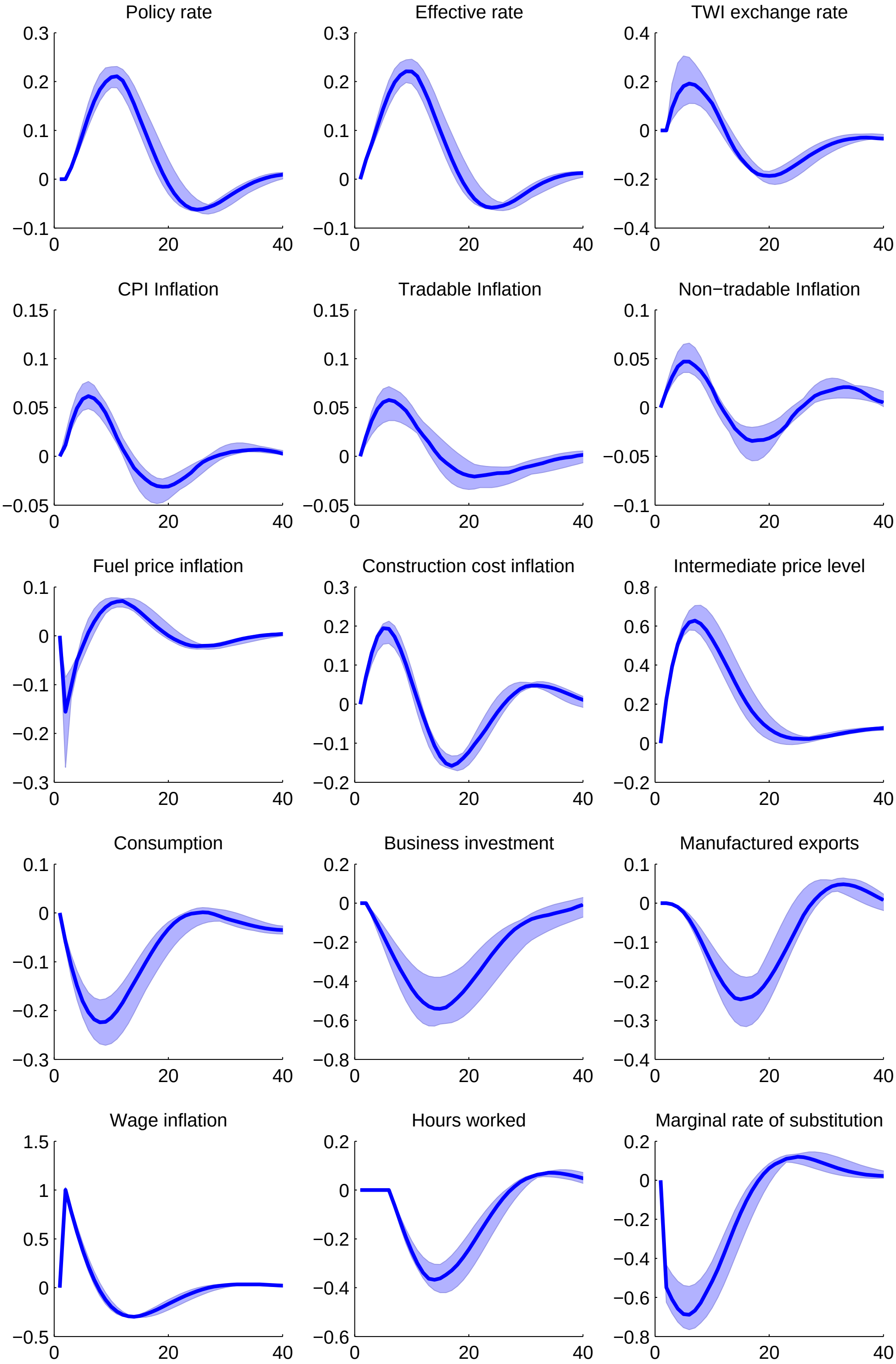
Once aggregate inflation is under control, the monetary authority is able to cut interest rates. The lower interest rates allow consumption and investment to increase until they are back at their steady state levels.

Wage cost-push shock

The wage cost push shock is analogous to a cost push shock in price Phillips curves. In this context, we can think of the Phillips curve as a labour supply function that maps labour supply into wage inflation. Figure 5.4 depicts the effects of a wage cost-push shock of sufficient magnitude to generate a 1 % increase in wage inflation in the initial period (depicted in the bottom left panel of the figure). The wage shock shifts labour supply such that, for a given level of wage inflation expectations and marginal rate of substitution (depicted in the bottom right panel of the figure), wage inflation will be higher. The new equilibrium in the labour market is thus consistent with higher wage inflation and lower labour demand with hours worked decreasing (see the middle panel in the last row of the figure).

Because labour is a key input to the production of the intermediate

Figure 5.4: Wage cost-push shock



Note that the impulses are constructed from the posterior mode.
The shaded area indicates a 90 % probability interval.

good, and because the intermediate input is used in all sectors of the economy, marginal costs in all monopolistically competitive sectors increase. Firms cannot substitute away from the intermediate good and their binding budget constraint forces lower demand for input factors and a production cutback.[2] Despite falling production, costs are still rising and are transmitted to price inflation. The monetary authority must respond by putting up interest rates.

The higher interest rates (the top-left panel of the figure shows that policy tightens by approximately 25 basis points) cause falls in consumption and investment, which further suppresses production. Demand for the intermediate good is forced to gradually drop (the demand curve shifts in), which brings the price of production factors, marginal costs of production, and prices of consumption goods back to their steady state levels. Finally, the exchange rate appreciates because the effect of the interest rate differential dominates the change in the relative price of exports.

Construction sector cost-push shock

Figure 5.5 depicts a positive shock that temporarily *decreases* installation costs in the housing construction sector. The shock is scaled in order to generate an immediate 1 % increase in residential investment.[3]

In response to the shock, the cost of constructing new houses drops, which generates demand for more residential investment, and increases the supply of new houses. The increase in demand for residential investment increases marginal costs in the construction sector. Construction firms pass on these increased costs as increased prices. Construction cost inflation increases (see the middle panel of the bottom row in figure 5.5) and so does the headline inflation rate, although this increase is not particularly pronounced. Monetary policy tightens in response to the inflationary pressure,

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[2] Strictly speaking, tradables firms can do some substitution away from labour using fuel in the input good, although this effect is very limited.

[3] As some context for the New Zealand data, residential investment comprises a little over five percent of real GDP, on average, since the start of 1990. Residential investment played a particularly strong role in the boom phase of the last business cycle.

provoking a small appreciation in the nominal exchange rate. Despite the increased supply of new houses, aggregate consumption and business investment initially drop due to the higher interest rates. Substitution effects change the mix of consumption, shifting consumption away from tradable and non-tradable consumption towards housing services.

As the construction costs level off, the primary inflation source disappears, and the interest rate eventually falls again. The supply of housing is still large enough at this point, and tradables and commodity imports are cheaper thanks to the appreciated exchange rate, to allow aggregate consumption to increase without inflationary pressures before returning back to its steady state.

5.1.2 Foreign shocks

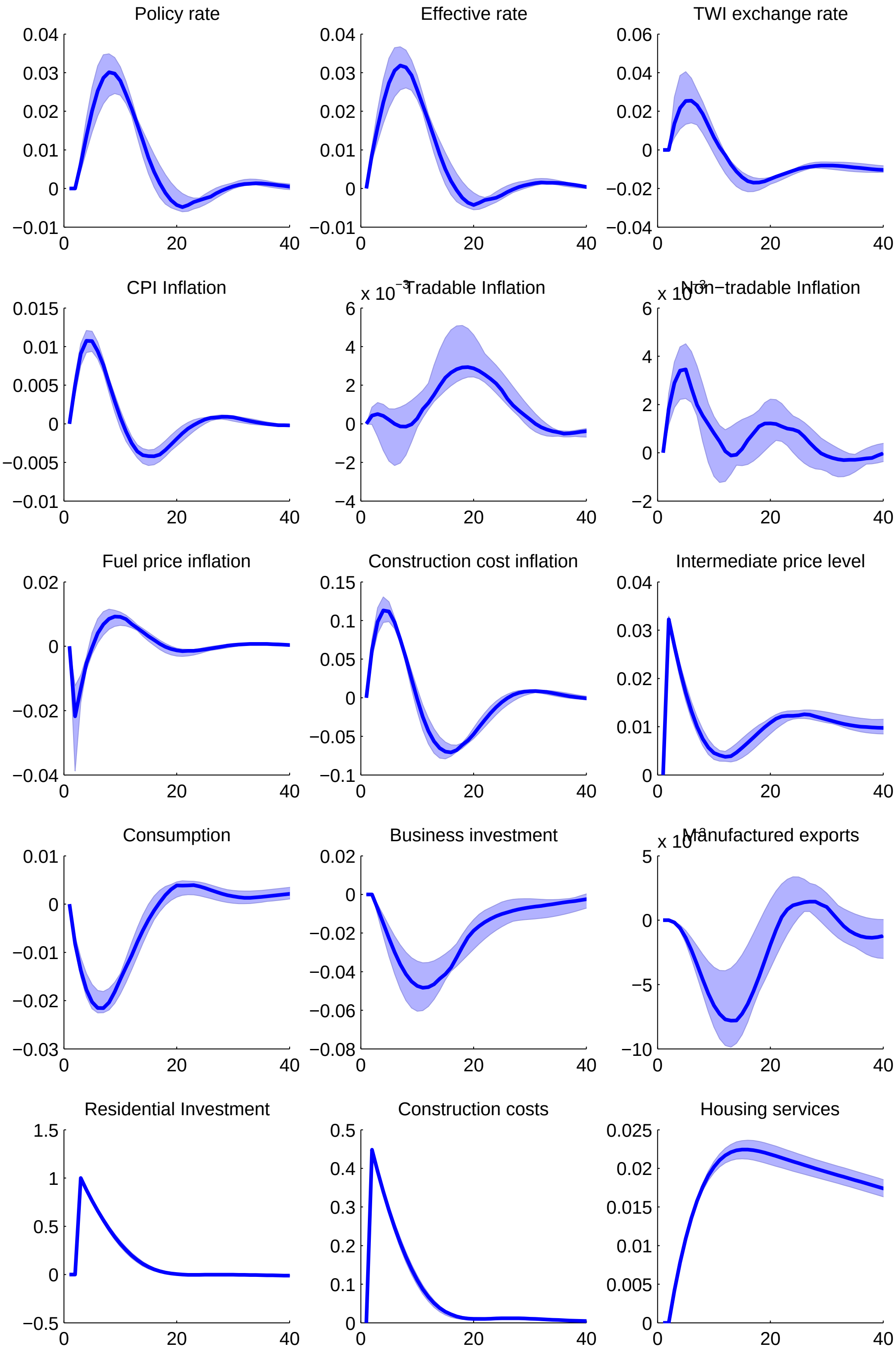
Foreign output shock

Figure 5.6 depicts how the economy responds to a foreign output shock. Because the foreign economy is described by simple AR(1) processes, the foreign output shock has no implications for either foreign interest rates or foreign inflation in the model. The specific shock examined in figure 5.6 is constructed to generate a 1 percent increase in foreign output.

In response to the foreign output shock, demand for both domestically produced commodity and non-commodity exports increases. Recall that demand for commodity goods are not price sensitive, while demand for non-commodity exports are sensitive to the price of non-commodity goods relative to the world price (the price of non-commodity goods relative to the implicit world GDP deflator).

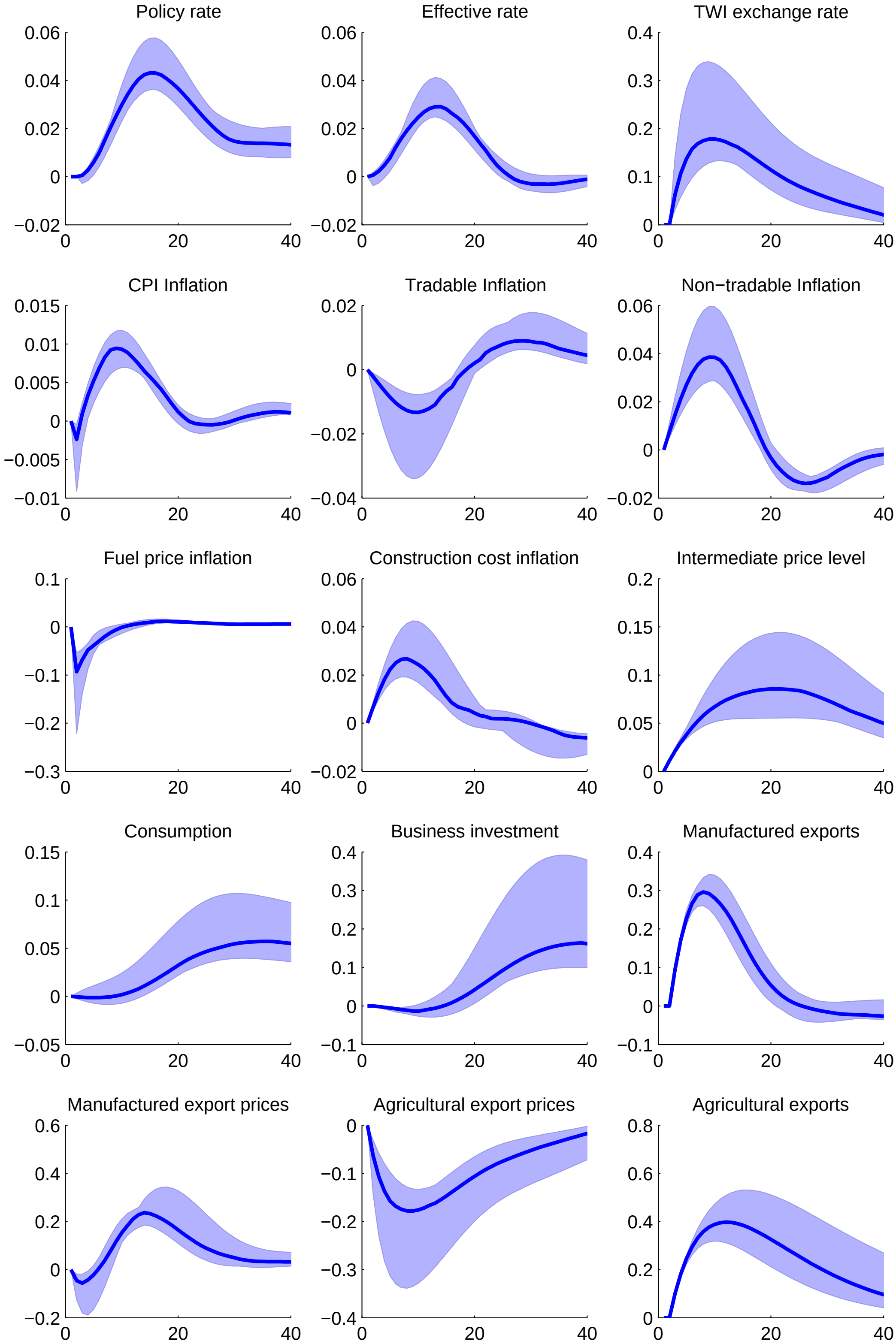
Firms that produce non-commodity export goods respond to the increase in demand by increasing output which generates upward pressure on marginal costs. The increased demand for the intermediate good in the non-commodity export sector effectively increases marginal costs for all sectors that use the intermediate good in production. This generates inflationary pressure across all sectors of the economy. Monetary policy responds to the increased inflationary outlook. This effect, combined with the

Figure 5.5: Construction cost-push shock



Note that the impulses are constructed from the posterior mode. The shaded area indicates a 90 % probability interval.

Figure 5.6: Foreign output shock



Note that the impulses are constructed from the posterior mode. The shaded area indicates a 90 % probability interval.

increase in the price of non-commodity exports, generates an appreciation of the exchange rate (see the top right panel of the figure).

The appreciation in the exchange rate produces an immediate fall in fuel price inflation. Over the medium term, this results in a decline in the relative price of imported goods, with domestic demand switching from domestically produced goods towards imports. Non-tradable consumption, housing consumption, and residential investment fall, while demand for imported investment and consumption goods increases. In aggregate, consumption increases. Despite the fall in non-tradable consumption, non-tradable output rises due to a rise in government spending (recall, the government consumes non-tradable goods in a fixed proportion to aggregate output).

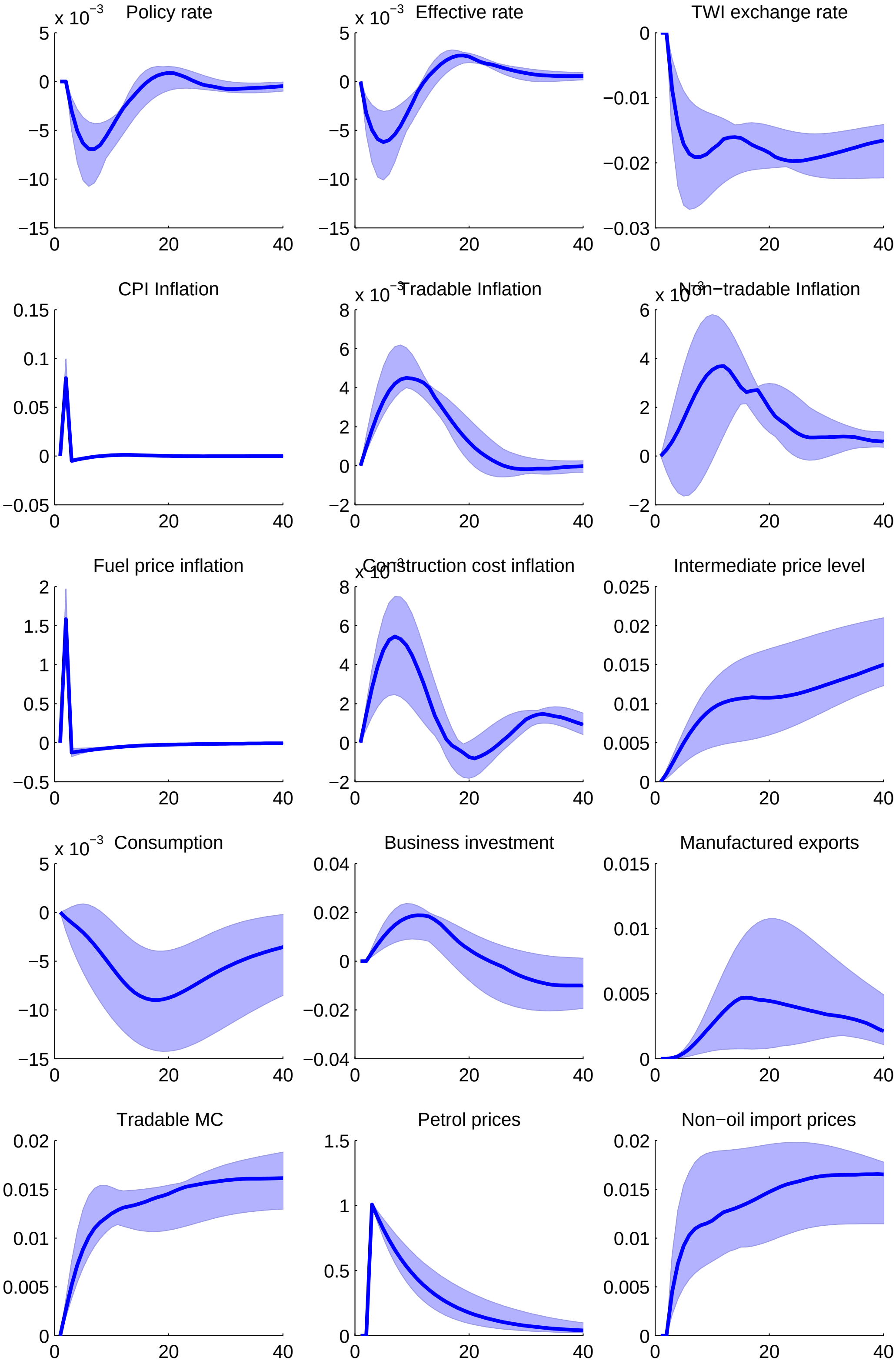
The increase demand for domestically-produced non-tradable goods and exports is met by an increase in the use of variable inputs by firms in these sectors. Labour demand, capital services, and fuel demand rise. To coax workers to provide additional labour effort wages rise. As a result, firms find that there is no alternative to increasing their productivity without accruing additional marginal costs.

Oil price shock

The magnitude of the world oil price shock depicted in figure 5.7 is constructed to generate a 1% increase in the domestic petrol price. An increase in world oil prices causes a sharp rise in domestic fuel prices. We assume that world oil prices are slow to adjust back to steady state following the shock. Thus, the initial rise in fuel price inflation is followed by a gentle decline over the following quarters. Headline inflation rises sharply initially. However, the impact of the decline in fuel prices following the shock is large enough to offset the consumer price inflation caused by higher marginal costs, and headline inflation quickly falls below target.

Although the shock has a large initial impact on headline inflation, the inflation profile across the medium term horizon is relatively benign. This is a direct result of the supply and demand channels through which oil

Figure 5.7: Oil price shock



Note that the impulses are constructed from the posterior mode. The shaded area indicates a 90 % probability interval.

affects the economy offsetting each other at longer horizons. The policy response is very small (less than one basis point). Indeed, the probability bands on the impulse responses for the effective rate households face span zero, suggesting the model is equivocal about whether households should face higher interest rates as a result of the shock.

As a result of the oil shock, firms face higher marginal costs since the cost of the intermediate good, in part constructed with petrol, increases. The marginal cost for producing the tradable good increases by slightly more than the other sectors partly because there is a small additional petrol input that is used in the production of the tradable good. These movements in marginal costs are inflationary, but are offset by the behaviour of the demand side of the economy.

While petrol only forms a small fraction of the bundle of goods that households consume (petrol is about 5% of the consumer price index) demand for petrol is particularly price inelastic such that households find it particularly difficult to substitute away from petrol in the short run. With binding budget constraints, higher petrol prices force households to consume less of other goods and aggregate consumption falls via the reduction in disposable income — a classic illustration of the income effect. In fact, consumption of petrol, housing services, non-tradable and tradable inflation all fall following the shock.

Because the oil price shock generates an increase in the domestic price level (relative to an unchanged foreign price level in the model) there is a small depreciation in the exchange rate. This leads to a slightly stronger decrease in tradable consumption since production of the tradables good relies partly on imported goods. Furthermore, the depreciation generates a small increase in manufactured exports.

Risk premium shock

Figure 5.8 depicts the result of a risk premium shock (see equation) that generates a one percent appreciation in the nominal exchange rate (see the top-right panel of the figure). The appreciation immediately results in cheaper imports, and marginal costs in the tradable sector drop dramatically because imported manufactured goods are used in the production of the tradable good.

Tradable inflation drops quickly but is returned to its target, after about five years. Non-tradable inflation falls, but the process for non-tradables inflation takes longer. Capital plays a large role in the production of the non-traded good and the investment goods required to produce capital are produced using imported materials in addition to the intermediate good and fuel. Thus, the sharp appreciation in the exchange rate leads to capital accumulating making non-tradables goods cheaper to produce in the medium-term. Further, because non-tradable marginal costs are expected to fall, non-tradable firms are reticent about increasing prices in the short term when the presence of adjustment costs restricts how quickly and by how much they can move prices in the future. Non-tradables inflation actually falls, even though both non-tradable consumption and the marginal cost of producing non-tradable goods are both higher in the short run.

5.1.3 Policy shocks

Disinflation shock

One metric for assessing the calibration of monetary policy in macroeconomic models is by analysing the dynamics following of the model following a one percentage point decrease in the inflation target. The sacrifice ratio, the ratio of the cumulative loss in consumption or output to the reduction in the target, is often computed in spite of evidence that estimates of the ratio vary widely Cecchetti and Rich (2001), may or may not be function of the degree of openness of the economy Temple (2002), and depend on the degree of central bank independence Daniels et al. (2005). Nevertheless, the shock is often presented and figure 5.9 depicts a one percentage point reduction in the inflation target.[4]

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[4] This exercise assumes the central bank is credible and that the change in the inflation target is believed by firms. Although the inflation target does not appear in the nominal adjustment costs and hence the Phillips curves, full indexation as we have in this model implies that the target is present (if we were to put it in it would just cancel out leaving the

Figure 5.8: Exchange rate shock

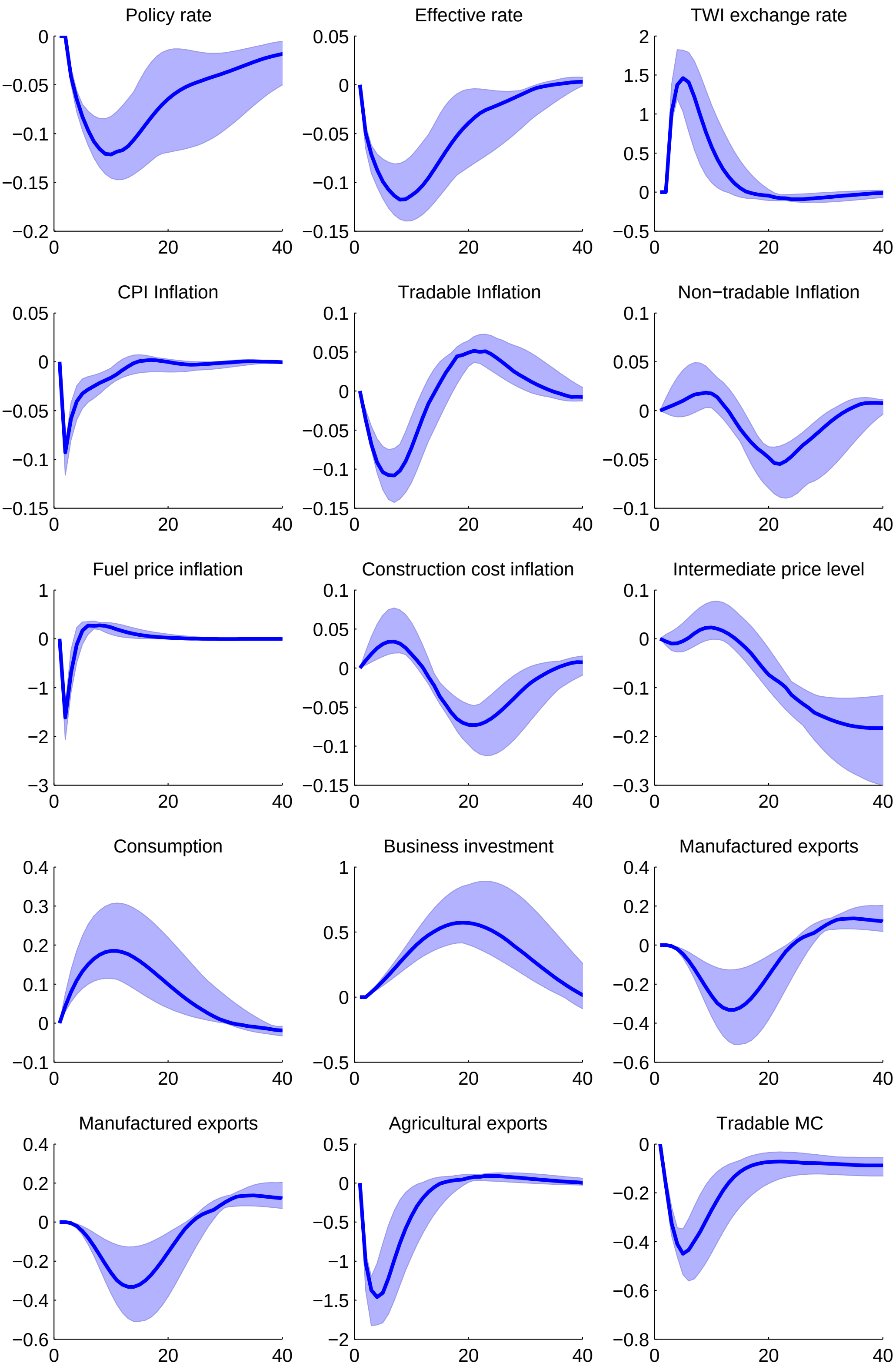
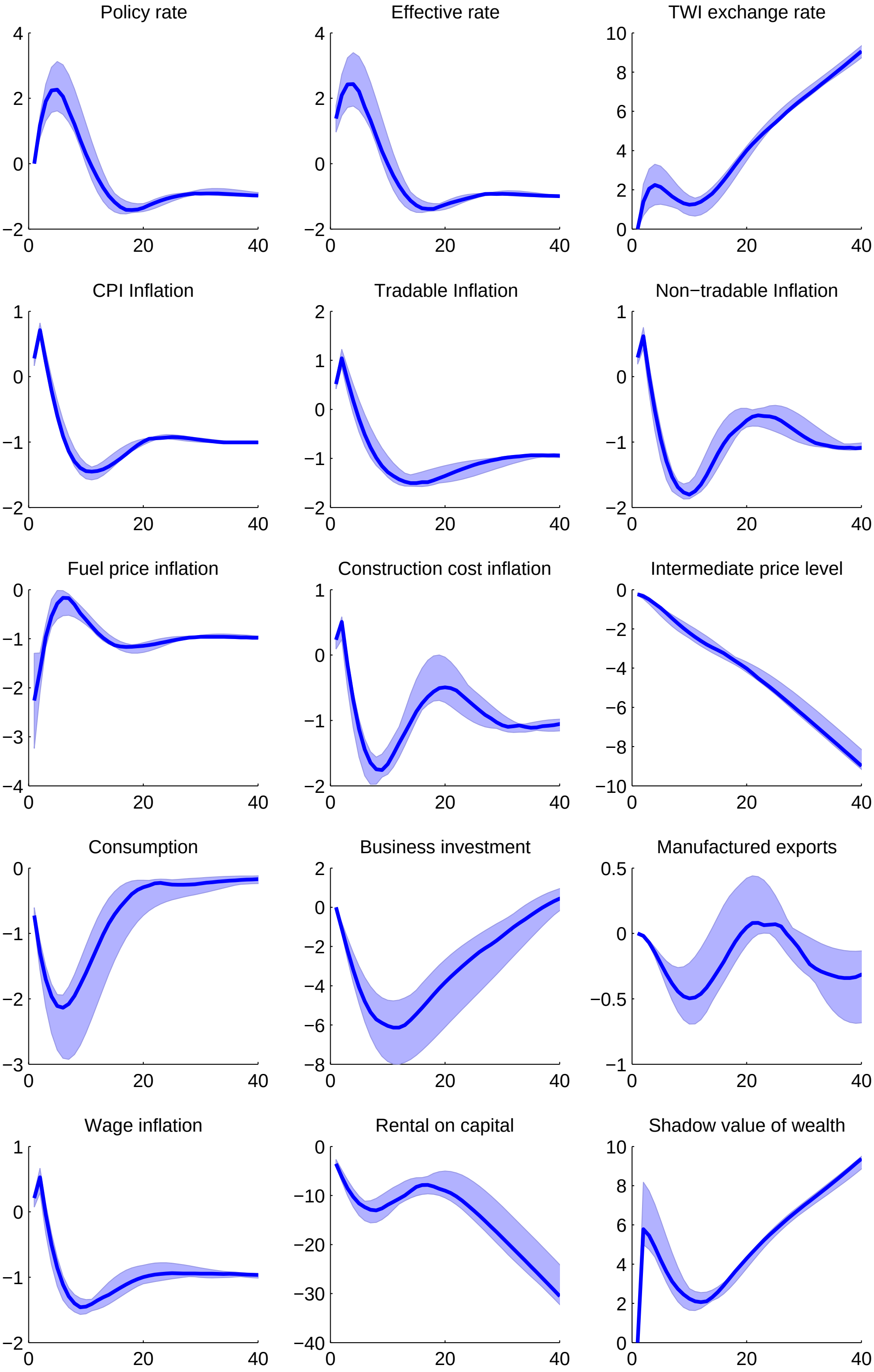


Figure 5.9: Inflation target shock



Note that the impulses are constructed from the posterior mode. The shaded area indicates a 90 % probability interval.

The top left panel shows that nominal interest rates must initially increase to drive inflation towards the lower target and remain elevated for approximately three quarters following the shock. The exchange rate immediately appreciates in response to the shock. Because the rate of change in the domestic price level is now always lower than its foreign counterpart, the exchange rate continues to appreciate at the rate determined by inflation differentials between exported and imported goods.

This appreciation, and the reduction in households' and firms' expectations about future inflation, reduces headline inflation immediately. Real interest rates increase and households postpone consumption. Consumption falls quickly but recovers to its trend growth rate after about four years. The fall in business investment induced by the shift in the target is about twice as large as the fall in consumption and takes some five years to return to its steady-state after the shock.

The decline in consumption, the initial appreciation of the exchange rate, and reductions in wage inflation all reduce firms' marginal costs. Headline inflation falls, and after some slight overshooting of the target, settles at the lower inflation target after about five years. Because the adjustment costs in non-tradable inflation are slightly stronger than tradable inflation, non-tradable inflation takes somewhat longer to return to its new level than tradable inflation.

Interestingly, the movements in inflation, consumption and interest rates are remarkably similar to the dynamics effects of the TOTEM model (see p. 79 in Murchison and Rennison, 2006).

Monetary Policy shock

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representation we have at the moment) in which case the change in the inflation target is believed by firms. Murchison and Rennison (2006) conduct a disinflation simulation that assumes firms take some time to learn or believe what the new inflation target is. We could conduct a similar exercise including the firm's belief of the inflation target (which would follow an AR process with a weight on the actual target) in the price adjustment cost term. When a disinflation occurs the firm's belief of the target would not drop out of the Phillips curve because it would be slowly adjusting to the actual target. The cost of disinflating under this setup would be higher.

Figure 5.10 depicts how the macroeconomy responds to a monetary policy shock. The shock considered is of sufficient magnitude to generate a one percent increase in the policy rate in the initial period (as depicted in the top left panel of the figure). This shock takes some time to dissipate, since there is considerable interest rate smoothing in the policy rule (recall that the posterior mean for ρ, the coefficient on the lag of the interest rate, is 0.884)

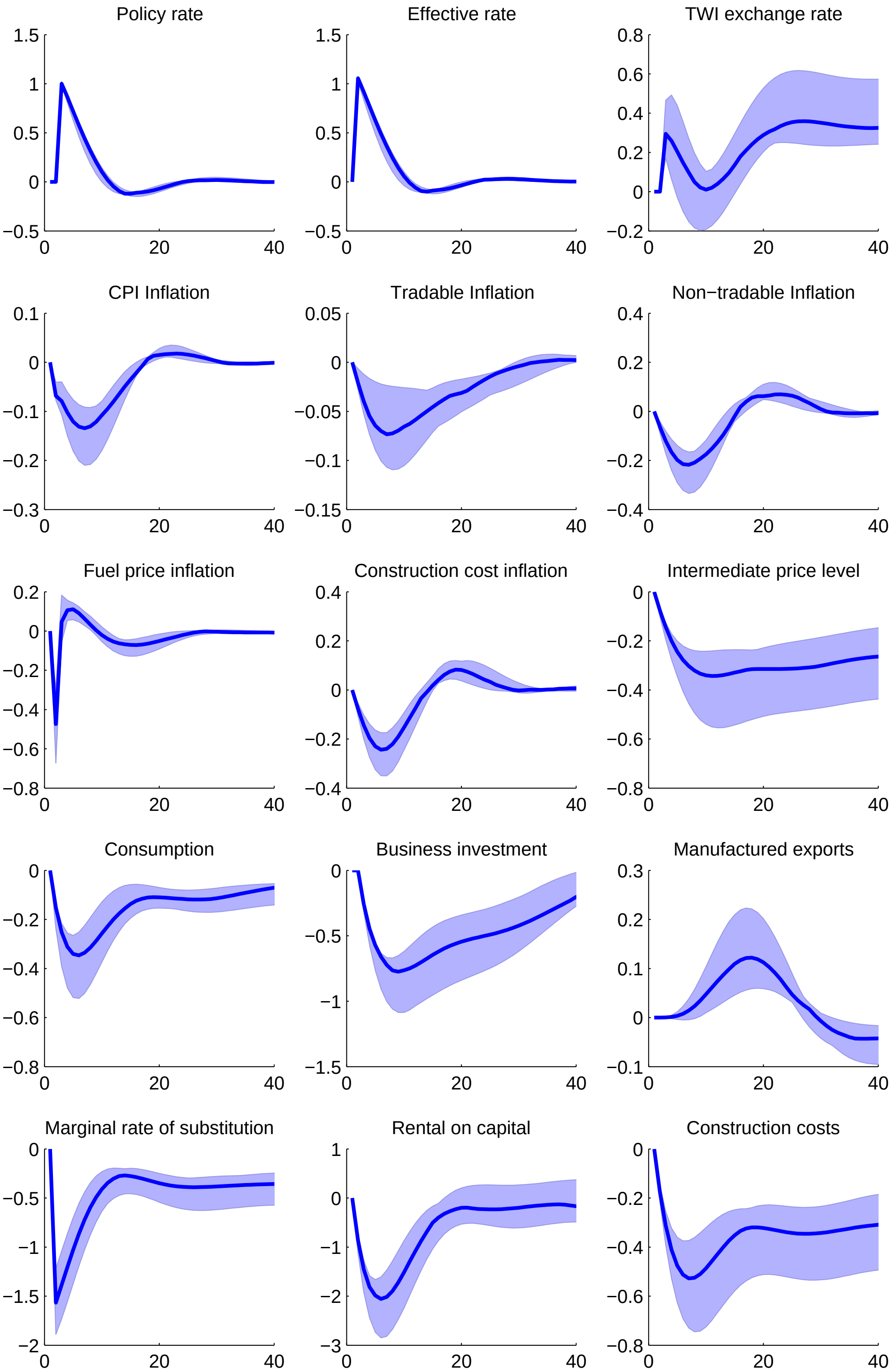
There are four main channels through which monetary policy operates in the model: (i) the consumption demand channel; (ii) the exchange rate channel; (iii) the investment demand channel; and (iv) the expectations channel (which works through the first three channels). The tighter monetary policy directly following the shock directly affects the consumption Euler equation, causing households to delay consumption as the price of borrowing rises. This also affects the marginal rate of substitution between consumption and leisure shown in the bottom left panel of the figure.

Figure 5.10 also shows that aggregate consumption decreases relatively quickly following the shock, falling by about 0.3 percent about three quarters after the shock. Firms reduce output to match the fall in demand and marginal costs fall, but the presence of adjustment costs prevents firms passing on lower prices immediately and inflation takes about five quarters to fall about 0.1 percent. Noticeably, non-tradable inflation falls more gradually than tradables inflation and takes longer (about five years) to return to its target value.

The increase in the interest rate appreciates the exchange rate by about 0.75 percent immediately following the shock. Because some agents in the economy set their future expectations of the exchange rate according to past lags (see equation 2.64), there is some persistence in the appreciation of the currency that puts downward pressure on import prices.

Finally, the increase in the policy rate also changes firms investment decisions. Firms discount by more the expected returns using the current interest rate in order to set investment. A higher interest rate thus reduces the expected return on investment, decreasing current investment. This reduces inflation pressure by reducing demand for the tradable good used

Figure 5.10: Monetary policy shock



Note that the impulses are constructed from the posterior mode. The shaded area indicates a 90 % probability interval.

for investment and reducing output.

Four quarter interest rate shock

A popular alternative to a one-quarter monetary policy to examining the transmission mechanism of monetary policy (see Harrison et al. (2005), for example) is to shock the interest rate by 100 basis points and leave policy fixed at the higher rate for four quarters. However, some care should be taken in interpreting this experiment since agents expect monetary policy to be set in every period according to the simple policy rule in equation (2.65) and are surprised for each successive quarter that policy is not set according to the rule.[5] This may have implications for how the impulse responses play out. That said, this type of policy shock has been used at the Reserve Bank of New Zealand to understand how FPS (the core Forecasting and Policy Model) behaves and is a useful point of comparison between the new, DSGE model and FPS.[6]

Figure 5.11 shows the results of the shock across the two models with the solid blue line used to represent KITT and the dashed red line used to represent FPS.

In terms of the interest rate (displayed in the top-left panel of the figure), both models show some persistence although interest rates fall more rapidly in FPS than in KITT. To an extent, this may be attributed to the fact that following the shock, inflation decreases more quickly in FPS than KITT and with both policy rules largely determined by expected future inflation, rates fall more rapidly in FPS which predicts lower future inflation.

Both models show relatively similar initial appreciations in the real exchange rate following the shock. However, since the decrease in inflation

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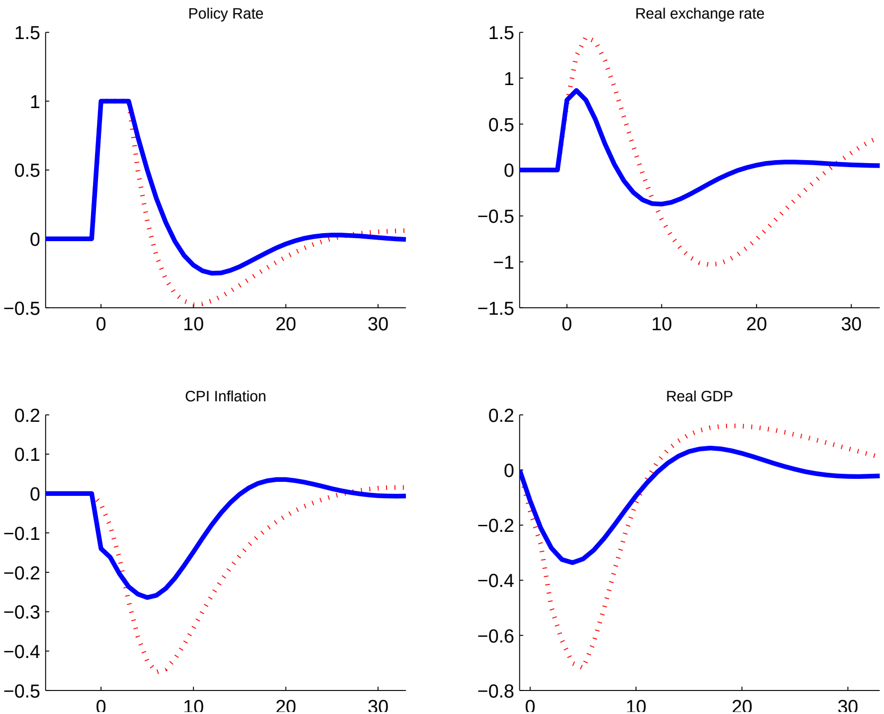
[5] Using euro area data Adolfson et al. (2007a) find interventions of this ilk can have non-trivial implications that render economic forecasts at particular horizons “meaningless”.

[6] Note that the shocks are unanticipated in KITT while they are anticipated in FPS, this reflects the primary operating mode of the modelling software. KITT is coded in IRIS while FPS is coded in TROLL. However because the agents in KITT are more backward looking the comparison seems fair.

Figure 5.11: Four quarter monetary policy shock: model comparison



Note that the impulses are constructed from the posterior mode.
The shaded area indicates a 90 % probability interval.

has left the domestic price level permanently lower that the foreign price level, the real exchange has a slight appreciation. There appears to be a secondary cycle in the real exchange rate in FPS but the real exchange rate ultimately returns to its long run constant steady-state level.

The bottom-right panel depicts the impact of the shock on the real economy. Real GDP falls further under the FPS model than KITT and appears to undergo a secondary cycle with output increasing above its initial level after approximately five years. However, broadly speaking, the properties of the two models are not too dissimilar.

Chapter 6

The Model in the Policy environment

6.1 Overview

Of course, operating a DSGE model in the forecast and policy environment requires more than simply presenting the forecasts from the model. Unsurprisingly policymakers want to understand and know the drivers of forecasts (see Pagan and Robertson, 2007, for example). Furthermore, policymakers are exposed to a whole range of information (from financial markets, business surveys, alternative macroeconomic models, regional experts) and want to reconcile this diverse information set. This section documents how the forecasting process works and provides examples of the the use of three specific tools: (i) forecast decompositions; (ii) density forecasts; and (iii) techniques for adding judgment; that help complement the use of the DSGE model. First, the following section details the forecasting process.

6.2 Forecasting process

Technically, each forecasting round begins with creating a database. The first step is to take the database of raw data, which was used during the previous forecasting round, and update it for the latest data releases.

Next, sector experts provide monitoring quarter information for data points where official data are not yet available, but are necessary for initialisation of the forecast. Typically, the official data on GDP are not available for current quarters, and they have to be estimated – so called now-casted. The now-cast is generated from a portfolio of univariate and multivariate forecasting models, and is subject to expert judgement.

Subsequently, the raw database is expanded to incorporate exogenous external projections such as the world GDP, world inflation rate, or world interest rate, that override the simple AR(1) forecasts from the estimated KITT model. Figure 6.1 shows how the updated database, the monitoring quarter information and the exogenous variables might be used to form a model-equivalent database.

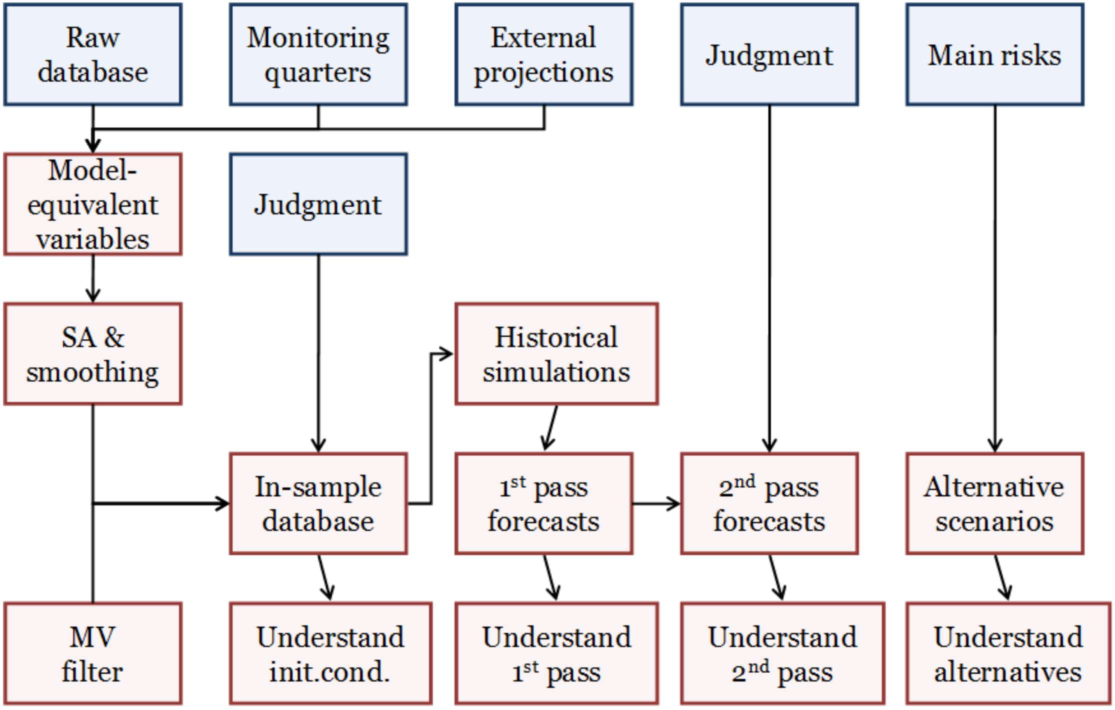
Then the transformed data are seasonally adjusted and smoothed using the X12 algorithm. Finally, using either a univariate of multivariate filter, the data are decomposed in to their permanent and cyclical components. The cyclical components establish the in-sample database. The trend components are extrapolated into the future, and saved for the variable reconciliation at the stages when forecasts are reported.

Before the in-sample database is ready to be used for historical simulations and the first-pass forecast, expert judgement might be required to adjust starting points for the forecast, e.g. an expert view on the current business cycle position. Any changes in the starting point are explored for their sensitivity of the forecasts to the initial conditions.

When the in-sample database is finalised, KITT is used to simulate the data, and provide their historical interpretation in terms of structural shocks. All model variables are decomposed into the contribution of individual shocks. We obtain the estimates of most recent shocks hitting the economy, and the cumulative effect of the past shocks. Their unique and robust estimation is important, because they provide a basis for the story that explains the forecast.

Subsequently the first-pass forecast is produced, and a series of exercises follow to interpret the forecast. To build up a consistent story, we use two

Figure 6.1: Forecasting process with KITT



sets of tools in particular: forecast decompositions and density forecasts. Decomposing the forecasts facilitates understanding of the drivers of the forecast. Density forecasts help provide a sense of the uncertainty that surrounds the first-pass forecast.

The first pass forecast is purely based on the model. To incorporate judgment about particular forecasts (from policymakers, forecasters and modellers alike), we can add reduced-form judgment using the techniques in Beneš et al. (2008). These techniques produce the conditional forecasts that recover the reduced-form judgement (such as a flat interest rate track) and are the most likely set of forecasts from the perspective of the DSGE model. Importantly, the degree of judgment can be monitored and compared to the extent of judgment that has been the historical norm.

Once the first-pass forecasts have been judgmentally adjusted, unsuprisingly these forecasts become second-pass forecasts. Several additional iterations may be required before the forecasts are published, and these iterations can use the same set of tools (forecast decompositions, density forecasts, judgment) that were applied to the first pass.

To illustrate the key ingredients that make up a particular model based forecast we can write the reduced form representation of KITT once it has been linearised and solved. The h step ahead forecast from such a representation can be written as:

$$\hat{y}_{t+h} = F^h y_t + \sum_{j=1}^{h} F^{h-j} B \varepsilon_{t+j} \tag{6.1}$$

where y_t is a vector of model variables, $\hat{y}_{t+h}$ is the h step ahead forecast of these variables, F is a matrix of reduced form coefficients, B is also a matrix of reduced form coefficients and ε_t is a vector of structural shocks.

From this representation we can see that the h step ahead forecast is a function of the reduced form coefficient matrices F and B, the initial conditions y_t, and a sequence of shocks $\{\varepsilon_{t+j}\}_1^h$. Furthermore, producing density forecasts is simply a case of drawing from the parameter distributes that describe F and B and from the distribution of shocks to generate a sequence of shocks $\{\epsilon_{t+j}\}_1^h$, in order to produce a sequences $\{y_{t+j}\}_1^h$.

Also, to produce a no judgement model based forecast requires knowledge of the coefficient matrix F and some initial conditions y_t so that:

$$\hat{y}_{t+h} = F^h y_t \tag{6.2}$$

we assume that agents expectations of shocks conditional of information at date t are zero and that there are no unexpected shocks.

However if we want to judgementally adjust forecasts we can see from (6.1) that there are three areas that judgement can be added, we can (i) change the initial conditions y_t to better reflect our views on the current situation, (ii) change structural parameters that affect the coefficient matrices F and B, or (ii) add sequences of shocks $\{\varepsilon_{t+j}\}_1^h$ over the forecast horizon.

6.3 Predictive densities

Predictive densities (or fan charts) have become a popular way of conveying forecast uncertainty to policy makers and the public. Indeed, many central banks routinely publish predictive densities in their inflation reports, as a way of aiding the communication of uncertainties around a particular policy prescription.[1] Not only can predictive densities aid in the communication of uncertainly, they can also be used to produce event probabilities, such as the probability of a recession, or the probability of inflation breaching the target band. In this section, we present predictive density forecasts and use the densities to assess the probability of inflation being outside the top of the target band.

Our predictive densities are constructed by simulating the model on the basis of the parameter and shock uncertainty estimated in section 3.

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[1] Some central banks that currently publish predictive densities (or fan charts) in their inflation reports include the Bank of England, Norges Bank and Sveriges Riksbank.

Explicitly, we randomly draw 1000 parameterisations of the model from the estimated posterior distribution. Then, for each of these parameterisations, we draw a set of random shocks for the period from 1992Q1 to 2008Q1. We then simulate the model, producing 1000 alternative forecasts, each conditional on a particular parameterisation of the model and a particular sequence of shocks.

Some of our predictive densities are displayed in figure 6.2. The solid line in each graph is the point (median) forecast and the bands around the point forecast represent the 90 percent, 70 percent and 50 percent probability intervals resulting from our simulations.

By themselves, the predictive densities appear to show a great deal of uncertainty around the point forecasts. For example, the densities show headline inflation to be somewhere between 2 and 4 percent with 90 percent confidence by the beginning of 2004 – only 1 year after the predictions were made. However, considering the relatively high volatility seen in the actual data over the forecast horizon helps to put the seemingly large amount of uncertainty in the predictive densities in perspective. The historical data tend to wander across most of the area covered the predictive densities, suggesting that they reflect the uncertainty in the forecasts reasonably well.

For an illustration of a probability experiment, we compute the probability of a 3 year moving average of inflation being outside the top of the target band for each quarter in the first year of the forecast horizon. The results of this experiment are displayed in figure 6.3.

The probability of a breach in the inflation target is around 10 percent in 2002Q4 and peaks at over 20 percent in 2003Q1. More generally, one can imagine a multitude of probability experiments that can be conducted with our estimated predictive densities, thus improving the breadth and quality of advice that can offered to policy makers.

Figure 6.2: Selected density forecasts from 2002Q4

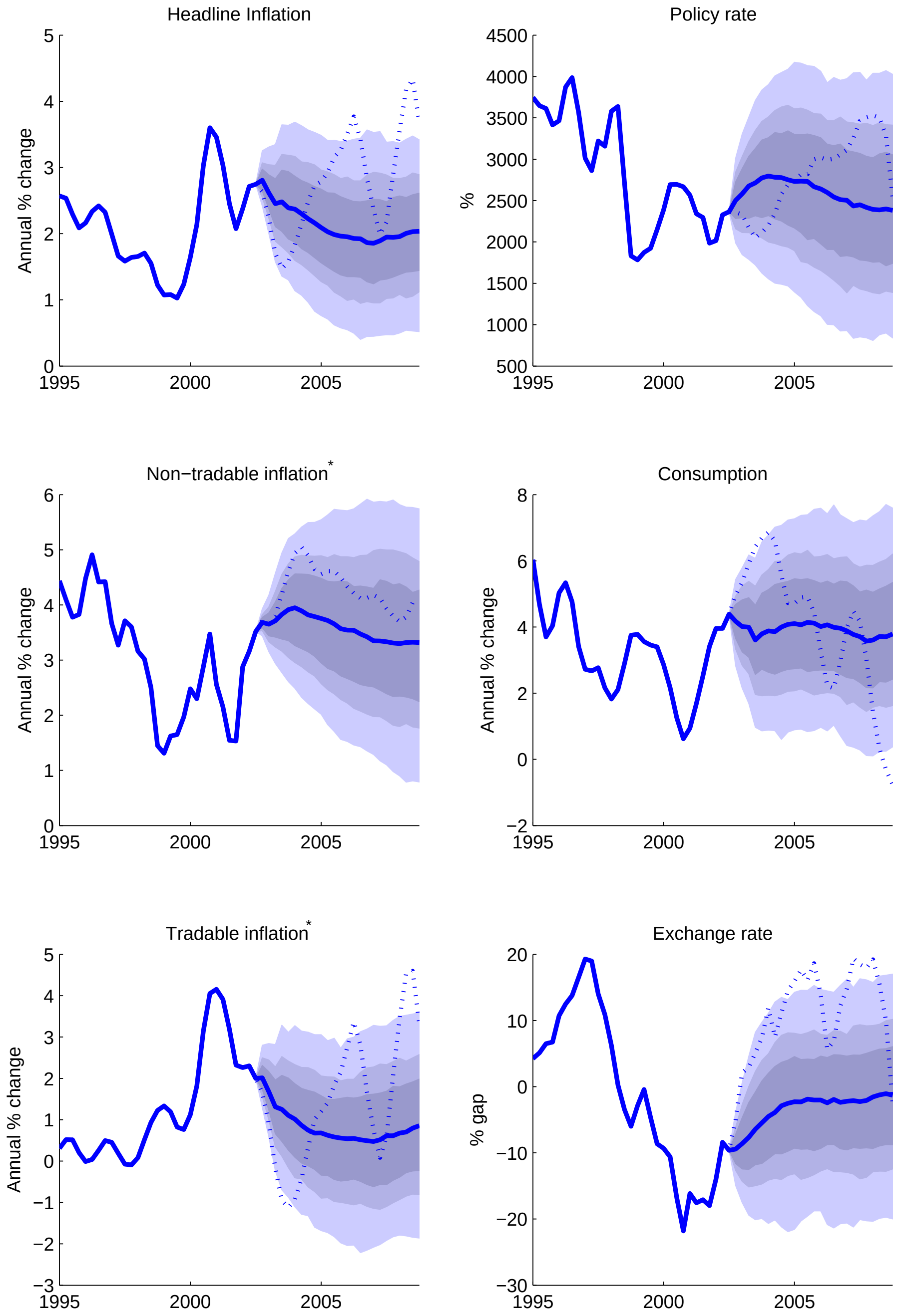
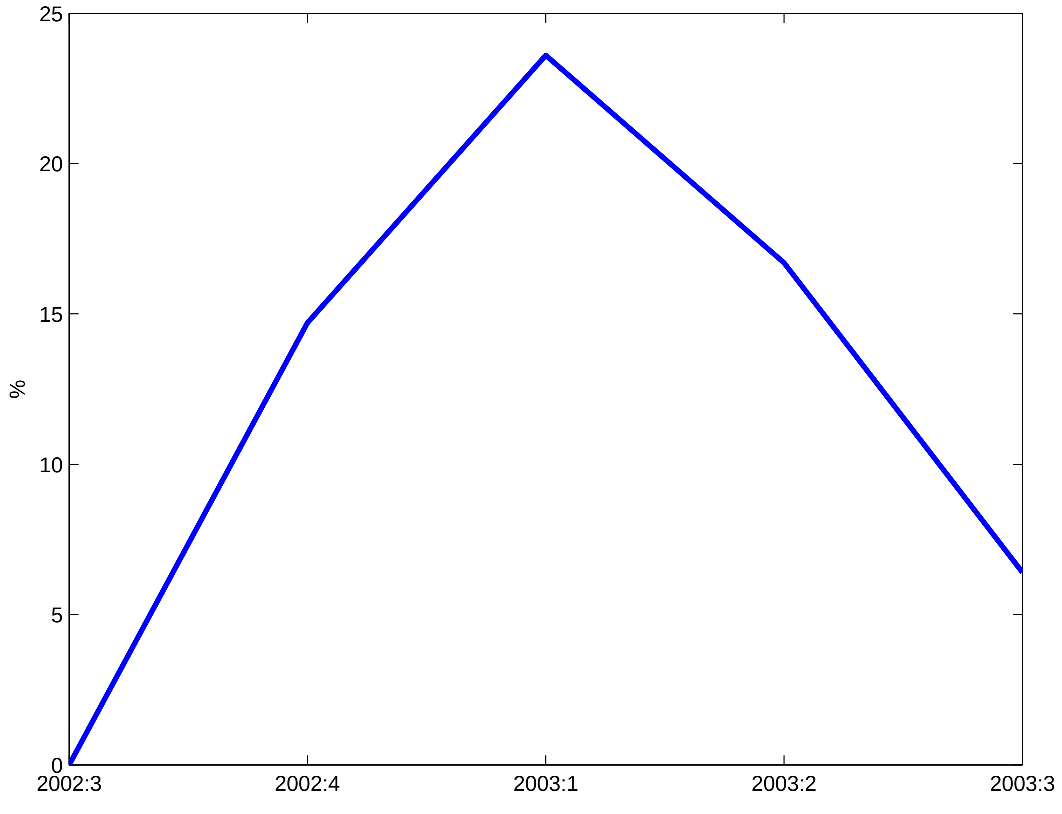


Figure 6.3: Probability of inflation above 3 percent



6.4 Forecast decompositions

In the sections bellow, we provide a forecast example. The purpose of this example is to demonstrate the devices that we use to interpret and present forecasts. We abstract from real time issues like adjusting trends and initial conditions, and thus the example does not provide a solid ground for judging the forecasting performance of KITT. We set the the forecasting experiment at 2002:Q2, and the forecasting horizon spans till 2008:Q2.

In this section, we describe the tools we use to reveal a forecast story, which is disciplined by the logic of economic theory incorporated in KITT. In figure 6.4, where the process is shown graphically, you can see that we take a top-down strategy to tell the story. We begin with revealing the implied future interest rate track, and then explain the driving factors.

The policy rate is decomposed into two basic components. The first part is the reaction to expected inflation deviation from the target. This is the most volatile part of the interest rate. The second part is the policy smoothing component, which reflects central bank's preferences to change interest rate gradually. In Figure 6.5, we show an example of such a decomposition. Note that it is performed on de-trended series. In the top panel, we see that the implied interest rate is increasing from 2002:Q2. The bottom panel provides an explanation why. The headline inflation rate is expected to deviate from the target on the forecasting horizon. Initially, the interest rate is driven by the reaction to it. But the increase is partially offset by the smoothing component. The inflationary pressures are foreseen to be persistent, and the reaction component slowly accumulates in to the smoothing component, which after two quarters gains on an increasing momentum and becomes the main component of the interest rate that brings the inflation rate back to the target.

The decomposition of the headline inflation creates the next layer in the story. We break down the inflation rate in to its four components: tradable, non-tradable, construction costs, and petrol price inflation rates. An example of the decomposition is in Figure 6.6. We can see that the price of non-tradables and construction costs are expected to be the main

Figure 6.4: Forecast decomposition: Overview

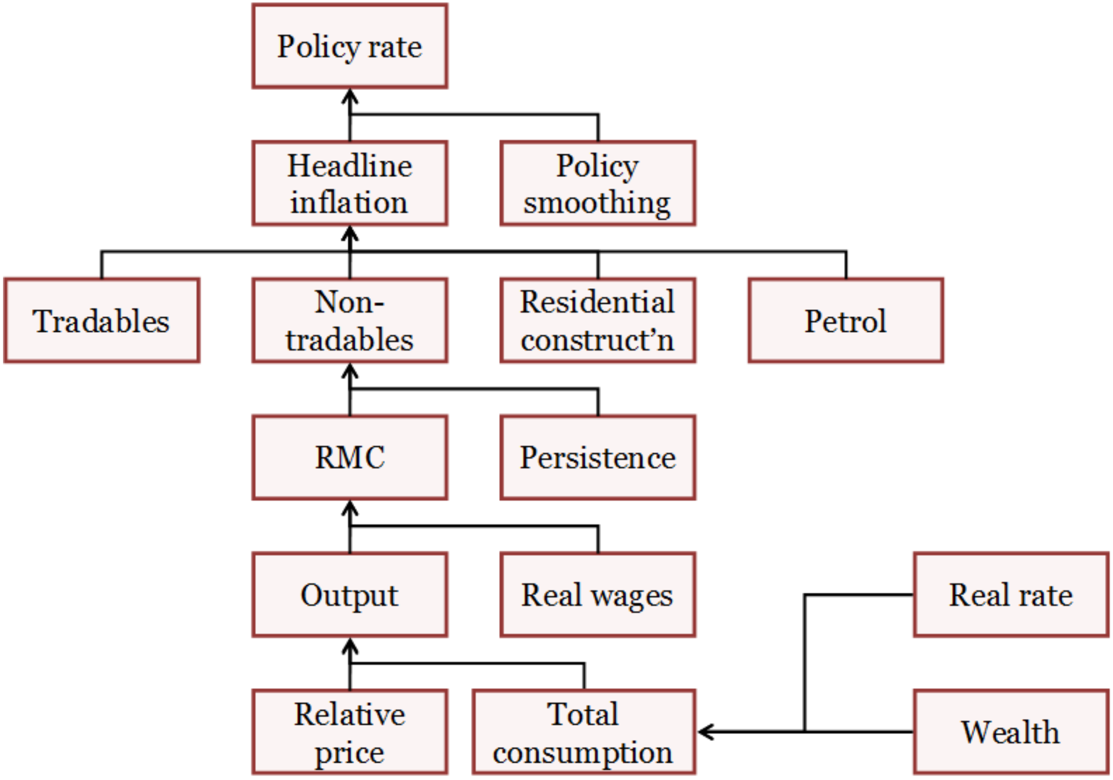
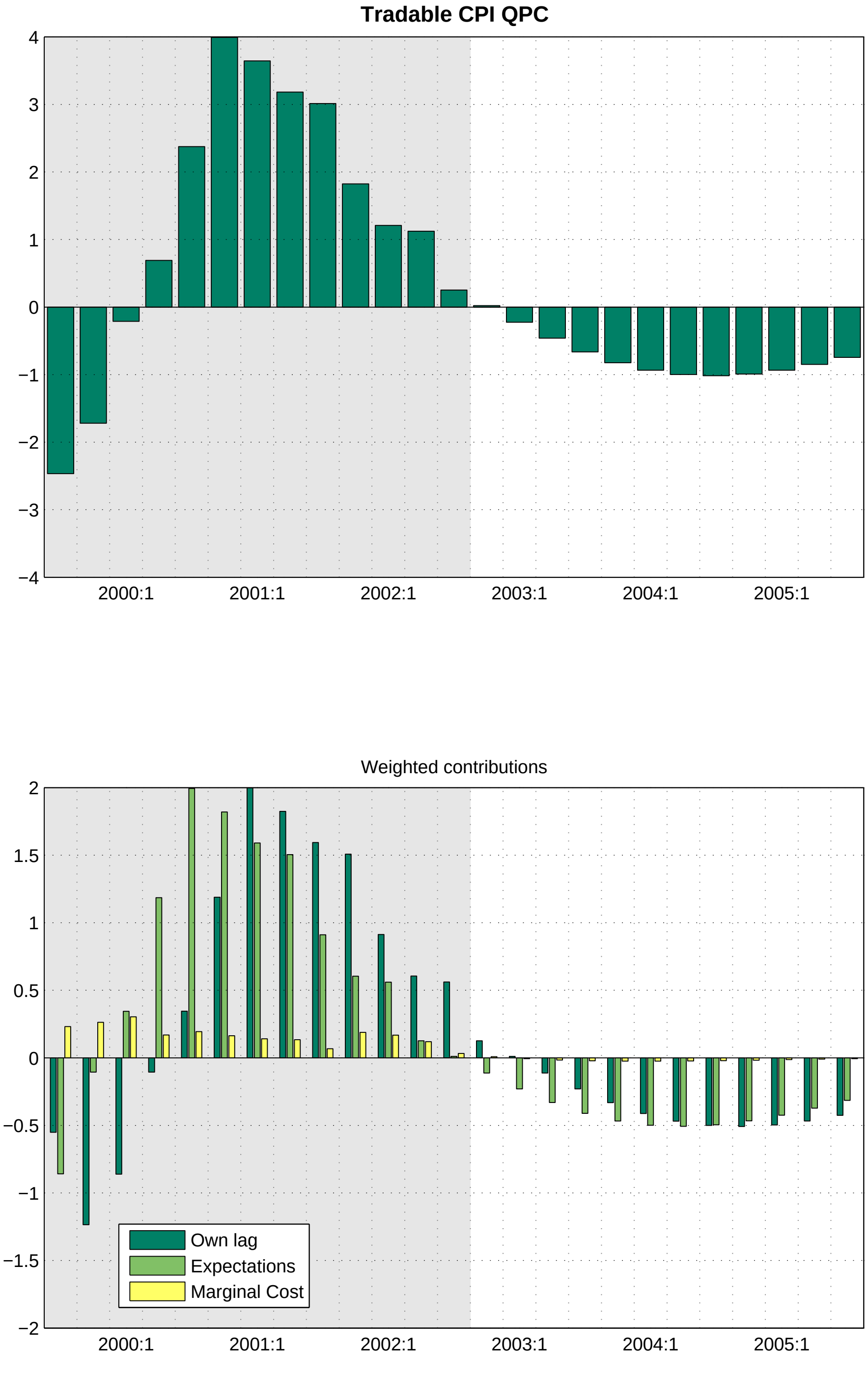


Figure 6.5: Forecast decomposition: Ninety-day interest rate

Figure 6.6: Forecast decomposition: headline inflation

Figure 6.7: Forecast decomposition: tradable inflation



sources of headline inflation, where as the price of tradables is deflationary, and the petrol price is neutral. Such a basic breakdown is a junction for a deeper structural story. At this point the main contributors of inflation are identified, and we continue down their sectoral lines to roll on the story.

For example, the non-tradable inflation is further decomposed in to the contribution of real marginal costs, (model based) inflation expectations, and the persistent part of inflation, which originates in the pricing behavior of the non-tradables producion firms. An example of such decomposition is in Figure 6.7.

Marginal cost, the main supply-side inflationary driver, is further decomposed into the contributions of the prices of production factors (like wages and fuel prices), and the total output. In the case of non-tradable sector, the output is linked to the demand side of the economy: relative price effects (i.e., how cheap the non-tradable good is relative to other consumption goods, and therefore how much people substitute away in favor of the non-tradable good), and the total consumption, which is determined by households' income and the real interest rate.

Similarly as the non-tradable inflation story, we reveal the story around the other main drivers of headline inflation. As you can see, the whole process is locked into the underlaying model logic, which imposes a strict discipline on the forecast interpretation.

6.5 Adding judgment

The forecasts presented to the Monetary Policy Committee and published in the *Monetary Policy Statement* are not viewed as model based forecasts, but as the Reserve Bank's forecasts. This reflects the addition of judgment to these forecasts, a consequence of the limitations of using model based forecasts. Models are simplifications of the real world, they are approximations of a much more complex data generating process. While we aim to build models that can explain and predict many of the features of the New Zealand economy, it is not possible for a model by itself to capture all information known by policy makers and sectoral analysts. For this reason we need to augment pure model based forecasts with sectoral analyst and policy maker judgement. The standard technique used by most model based forecasters is to add a sequence of shocks over the forecast horizon. In this section we consider and outline some of the different approaches that can be used for choosing and adding the sequence of shocks.

From equation 6.1 the h step ahead forecast from DSGE model in reduced form, can be written in the following way:

$$\hat{y}_{t+h} = F^h y_t + \sum_{j=1}^{h} F^{h-j} B \varepsilon_{t+j}$$

We want to change the forecast paths $\{\hat{y}_{t+j}\}_1^h$ by choosing a sequence of shocks $\{\varepsilon_{t+j}\}_1^h$. We consider two modes, for adding shocks over the forecast horizon, and two approaches for choosing the shocks. Shocks over the forecast horizon can either be expected, or unexpected. When the sequence of shocks is expected, in the first period after the end of history, agents are surprised by the entire sequence of shocks that are expected to hit the economy over the forecast period. At all future periods over the forecast horizon, agents have perfect foresight and can perfectly anticipate the shocks yet to hit the economy. Their responses and expectations of the future paths of variables are entirely consistent with them perfectly foreseeing the shocks that have yet to hit the economy. When the sequence of shocks is unexpected, agents are surprised in every period they are hit by the contemporaneous shocks. At any point over the forecasting horizon, they do not foresee the remaining shocks yet to hit the economy, hence their expectations for the future paths of variables is consistent with no further shocks hitting the economy. As a consequence of the different assumptions underlying these modes, given a shock of a particular size, the responses can be quite different under each of the setups.

We also consider two setups for choosing the sequence of shocks, we refer to these different methods as adding structural judgement, and adding reduced form judgement. Adding structural judgement involves 'hard-typing' the particular value of the shocks. For example the policymaker or the model operator may know the type and size of shock they expect to hit the economy over the forecast horizon. This requires knowledge of the particular type and size of the shock expected. Adding reduced form judgement involves choosing the path for one or more variables (these variables are hard-tuned) over the forecast horizon and then backing out the shocks that are consistent with these paths. When the number of shock types equals the number of variables being tuned, the exercise is a trivial one, because there is a unique mapping between the shocks and the judgement. When we choose from more types of shocks, than there are variables we are choosing, the judgement will not be unique, there will be infinitely many combinations of shocks that will be consistent with the tuned tracks. To get around this problem, we use the Waggoner-Zha algorithm (see Beneš et al. (2008) and Waggoner and Zha (1999)). The Waggoner-Zha algorithm is a least squares procedure for choosing the combination of shocks with the smallest variance. In probabilistic sense these shocks are going to be the most likely conditional on the model and the conditioning information.

Using reduced form judgement has the advantage, that the policymaker or model operator does not need to know the exact size of the shock and in some cases the types of shocks required.[2] It is more likely that a policymaker would approach the model operator with specific requests about the paths of particular variables, with some knowledge about the source of this judgement, but nothing concrete, while at the same time they may not hold any particular views on the paths of other variables. In this particular situation the Waggoner-Zha algorithm will tell the policymaker what the most likely path is (conditional on the model) for those variables that were not hard-tuned.

As part of the Waggoner-Zha algorithm we are able to generate a metric based on the sum of squared residuals that informs us of how much judgement has been added. This may prove useful during particular fore-

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[2] If a particular shock in the subset to be estimated using the Waggoner-Zha algorithm is not very important, the algorithm will only choose negligible values for this shock.

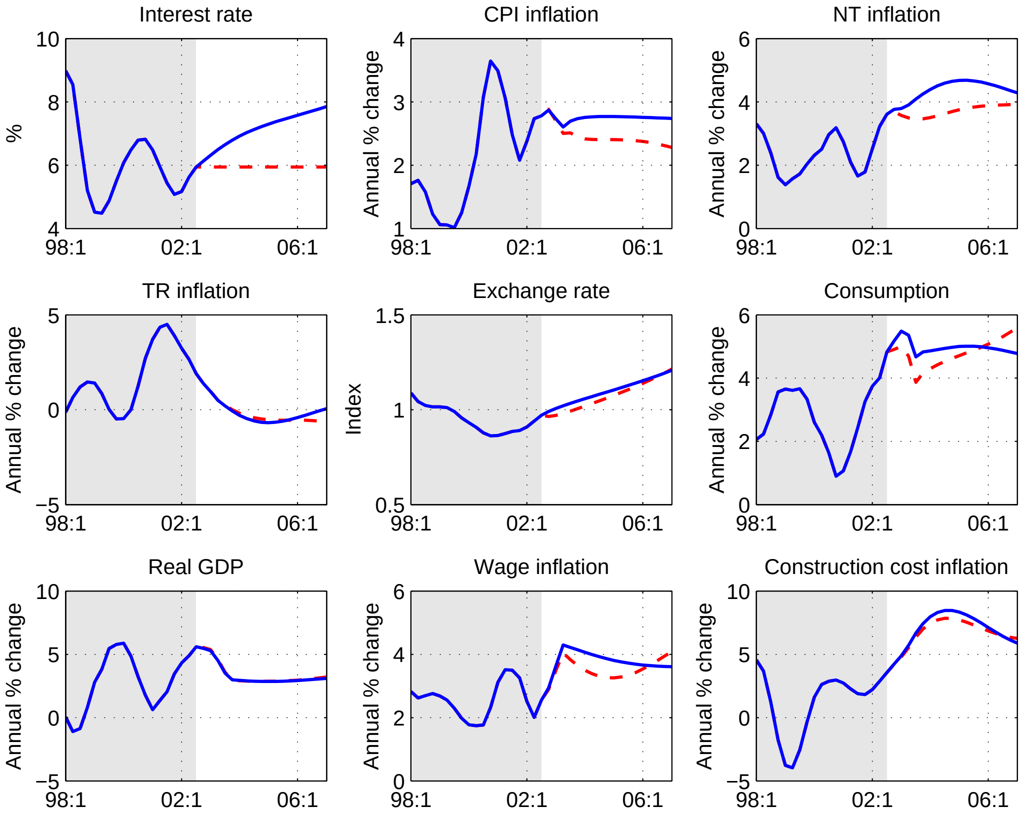
cast rounds where this number is particularly high. It could force policy makers and sectoral analysts to question the assumptions underlying their judgement, given the model is a reasonable representation of the historical data generating process.

We demonstrate the Waggoner-Zha algorithm with an example. We perform a forecast from the third quarter 2002 to the first quarter 2007. The particular reduced form judgement we want to incorporate is a flat interest rate track. We believe that this is due to four types of shocks; monetary policy shocks, consumption shocks, non-tradable cost push shocks and tradable costs push shocks. We also believe agents can perfectly foresee these shocks. So we set up the Waggoner-Zha algorithm to find the combination of these four shocks with the smallest variance.

The interest rate rule in KITT is a function of a smoothing term, a reaction term, and a monetary policy shock. Hard-tuning in a particular interest rate track using the monetary policy shock plus additional types of shocks allows the interest rate rule to respond semi-endogenously to inflation pressures in the model. Imposing a flat track, that is otherwise lower than the no judgement projection requires lower inflation over the forecast horizon, if the interest rate rule is to respond endogenously. To get lower inflation requires negative cost push shocks to both tradable and non-tradable inflation. The Waggoner-Zha algorithm chooses quite large cost shocks initially. These shocks decline into the forecast horizon because inflation expectations are falling which helps bring down inflation. However inflation expectations begin to increase toward the end of the forecast horizon as we near the end of the hard-tune. This is because agents are forward looking and see that inflation rises after the period of the hard-tune because there will be no further negative cost push shocks to hold inflation down. This requires larger cost push shocks at the end of the forecast horizon to work against the rising inflation expectations.

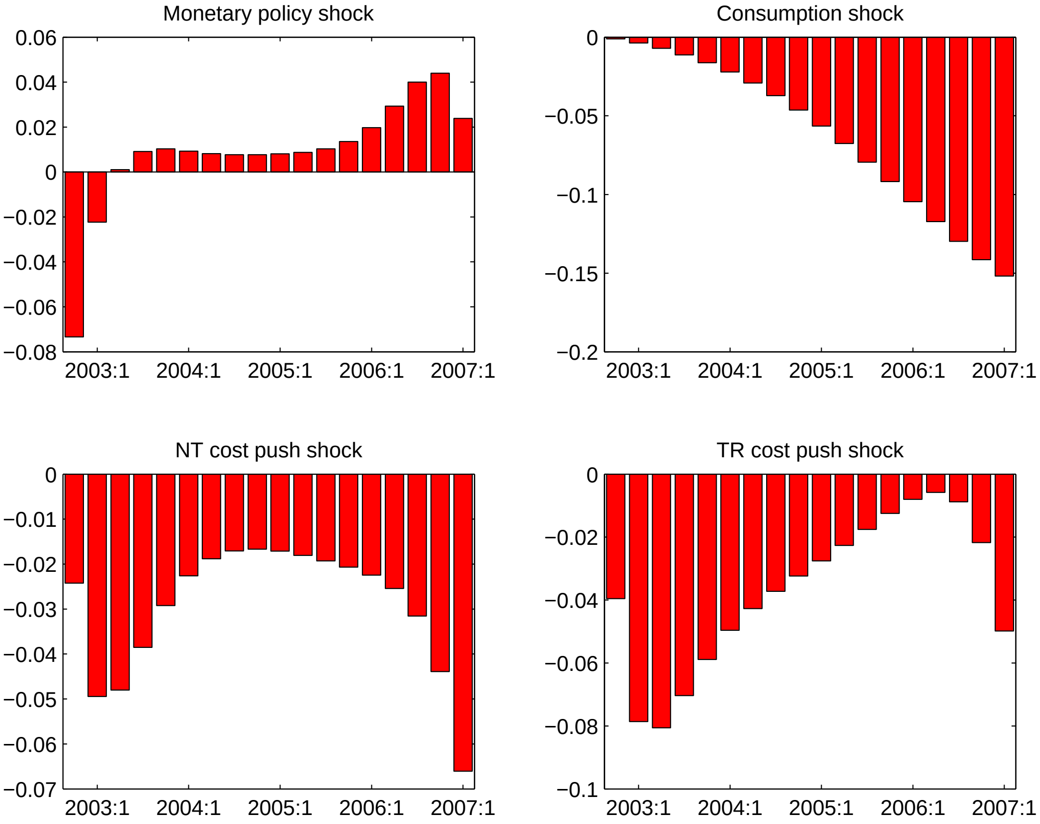
We also allow the Waggoner-Zha algorithm to choose consumption shocks. These shocks have to work indirectly through the reaction component of the interest rate rule. These shocks need to bring down inflation so that the monetary authority can respond by setting a lower interest rate. The

Figure 6.8: Judgmentally adjusted forecasts: A flat interest rate track



NB. The solid blue line represents the no judgement forecast, and the dashed red line represents the judgementally adjusted forecasts.

Figure 6.9: Judgementally adjusted forecasts: Shocks



Waggoner-Zha algorithm chooses negative consumption shocks that increase in absolute value over the forecast horizon. Negative consumption shocks bring down consumption which is required to lower inflation. The increasing nature of the shock profile is due to nature of the Euler equation and habit formation. Because the discount factor almost offsets the interest rate in the Euler equation,[3] the shock terms are not discounted (at least not by much). This means that an expected consumption shock that occurs 10 quarters into the future will have the same direct impact on the Euler equation as an expected consumption shock that occurs 2 quarters into the future.[4] In addition to this, the Euler equation encourages consumption smoothing, if households expect lower consumption in the future, they will start to cut consumption today. The anticipation of a sequence of negative consumption shocks hitting the economy causes households to cut their consumption immediately, the introduction of habit formation makes it easier for households to cut their consumption immediately. This is because as they progress through the forecast period their consumption reference point also falls amplifying the effect of the anticipated shocks. The combination of the consumption shock weighting, consumption smoothing and habit formation mean we get more bang for our buck by putting in larger consumption shocks further into the future.

Finally we allow for the Waggoner-Zha algorithm to choose monetary policy shocks. This is essentially the exogenous component of the interest rate response. The Waggoner-Zha algorithm chooses some large negative shocks in the first couple of quarters to hold the interest rate below where it would otherwise want to go (see the response of the no judgement interest rate path). However further into the future, positive interest rate shocks are added, peaking towards the end of the forecast horizon. This is because inflation expectations have fallen over this period, and the nominal interest rate is fixed at this flat level. From a Fisher equation perspective, we would need higher real interest rates when inflation expectations are falling to maintain a flat interest rate track. This is achieved by adding

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[3] They exactly offset each other on the balanced growth path.

[4] The indirect impact could be quite different.

positive interest rate shocks. Because the interest rate rule only looks at inflation expectations one step ahead (compared to the Phillips curves that look determine inflation as the infinite sum of the discounted future stream of real marginal cost gaps) the rising inflation expectations outside the forecast horizon do not impact the need for higher real interest rates toward the end of this period.

Chapter 7

Conclusion

There were several over-arching objectives that we aimed for with the development of the Reserve Bank of New Zealand's DSGE model. We feel that the model has been successful in achieving these initial objectives and that this book attests to that fact. However, there are weaknesses in the model. In our view, the failure to estimate of a model that encapsulates the full set of trends required in our multi-sector model is unfortunate and leaves room for future work. This would allow a richer understanding of the role of technology shocks and other shocks that affect the underlying trends in the economy in New Zealand's recent macroeconomic story. That said, there are large benefits from being pragmatic enough to pursue the implementation of the gaps model. We believe the model will prove very helpful in honing debate during forecasting and policy rounds.

This book also details a large number of tools (including forecast decompositions, density forecasts, methods to add judgment) to improve the payoff from using the DSGE model. Model development paid particular attention to the need for the model to be used in the forecasting and policy environment. Our desire is that the model and technology proves useful for forecasters. Moreover, we think there are many questions that should reinvigorate interaction between central bank forecasters and researchers and hope that the model can prove useful in this regard.

There also remain model development tasks, some of which are smaller, for example, increasing the persistence of imports through additional adjustment costs, while some are much larger.

In addition, we have used a particularly simple interest rate rule in the model. The rule appears to have a reasonable description of the Reserve Bank of New Zealand's behaviour but the rule is not based on any optimising behaviour or designed to address model or parameter uncertainty avenues. Not only has the model changed from FPS, New Zealand's inflation targeting remit has been modified at times since the inception of inflation targeting. Establishing a new policy rule appears a useful research task.

Also, the real-time forecast performance of the model needs to examined. This is a non-trivial task. On a quarterly basis, the log-linearisation of the steady-state takes computational time and the Bayesian estimation requires intensive cpu power to obtain a chain (or indeed, multiple chains) to confirm the convergence of the posterior mode. Further, the model should be re-estimated with the data available to the central bank at each quarter without assuming any knowledge of the future trends of these variables.

However, the primary focus of the model is to provide insights into the forecasting and policy environment. The model should provide rigour to the debate, challenge thinking, but not constrain. The model also provides empirical content and this should be used to discern the likelihood of competing economic hypotheses.

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Log-Linear model

.1 Log-linearisation

Households: shoppers

$$\begin{aligned}
\hat{\lambda}_t &= \hat{r}_t + \hat{\lambda}_{t+1} - \varepsilon_t^c && (1)\\
\hat{p}_t^n + \frac{\hat{c}_t^n}{1-\chi} + \hat{\lambda}_t &= \left(\frac{\chi}{1-\chi}\right)\hat{c}_{t-1}^n && (2)\\
\hat{p}_t^\tau + \frac{\hat{c}_t^\tau}{1-\chi} + \hat{\lambda}_t &= \left(\frac{\chi}{1-\chi}\right)\hat{c}_{t-1}^\tau && (3)\\
\hat{p}_t^f + \frac{\hat{c}_t^f}{1-\chi} + \hat{\lambda}_t + \phi_c\left(\hat{c}_t^f - \hat{c}_{t-1}^f\right) &= \left(\frac{\chi}{1-\chi}\right)\hat{c}_{t-1}^f + \\
&\quad \left(\frac{\beta\phi_c}{\bar{p}^f\bar{\lambda}}\right)\left(\hat{c}_{t+1}^f - \hat{c}_t^f\right) && (4)
\end{aligned}$$

Households: workers

$$\begin{aligned}
\epsilon\hat{l}_t &= \hat{\phi}_t^w + \hat{\lambda}_t && (5)\\
\pi_t^w - \pi_{t-1}^w &= \xi_w\left(\hat{\gamma}_t + \epsilon\hat{l}_t - \hat{w}_t\right) + \beta\left(\pi_{t+1}^w - \pi_t^w\right) && (6)
\end{aligned}$$

Households: landlords

$$\begin{aligned}
\hat{c}_t^s &= \hat{h}_{t-1} + \varepsilon_t^{cs} && (7)\\
\hat{\lambda}_t + \hat{\phi}_t^h + \delta_h\hat{c}_{t+1}^s &= (1-\delta_h)\left(\hat{\lambda}_{t+1} + \hat{\phi}_{t+1}^h + \varepsilon_t^{\phi h}\right) && (8)\\
\hat{h}_t &= \delta_h\gamma_h\hat{i}_t^h + (1-\delta_h)\,\hat{h}_{t-1} && (9)\\
\hat{\phi}_t^h + (\gamma_h - 1)\,\hat{i}_t^h - \hat{p}_t^c &= \iota_h\left(\hat{i}_t^h - \hat{i}_{t-1}^h - \varepsilon_t^{ih}\right) && (10)
\end{aligned}$$

Households: investors

$$
\begin{aligned}
\hat{\lambda}_t + \hat{\phi}_t^k &= \hat{\lambda}_{t+1} + \delta_k \hat{R}_{t+1} + (1-\delta_k)\,\hat{\phi}_{t+1}^k && (11)\\
\hat{k}_t &= \delta \hat{i}_t^k + (1-\delta_k)\,\hat{k}_{t-1} && (12)\\
\hat{\phi}_t^k - \hat{p}_t^\tau &= \iota_k \left(\hat{i}_t^k - \hat{i}_{t-1}^k - \varepsilon_t^{ik}\right) && (13)
\end{aligned}
$$

Supply of domestic factor services

$$
\begin{aligned}
\hat{z}_t &= \hat{a}_t + \gamma_{z1}\left(\tfrac{\bar{L}}{\bar{L}-L_0}\right)\hat{l}_t + \gamma_{z2}\hat{k}_{t-1} + \\
&\quad (1-\gamma_{z1}-\gamma_{z2})\,\hat{f}_t && (14)\\
\hat{p}_t^z + \hat{z}_t &= \tfrac{\bar{W}\bar{L}}{\gamma_{z1}\bar{P}^z\bar{Z}}\left(\hat{w}_t + \hat{l}_t\right) - \tfrac{\bar{W}L_0}{\gamma_{z1}\bar{P}^z\bar{Z}}\hat{w}_t && (15)\\
\hat{p}_t^z - \hat{z}_t &= \hat{r}_t - \hat{k}_{t-1} && (16)\\
\hat{p}_t^z + \hat{z}_t - \hat{p}_t^f - \hat{f}_t^z &= \phi_z\left(\hat{f}_t^z - \hat{f}_t - \hat{z}_t + \hat{z}_{t-1}\right) && (17)
\end{aligned}
$$

Non-tradables firms

$$
\begin{aligned}
\hat{y}_t^n &= \hat{a}_t^n + \gamma_n \hat{z}_t^n && (18)\\
\hat{\phi}_t^n + \hat{y}_t^n &= \hat{p}_t^z + \hat{z}_t^n && (19)\\
\pi_t^n - \pi_{t-1}^n &= \xi_n\left[\hat{p}_t^{z/n} + \left(\tfrac{1}{\gamma_n}-1\right)\hat{y}_t^n\right] + \beta\left(\pi_{t+1}^n - \pi_t^n\right) + \varepsilon_t^{pn} && (20)
\end{aligned}
$$

Construction firms

$$
\begin{aligned}
\hat{i}_t^h &= \hat{a}_t^c + \gamma_c \hat{z}_t^c && (21)\\
\hat{\phi}_t^c + \hat{i}_t^h &= \hat{p}_t^z + \hat{z}_t^c && (22)\\
\pi_t^c - \pi_{t-1}^c &= \xi_n\left[\hat{p}_t^{z/c} + \left(\tfrac{1}{\gamma_c}-1\right)\hat{y}_t^c\right] + \beta\left(\pi_{t+1}^c - \pi_t^c\right) + \varepsilon_t^{pn} && (23)
\end{aligned}
$$

Tradables firms

$$\begin{aligned}
\hat{y}_t^{\tau} &= \hat{a}_t^{\tau} + \gamma_{\tau 1}\hat{z}_t^{\tau} + \gamma_{\tau 2}\hat{m}_t^{q} + \left(1 - \gamma_{\tau 1} - \gamma_{\tau 2}\right)\hat{f}_t^{\tau} && (24)\\
\hat{\phi}_t^{\tau} + \hat{y}_t^{\tau} &= \hat{p}_t^{z} + \hat{z}_t^{\tau} && (25)\\
\hat{\phi}_t^{\tau} + \hat{y}_t^{\tau} &= \hat{p}_t^{q} + \hat{m}_t^{q} && (26)\\
\hat{\phi}_t^{\tau} + \hat{y}_t^{\tau} - \hat{p}_t^{f} - \hat{f}_t^{\tau} &= \phi_{\tau}\left(\hat{f}_t^{z} - \hat{f}_{t-1}^{z} - \hat{y}_t^{\tau} + \hat{y}_{t-1}^{\tau}\right) && (27)\\
\pi_t^{\tau} - \pi_{t-1}^{\tau} &= \xi_{\tau}\left[\gamma_{\tau 1}\hat{p}_t^{z/\tau} + \gamma_{\tau 2}\hat{r}_t^{z/\tau} + \left(1 - \gamma_{\tau 1} - \gamma_{\tau 2}\right)\hat{w}_t^{z/\tau}\right] + \\
&\quad \beta\left(\pi_{t+1}^{\tau} - \pi_t^{\tau}\right) + \varepsilon_t^{p\tau} && (28)
\end{aligned}$$

Manufactured exports

$$\begin{aligned}
\hat{x}_t &= \hat{a}_t + \gamma_v \hat{z}_t^{v} && (29)\\
\hat{\phi}_t^{v} + \hat{x}_t^{v} &= \hat{p}_t^{v} + \hat{z}_t^{v} && (30)\\
\pi_t^{v*} - \pi_{t-1}^{v*} &= \xi_v\left[\hat{p}_t^{z/v*} + \left(\tfrac{1}{\gamma_{v*}} - 1\right)\hat{x}_t^{v*}\right] + \\
&\quad \beta\left(\pi_{t+1}^{v} - \pi_t^{v}\right) + \varepsilon_t^{v*} && (31)\\
\hat{p}_t^{w*} + \hat{y}_t^{*} - \hat{p}_t^{v} - \hat{s}_t - \hat{x}_t^{v} &= \eta_v\left(\hat{x}_t^{v} - \hat{x}_{t-1}^{v} - \varepsilon_t^{xv}\right) && (32)
\end{aligned}$$

Commodity exports

$$\hat{y}_t^{*} - \hat{x}_t^{d} = \eta_d\left(\hat{x}_t^{d} - \hat{x}_{t-1}^{d} - \varepsilon_t^{xd}\right) \qquad (33)$$

International flows

$$\begin{aligned}
&\hat{b}_t = \bar{r}\left(\hat{r}_{t-1} + \hat{b}_{t-1} + \varepsilon_t^{b}\right) - \\
&\left[\left(\tfrac{\bar{P}^{v}\bar{X}^{v}}{\bar{B}}\right)\left(\hat{p}_t^{v} + \hat{x}_t^{v}\right) + \left(\tfrac{\bar{P}^{d}\bar{X}^{d}}{\bar{B}}\right)\left(\hat{p}_t^{d} + \hat{x}_t^{d}\right) - \left(\tfrac{\bar{P}^{q}\bar{M}^{q}}{\bar{B}}\right)\left(\hat{p}_t^{q} + \hat{m}_t^{q}\right) - \left(\tfrac{\bar{P}^{o}\bar{M}^{o}}{\bar{B}}\right)\left(\hat{p}_t^{o} + \hat{m}_t^{o}\right)\right]
\end{aligned} \qquad (34)$$

$$\begin{aligned} \hat{r}^h_t &= \hat{r}_t + \zeta\lambda\left(\hat{b}_t - \hat{\phi}^h_t - \hat{h}_t\right) && (35) \\ \hat{r}_t - \hat{r}^*_t + \hat{s}_{t+1} - \hat{s}_t &= \theta\left(\hat{r}_{t-1} - \hat{r}^*_{t-1}\right) + \varepsilon^s_t && (36) \end{aligned}$$

Central bank

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1-\rho_i)\left(\bar{\pi} + \kappa\left(\pi_{t+1} - \bar{\pi}\right)\right) \qquad (37)$$

$$\hat{p}_t = \left(1 - \nu_c - \nu_\tau - \nu_f\right)\hat{p}^n_t + \nu_c\hat{p}^c_t + \nu_\tau\hat{p}^\tau_t + \nu_f\hat{p}^f_t \qquad (38)$$

Government

$$\hat{g}_t = \rho_g\hat{g}_{t-1} + (1-\rho_g)\left(\hat{ngdp}_t - \hat{p}^n_t\right) \qquad (39)$$

Market clearing

$$\begin{aligned} \hat{z}_t &= \left(\frac{\bar{Z}^n}{\bar{Z}}\right)\hat{z}^n_t + \left(\frac{\bar{Z}^c}{\bar{Z}}\right)\hat{z}^c_t + \left(\frac{\bar{Z}^\tau}{\bar{Z}}\right)\hat{z}^\tau_t + \left(\frac{\bar{Z}^v}{\bar{Z}}\right)\hat{z}^v_t && (40) \\ \hat{y}^n_t &= \left(\frac{\bar{C}^n}{\bar{Y}^n}\right)\hat{c}^n_t + \left(\frac{\bar{G}}{\bar{Y}^n}\right)\hat{g}_t && (41) \\ \hat{y}^\tau &= \left(\frac{\bar{C}^\tau}{\bar{Y}^\tau}\right)\hat{c}^\tau_t + \left(\frac{\bar{I}^k}{\bar{Y}^\tau}\right)\hat{i}^k_t && (42) \\ \hat{m}^o_t &= \left(\frac{\bar{F}^\tau}{\bar{M}^o}\right)\hat{f}^\tau_t + \left(\frac{\bar{F}^z}{\bar{M}^o}\right)\hat{f}^z_t + \left(\frac{\bar{C}^f}{\bar{M}^o}\right)\hat{c}^f_t && (43) \end{aligned}$$

$$\begin{aligned} \hat{p}^{w*}_t &= \rho_{p^{w*}}\hat{p}^{w*}_{t-1} + \varepsilon^{p^{w*}}_t && (44) \\ \hat{p}^{o*}_t &= \rho_{p^{o*}}\hat{p}^{o*}_{t-1} + \varepsilon^{p^{o*}}_t && (45) \\ \hat{p}^{d*}_t &= \rho_{p^{d*}}\hat{p}^{d*}_{t-1} + \varepsilon^{p^{d*}}_t && (46) \end{aligned}$$