HOW FAIR IS THE DUCKWORTH/LEWIS ADJUSTMENT IN ONE DAY INTERNATIONAL CRICKET?

Brooker, Scott

Department of Economics and Finance
University of Canterbury
srbrooker@gmail.com
+64 21 104 3700

Hogan, Seamus

Department of Economics and Finance
University of Canterbury

Abstract

It is necessary to adjust the target score when a game of cricket is interrupted by bad weather. We estimate the probability of each team winning from any game situation and propose adjusting the target score in such a way that the probability of winning is preserved across the interruption. Our method is compared to the Duckworth/Lewis system currently used in international cricket to investigate the fairness of the adjustment in different game situations. We decompose the difference between the two methods into potential contributing factors; for example, the ground conditions present on the day of the match.

_____

1   Introduction

A full-length game of One Day International (ODI) cricket consists of each team sequentially batting for an innings of 300 balls, divided into 50 "overs" of six balls each. However, it is generally considered unreasonable by cricket officials to play during poor weather and/or ground surface conditions. Rain before or during the game can cause the length of the game to be reduced or even abandoned altogether. This causes no significant problem if there is simply a delay in getting the game underway; in this case the total number of overs is reduced equivalently for both teams. The situation where the match is interrupted in the middle is of far more concern. In this case we need a rule to determine whether and by how much the target for the team batting second should be adjusted due to the interruption. The rule also needs to determine the winner of the match if continued bad weather causes it to be abandoned.

International cricket has employed several target-adjustment mechanisms (known as "rain rules") over the history of ODI cricket and its more recent shorter format, Twenty20 cricket[1]. In this paper we outline a selection of these rain rules, including the incumbent "Duckworth/Lewis method". We propose and develop an alternative method and analyse how well each method predicts actual game outcomes.

---

[1] While a game of One Day International cricket takes around eight hours to complete, a game of Twenty20 cricket takes around three hours.

2 Review of selected target adjustment methods

2.1 Pre-Duckworth/Lewis Methods

The early methods of target adjustment in the event of rain were basic to say the least. The Average Run Rate method (ARR) simply asked the team batting second (Team 2) to score at a faster rate than the team batting first (Team 1) over the number of overs that each team had available. Let T_{ARR} be Team 2's revised target, S_1 be Team 1's score and O_1 and O_2 be the number of overs available to Team 1 and 2, respectively. The revised target score is as follows:

$$T_{ARR} = \frac{S_1}{O_1} \bullet O_2 + 1$$

The Most Productive Overs method (MPO) involved the overs faced by Team 1 being ordered from the smallest number of runs scored to the largest. The reduced number of overs available to Team 2 was accounted for simply by removing Team 1's least productive overs, in terms of run scoring, from the target. That is:

$$T_{MPO} = \sum_{m=1}^{O_2} P_m + 1$$

Where P_m is the runs scored from the mth highest-scoring over in Team 1's innings.

Both the ARR and the MPO methods have substantial limitations. Neither method takes into account the number of wickets lost by a team at the time of an interruption. Additionally, no attention is paid to the timing of an interruption in a team's innings. Losing their last ten overs unexpectedly is clearly going to negatively affect a team's scoring ability

far more than losing their first ten overs, where they could plan their entire innings around having 40 overs. In our view the largest drawback of the ARR method is that it is simply more difficult to score at a high run rate when you need to spread your ten wickets over fifty overs than if those wickets need only be spread over a reduced number of overs. Team 2, in a shortened match, can take a higher level of risk and expect to score at a faster rate, due to the reduced possibility of running out of batsmen before the end of their allocation of overs. Team 2 has a substantial advantage under the ARR method. The MPO method clearly advantages Team 1 as it only counts the overs of the first innings where they performed best.

2.2   The Duckworth/Lewis Method

Duckworth and Lewis (1998)[2] proposed a method for calculating an appropriate target for the team batting second (Team 2). Weather-affected ODI games have for more than a decade adopted the Duckworth/Lewis method. To outline the basic idea of their method, consider the most common situation where two teams play a full length game. Each team has 100% of the resources (50 overs, ten wickets) of a full length game available to them. Team 2 simply has to beat Team 1's score, without adjustment. Now consider a game where Team 1 has an uninterrupted 50 over innings, but at some point in Team 2's innings it rains and ten overs are lost. Team 2 now only bats for a total of 40 overs. This clearly hurts their chances of beating Team 1's score so an adjustment is needed. The Duckworth/Lewis system makes this adjustment based on the ratio of resources available to Team 2 to the resources available to Team 1. The resources lost depend on the number of overs and wickets remaining at the

---

[2] FC Duckworth and Tony Lewis (1998), "A fair method for resetting the target in interrupted One-Day cricket matches", *The Journal of the Operational Research Society*, Vol. 49, No. 3 (Mar. 1998), pp. 220-227.

time of the interruption as shown in Figure 2.1 below. Defining the resource percentages available to Team 1 and Team 2 as R_1 and R_2, respectively, the formula for the Duckworth/Lewis target for Team 2 is as follows:

$$T_{DL} = \frac{R_2}{R_1} \bullet S_1 + 1$$

Figure 2.1: Duckworth/Lewis Resource Percentages

2.3   Proposed Methods

There are several considerations when constructing a rain rule. Firstly, the criterion of fairness must be determined. Duckworth and Lewis use a "resources lost" approach, where teams are compensated for the lost time with an adjustment reflecting the percentage of the target that teams would need to score, on average, from that lost time resource. On the other hand, Preston and Thomas (2002)[3] outline an approach where the probability of winning is preserved across any rain interruption. They calculate these probabilities using a model of actual strategies, as described as Preston and Thomas (2000)[4]. Secondly, there is the choice of estimation method, of which two prominent ideas are direct estimation of scoring patterns from the data and indirect estimation via the calculation of transition probabilities in a dynamic programming framework. Thirdly, the variables to be included in the modelling process need to be determined.

Carter and Guthrie (2004)[5] build on the work of Preston and Thomas (2002), arguing that the Duckworth/Lewis method is unfair in some situations as it does not preserve the probability of winning of each team before and after the break. They discuss an example where Team 2 is playing very well and is obviously well ahead before an interruption and then later in the day further play is possible. In some situations the Duckworth/Lewis method determines that Team 2 is already ahead of their revised target score and therefore is declared the winner without any further play taking place. It must be the case, however, that Team 1

---

[3] Ian Preston and Jonathan Thomas (2002), "Rain rules for limited overs cricket and probabilities of victory", *The Statistician (2002), Part 2, pp. 189-202.*

[4] Ian Preston and Jonathan Thomas (2000), "Batting Strategy in Limited Overs Cricket", *Journal of the Royal Statistical Society, Series D (The Statistician), Vol. 49, No. 1 (2000), pp. 95-106.*

[5] Carter M and Guthrie G (2004). "Cricket interruptus: fairness and incentive in limited overs cricket matches", *The Journal of the Operational Research Society*, Vol. 55, No. 8 (Aug. 2004), pp. 822-829.

had some probability greater than zero of winning this game before the interruption but the adjustment hands them a loss with certainty. Carter and Guthrie use a dynamic programming approach to determining their probability of winning model, using the data from the 1999 World Cup in England.

We agree with the views of Carter and Guthrie on both the fairness of a probability-preserving adjustment rule and the dynamic programming approach. In our view, a team should have the same chance of winning after a rain interruption as they did before the rain. We also believe that a dynamic programming approach to the construction of the model is likely to result in a more accurate model, particularly in rare situations where there is little actual match data. Accordingly, we extend the work of Carter and Guthrie by creating a model of the second innings and include two significant additional variables; the run rate required and the ground conditions existing on the day of the match. Our model is created using a substantially larger data set than that used by Carter and Guthrie. Estimating the model using these additional variables and a larger data set is one of two main contributions of this paper. The second contribution involves the testing of different methods by creating artificial interruptions in completed matches and analysing the ability of each method to correctly predict the eventual outcome of the match.

3   The Dynamic Programme

Our ball-by-ball data consists of 311 One Day Internationals (ODIs) played between 2001 and 2008. Using this data we build a dynamic programme to estimate π, the probability of Team 2 winning, based on estimating four variables on any particular ball of the second

innings; r_m, the probability of scoring m runs from a fair delivery; γ, the probability of bowling a wide or no ball; ω_q, the probability of scoring q runs from a wide or no ball; and λ, the probability of being dismissed. In our view, there are four important, estimable factors which determine the values of these four variables. These are;

i, the ball of the innings that is about to be bowled   *i = [1,..,300]*

j, the number of wickets lost by the batting team   *j = [0,..,10]*

k, the run rate required from this point on to win the game   *k = [0,..,∞]*

c, a variable for the ground conditions on the day of the game   *c = [0,.., ∞]*

The ground conditions variable is defined as the average number of runs that the average batting team would score in those conditions against the average bowling team. As one of the goals of this analysis is to investigate the effect that ground conditions have on the nature of the adjusted target, we prepare two models with the slightly simpler model differing from the full model in that it does not include the conditions variable.

Below we present the Bellman equation for the full model. Note that we make the simplifying assumption that batsmen are not dismissed from a wide or no ball. In reality this can occur but extremely rarely.

$$\pi(i, j, k, c) = (1 - \gamma)((1 - \lambda) \bullet \sum_{m=0}^{6} r_m(\pi(i+1, j, ((300-i)k - m)/(300-(i+1)), c)$$

$$+ \lambda\pi(i+1, j+1, k, c)) + \gamma \bullet \sum_{q=1}^{7} \omega_q \pi(i, j, ((300-i)k - q)/(300-i), c)$$

We use regression models to estimate values for r, γ, λ and ω. The models for r and ω are ordered Probit models while the models for γ and λ are regular probit models. We create dummy variables for each level of the wicket variable j in order to account for any non-linear relationship in this variable. The conditions variable, c, is estimated by comparing the difference between the variance of first innings scores and the distribution of scores implied by a Probit model of the probability of Team 2 chasing each score. In this way we are able to use Bayes Theorem to estimate a distribution for c which is conditional on the first innings score and the result of the match. Our method for estimating the conditions distribution is fully described in Brooker and Hogan (2009)[6].

We investigate different models with different interactions of the four variables and select the most appropriate models using a combination of statistical significance of coefficients and cricket knowledge, ensuring that we are not left with models that do not make intuitive sense.

Figure 3.1 shows the probabilities of scoring each number of runs, r, under approximately average conditions of 250 and when an average run rate of five runs per over

[6]Brooker S and Hogan S (2009). "Inferring the contribution of ground conditions variability", *A paper especially prepared for the New Zealand Association of Economists Conference 2009.*

are required. The graph shows an increase in scoring rates throughout the innings, although there is a structural break indicating the end of the fielding restrictions overs[7]. When the field goes back there are far fewer dot balls and boundaries and more singles. Figure 3.2 shows the probability of losing a wicket under the same game circumstances. This probability decreases to begin with as the threat of the new ball is seen off and then rises as the batsmen become more aggressive. The shape of these curves are fairly typical results.
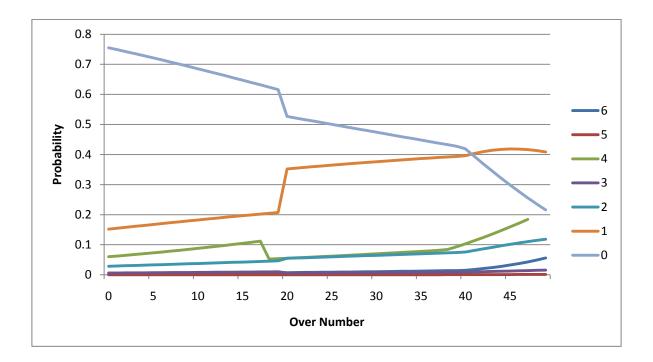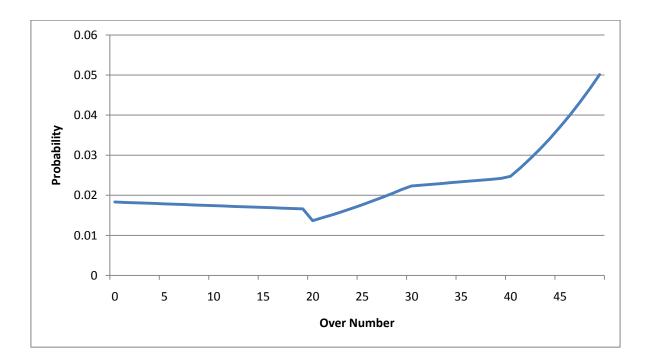
Figure 3.1: r distribution, *j=2, c=250, k=5/6*

---

[7] The games in our data set have been played under two different rules relating to the fielding restrictions. For illustrative purposes we show the more recent rules, under which fielding restrictions almost exclusively lasted for the first 20 overs.

Figure 3.2: λ, $j=2$, $c=250$, $k=5/6$

When calculating the probability of winning it is necessary to know the probability of scoring each number of runs from a given ball; however, from an illustrative point of view it is useful to take expectations to get a value for Expected Runs. We use this to show the effect of changing the values of some of the variables. Figure 3.3 shows a comparison between being two and three wickets down while Figure 3.4 shows a comparison between chasing three runs per over and chasing eight runs per over. Finally, Figure 3.5 shows a comparison between batting in conditions worth 200 and conditions worth 300 runs.

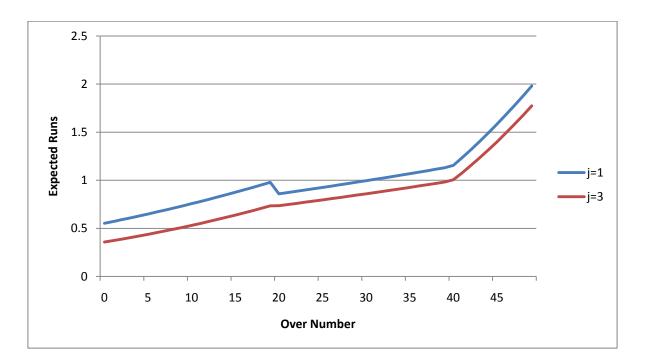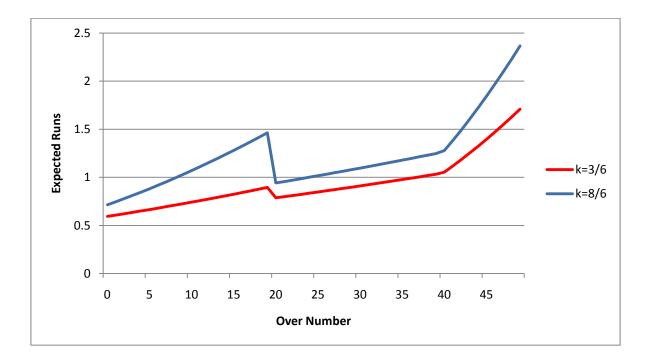**Figure 3.3: Expected Runs, *c=250, k=5/6***

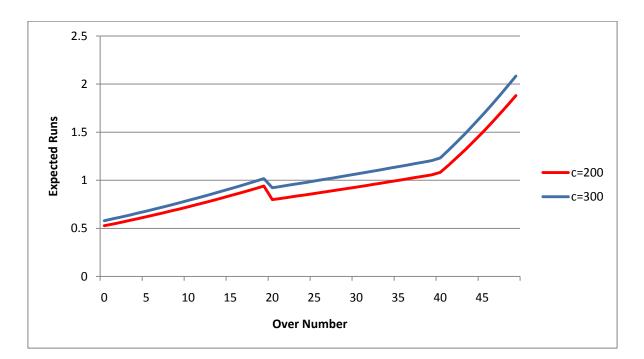**Figure 3.4: Expected Runs, *c=250, j=1***

Figure 3.5: Expected Runs, *k=5/6, j=1*
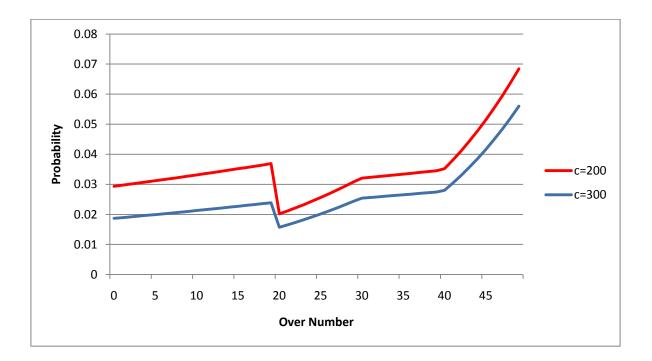

Figure 3.6: λ, *k=5/6, j=1*

Figure 3.6 shows the rather large difference in the probability of getting out when batting in conditions where the average score is expected to be 200 and conditions where the average score is expected to be 300.

After solving the dynamic programme by backward induction we are able to see the probability of winning for Team 2 in any situation that they might find themselves in. In Figure 3.7 we show the relationship between run rate required (k) and probability of winning after 20 overs of the innings, for a team that has lost one wicket and is playing in conditions worth 250 runs. Figure 3.8 shows the difference between the situation described above and the identical situation with three wickets lost, rather than one. Clearly the extra two wickets make a large difference in the probability of winning. Finally, Figure 5.9 shows how the situation in Figure 3.7 would change if conditions were worth 300 runs, rather than 250. As we would expect, Team 2 can now successfully chase higher targets.
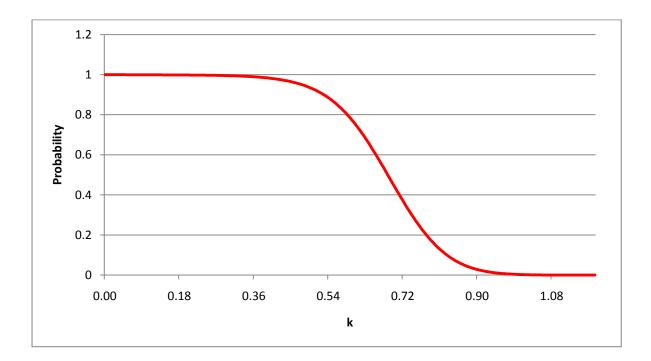
Figure 3.7: π, i=120, j=1, c=250

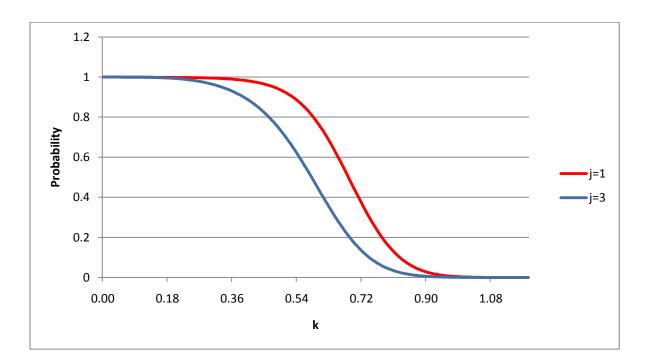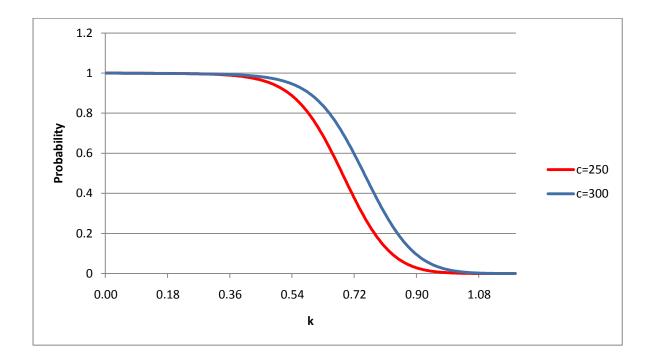**Figure 3.8: π, i=120, c=250**

Figure 3.9: π, i=120, j=1

4   The Adjustment Method

We operate our adjusted-target rule as follows:

- If the players leave the field, the probability of Team 2 winning is calculated.

- If the players do not return to the field, then Team 2 wins if their probability is greater than 0.5 and loses if their probability is less than 0.5. In the extremely unlikely situation where the probability of winning is exactly equal to 0.5, the game would be declared a tie.

- If the players return to the field, the new target is selected as the target that maintains Team 2's probability of winning at the level it was at before the interruption, rounded off to an integer number of runs.

5   Choosing the Best Method

In order to compare the target-adjustment methods, we create artificial interruptions in every game that we have in our data set (a total of 310 games). We create an interruption at the end of every over from the 20th over to the 49th over. We note that interruptions are possible at any stage of the match; however, for simplicity, we restrict our analysis to interruptions in Team 2's innings that occur at or after the minimum number of balls that allow for a winner to be declared, according to the rules of cricket.

When a game is interrupted by weather, the interruption may be temporary (allowing play later in the game) or permanent (causing the game to be abandoned). The most interesting aspect of any target-adjustment method is the target set in a resumed game;

however, abandoned games provide us with a convenient way of comparing methods. Using our artificial interruptions, we are able to look at which team would have been awarded the win had the game actually been abandoned at each point and compare this outcome with the true outcome of the game (whether Team 1 won, Team 2 won or the match was a tie). We are then able to calculate the percentage of matches in which each method corrected predicted the winner, for each possible interruption point.

5.1   The selected methods for comparison

- A simple coin toss (CT)

- Average Run Rate method (ARR)

- Duckworth/Lewis method (DL)

- Proposed Brooker/Hogan method, without the variable for conditions (BH)

- Proposed Brooker/Hogan method, with the variable for conditions (BH*)

Our model without the conditions variable is included for two main reasons. Firstly, we would like to assess the difference that including a measure of conditions makes to the accuracy of the method. Secondly, when considering the actual implementation of our model in a game of cricket, we note that our BH* model requires the value for conditions to be estimated for the game, adding a complication that may seem undesirable to those involved in the game.

We expect that the coin toss method, by definition, will score a correct prediction percentage of approximately 50% in all games that are yet to be decided. We include it to indicate the progress that we make by using any of the other methods, as well as to give a guide to the variability naturally existing in our assessment approach. Note that we consider games with known outcomes at the point of interruption to be a successful prediction for each of the methods. This is to avoid discontinuities as most games reach probabilities very close to zero or one just before the result is decided and including these extreme values before having them disappear from the series completely makes the interpretation of the graphed series more difficult.
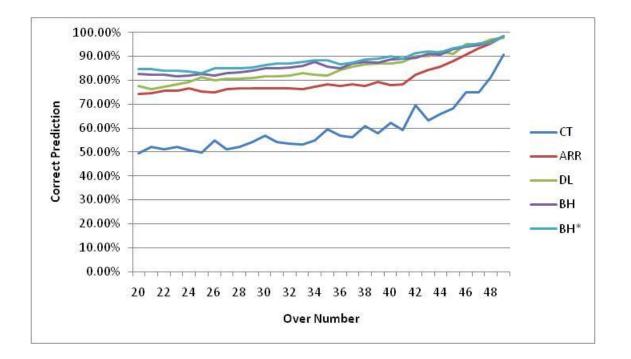
Figure 5.1: Correct Prediction Rates of the five methods

Figure 5.1 shows the correct prediction rates of each of the five methods. As expected the CT method is very poor, indicating that even the basic ARR method is a significant improvement over simply deciding the game by chance. The DL method is a significant

improvement over the ARR method throughout the majority of the innings, while the BH method performs better than the DL method, particularly in the earlier stages of the innings. Finally, the BH* method outperforms the standard BH method, indicating that a variable for conditions is a worthwhile addition in our quest to develop the best possible model.

6   A Case Study

In this section we look at, arguably, the most disastrous application of a rain rule in One Day International history. We investigate the impact that alternative methods may have had on this match.

The match in question is the 1992 World Cup semi final, played between England and South Africa in Sydney, Australia. The Most Productive Overs (MPO) method was the rain rule in place for this tournament. England batted first, scoring 252/6 from 45 overs before the remainder of their innings was abandoned due to time constraints[8]. South Africa's target score, at the beginning of their innings, was 253 runs from 45 overs as there was no provision in the rules to compensate England for unexpectedly losing five overs at the end of their innings. Both the Duckworth/Lewis (DL) and Brooker/Hogan (BH) methods would have required that South Africa's target be increased at the innings break; however, since this did not happen, we will assume the 253 target for the rest of this analysis as that is the number that South Africa believed they were chasing. This is equivalent to assuming that England knew their innings would be only 45 overs from the outset.

---

[8] Note that it was not raining at the time that England's innings was ended, they simply abandoned it because it took too long, something that would not occur under the current regulations.

South Africa reached 231/6 after 42.5 overs, requiring a further 22 runs from the remaining 13 balls; however, it rained at that moment and two overs were lost. Under the MPO rule, South Africa's target score was reduced by just a single run, meaning they had to chase an impossible 21 runs from one ball.

Had this game been played under the Average Run Rate (ARR) method, South Africa's target would have been set at 240; therefore, they would have needed nine runs from one ball, which is still virtually impossible[9]. Under the Duckworth/Lewis method, the target would have been 235, so a boundary from the last delivery would have won the game for South Africa.

The Brooker/Hogan method (without conditions since we cannot estimate what conditions were like on that day) suggests the following:

- Six wickets down and needing 22 from 13 balls, South Africa's probability of winning was 34.9%.

- With South Africa still six wickets down and with just a single ball remaining, a requirement of two further runs would have given South Africa a probability of winning closest to 34.9%.

This case study is an example of the substantial improvement that the Duckworth/Lewis rule has made to rain-interrupted games, while also outlining the impact of

---

[9] Any score is technically possible with enough wides or no balls but since he is unlikely to risk bowling these deliveries when more than six runs are required we assume required run rates in excess of six runs per over are impossible to chase.

a probability-maintaining adjustment mechanism. South Africa had some chance of winning before it rained and therefore we believe it is fair that they should have that same chance of winning if indeed a resumption of play is possible.

References

Brooker, S. and Hogan, S. (2009). "Inferring the contribution of ground conditions variability*", New Zealand Association of Economists Conference 2009 – Paper Submission,* http://www.nzae.org.nz/conferences/2009/pdfs/Inferring_the_contribution_of_ground_conditions_variability_27_May_09.pdf.

Carter, M. and Guthrie, G. (2004). "Cricket interruptus: fairness and incentive in limited overs cricket matches", *The Journal of the Operational Research Society*, Vol. 55, No. 8 (Aug. 2004), pp. 822-829.

Duckworth, FC. and Lewis, Tony (1998), "A fair method for resetting the target in interrupted One-Day cricket matches", *The Journal of the Operational Research Society*, Vol. 49, No. 3 (Mar. 1998), pp. 220-227.

Preston, Ian and Thomas, Jonathan (2000), "Batting Strategy in Limited Overs Cricket", *Journal of the Royal Statistical Society, Series D (The Statistician), Vol. 49, No. 1 (2000), pp. 95-106.*

Preston, Ian and Thomas, Jonathan (2002), "Rain rules for limited overs cricket and probabilities of victory", *The Statistician (2002), Part 2, pp. 189-202.*